

Current Mode Fractional Order Filters Using VDTAs with Grounded Capacitors

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Abstract—In this work, the design of current mode Fractional order filter using VDTAs (Voltage differencing trans-conductance amplifier) as an active element with grounded capacitors has been proposed. The approximate transfer functions of low and high pass filters of fractional order on the basis of the integer order transfer has been shown and the form of those functions of filters is also implemented using VDTA as an active building block. In this work, filters of the different sequence have been realized. The frequency domain simulation results of the proposed filters are obtained on Matlab and PSPICE with TSMC CMOS 180 nm technology parameters. Stability and sensitivity is also verified

Keywords—VDTAs, fractional order and filter

I. INTRODUCTION

THE submitted Fractional calculus in mathematics deals with the integration and differentiation of non integer order functions. Since last decade the fractional order calculus is widely used for the mathematical modelling of the many engineering branches. As a result, a number of researchers are focusing on the fractional calculus and its application in various fields such as designing of control systems, electrical and electronics circuits etc. Infinite memory is the characteristics of fractional sequence and finite memory is the main characteristics of integer sequence system. Due to this reason, the major focus of researchers is on fractional order calculus to model and resolve the new design problems in last few years [1-4].

The time domain representation of voltage for the fractance device (capacitor) can be expressed by fractional order integration, as:

$$v_c^\alpha(t) = \frac{1}{CT(\alpha)} \int_0^t \frac{i(\tau)}{(t-\tau)^{1-\alpha}} d\tau \quad (1)$$

Where 'a' the starting time, 'C' is the capacitance and i(t) is the current through the device. Fractance device impedance is shown by equation (2).

$$Z(s) = K_0 S^\alpha = K_0 (j\omega)^\alpha \quad (2)$$

Where α is the order of fraction and the constant is represented by K_0 . Then, the polar form of Z becomes:

$$|z| = K_0 \omega^\alpha \quad \angle Z = \alpha \frac{\pi}{2} \quad (3)$$

From (2), it may be found that for different values of α , impedance Z will show different element characteristics like for $\alpha = 1$ inductor, for $\alpha = 0$ resistor and -1 capacitor. Therefore the

detail of the passive circuit that offers a regular section attitude with frequency can be referred to as a Fractional Order Element (FOE) which is the generalized detail of the already present electrical circuit factors [5].

In fractional calculus, fractional order elements (FOEs) are the main active blocks to process the signal in signal processing. Due to unavailability of these elements, approximate arrangement of RC networks is used in place of fractional order elements. Following this approach various active building blocks like Op-Amps, Current Conveyor-I (CCI), Current Conveyor-II (CCII), operational trans-conductance amplifier etc (OTA) etc. are used [6-12], current mode fractional order band pass and band stop filters using voltage differencing trans-conductance amplifier (VDTA) is given in [13].

In this paper, realization, performance and design of the proposed filters of various orders using VDTAs as an active building block has been describe. Grounded capacitors are used to realize the filter circuits because it is easy to fabricate the grounded capacitors [14]. VDTA have some advantages like works in both voltage mode and current mode, works at high frequency operations and free from parasitic capacitance.

The proposed design has features like it can be used to realize higher order circuits, having less passive components and uses of fractional capacitor will improve the pliability.

This paper is arranged in the sequence as: in section 2 designed configuration and simulation results of low pass and high pass filters of non integer order $(1+\alpha)$ for distinct values of order α using MATLAB is presented. In section 3 stability of the proposed design is discussed. The circuit design using pspice and its simulation results for the verification of proposed design have been shown in section 4. The sensitivity of the design is analyzed in section 5 and at the end, summary of this proposed work with some important points are discussed in conclusion.

II. PROPOSED CONFIGURATION

In this section the design equations of non integer order high pass and low pass filters has been given. The performance parameters are calculated using Matlab and its plots are also shown. Analysis shown in[15], provide that if the $(x+\alpha)<2$ condition satisfies then the straight implementation of the filter of non integer order, $x+\alpha$ will be stable, where α is a fraction number and $0<\alpha<1$ and 'x' is an integer number.

A. Low pass filter

Equation (5) represents the transfer function of non integer low pass filters of order $1+\alpha$.

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$$T_{\text{FLPF}} = \frac{K_1}{(\tau s)^{1+\alpha} + K_3(\tau s)^\alpha + K_2} \quad (4)$$

$$|A(j\omega)| = \frac{K_1}{\sqrt{\left(\frac{\omega}{\omega_0}\right)^{2(1+\alpha)} - 2K_2\left(\frac{\omega}{\omega_0}\right)^{(1+\alpha)} \sin\left(\frac{\alpha\pi}{2}\right) + K_3^2\left(\frac{\omega}{\omega_0}\right)^{2\alpha} + 2K_2K_3\left(\frac{\omega}{\omega_0}\right)^\alpha \cos\left(\frac{\alpha\pi}{2}\right) - K_2^2}} \quad (5)$$

$$\angle A(j\omega) = \angle K_1 - \tan^{-1}\left(\frac{\left(\frac{\omega}{\omega_0}\right)^{1+\alpha} \cos\left(\frac{\alpha\pi}{2}\right) + K_3\left(\frac{\omega}{\omega_0}\right)^\alpha \sin\left(\frac{\alpha\pi}{2}\right)}{-\left(\frac{\omega}{\omega_0}\right)^{1+\alpha} \sin\left(\frac{\alpha\pi}{2}\right) + K_3\left(\frac{\omega}{\omega_0}\right)^\alpha \cos\left(\frac{\alpha\pi}{2}\right) + K_2}\right) \quad (6)$$

$$\left(\frac{\omega_c}{\omega_0}\right)^{2(1+\alpha)} - 2K_2\left(\frac{\omega_c}{\omega_0}\right)^{(1+\alpha)} \sin\left(\frac{\alpha\pi}{2}\right) + K_3^2\left(\frac{\omega_c}{\omega_0}\right)^{2\alpha} + 2K_2K_3\left(\frac{\omega_c}{\omega_0}\right)^\alpha \cos\left(\frac{\alpha\pi}{2}\right) - K_2^2 = 0 \quad (7)$$

B. High Pass Filter

Equation (8) represents the transfer function of non integer high pass filters of order $1+\alpha$.

$$T_{\text{FHPF}} = \frac{K_1 s^{1+\alpha}}{(\tau s)^{1+\alpha} + K_3(\tau s)^\alpha + K_2} \quad (8)$$

$$|A(j\omega)| = \frac{K_1 \left(\frac{\omega}{\omega_0}\right)^{1+\alpha}}{\sqrt{\left(\frac{\omega}{\omega_0}\right)^{2(1+\alpha)} - 2K_2\left(\frac{\omega}{\omega_0}\right)^{(1+\alpha)} \sin\left(\frac{\alpha\pi}{2}\right) + K_3^2\left(\frac{\omega}{\omega_0}\right)^{2\alpha} + 2K_2K_3\left(\frac{\omega}{\omega_0}\right)^\alpha \cos\left(\frac{\alpha\pi}{2}\right) - K_2^2}} \quad (9)$$

$$\angle A(j\omega) = \angle K_1 + \frac{(1+\alpha)\pi}{2} - \tan^{-1}\left(\frac{\left(\frac{\omega}{\omega_0}\right)^{1+\alpha} \cos\left(\frac{\alpha\pi}{2}\right) + K_3\left(\frac{\omega}{\omega_0}\right)^\alpha \sin\left(\frac{\alpha\pi}{2}\right)}{-\left(\frac{\omega}{\omega_0}\right)^{1+\alpha} \sin\left(\frac{\alpha\pi}{2}\right) + K_3\left(\frac{\omega}{\omega_0}\right)^\alpha \cos\left(\frac{\alpha\pi}{2}\right) + K_2}\right) \quad (10)$$

$$\left(\frac{\omega_c}{\omega_0}\right)^{2(1+\alpha)} + 2K_2\left(\frac{\omega_c}{\omega_0}\right)^{(1+\alpha)} \sin\left(\frac{\alpha\pi}{2}\right) - K_3^2\left(\frac{\omega_c}{\omega_0}\right)^{2\alpha} - 2K_2K_3\left(\frac{\omega_c}{\omega_0}\right)^\alpha \cos\left(\frac{\alpha\pi}{2}\right) - K_2^2 = 0 \quad (11)$$

For the transfer function of the fractional order low pass and high pass filter given in equations (4) and (8) respectively. ‘ τ ’ is the time constant and the factor K_i is included in the equation to lessen the errors in gain Vs frequency. The expression of the magnitude and the phase of the non integer order high and low pass filter is given in the equation (5) (6) (9) and (10) respectively [16].

The -3 dB frequency ω_c (also called 3-dB frequency or half power point) is the important parameters to design a filter. The -3 dB frequency for the high and low pass fractional order of $(1+\alpha)$ can be calculated by using equation (11) and (7) respectively. Fig (1) Shows the MATLAB plot of variations of the cut off frequency of non integer high and low pass filters for distinct values of α .

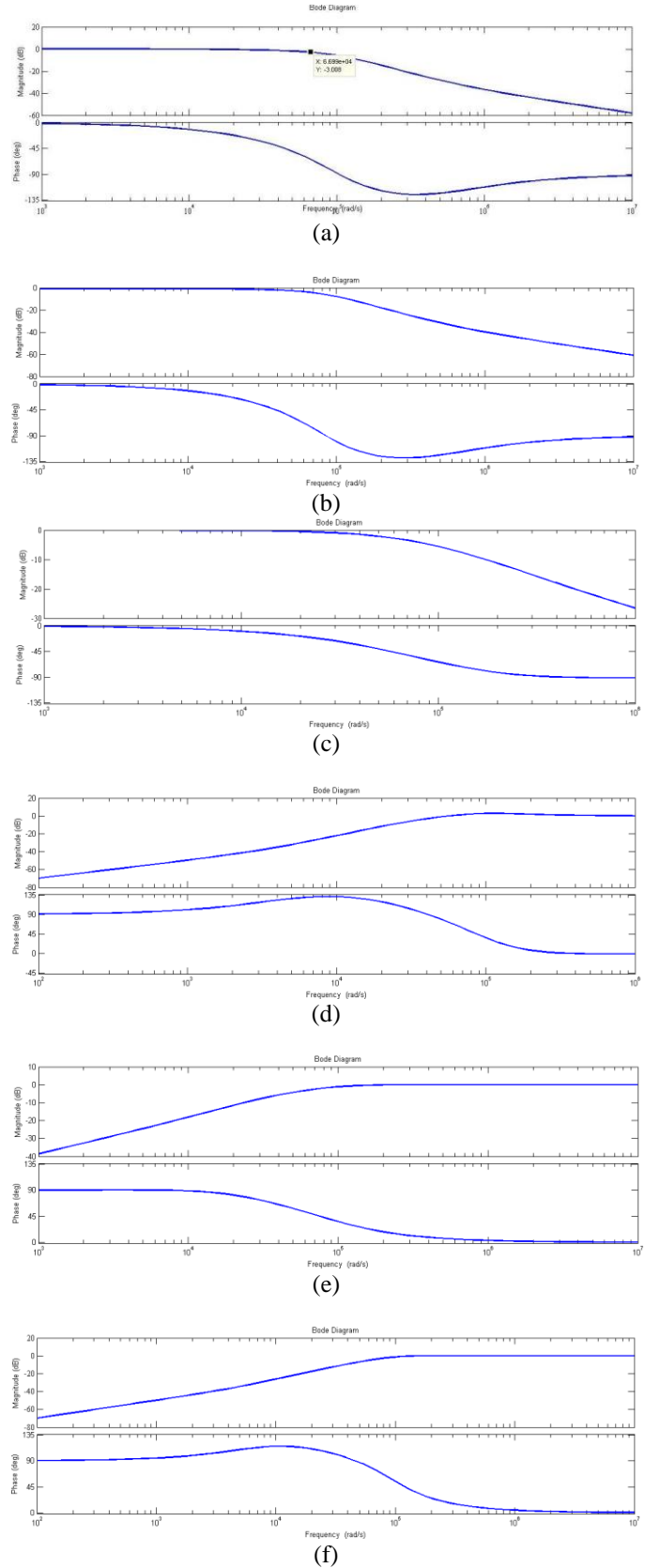


Fig.1. Gain Vs frequency curve of the (a) low pass filter of 1.1 orders (b) low pass filter of 1.5 orders (c) low pass filter of 1.8 orders (d) high pass filter of 1.1 orders (e) high pass filter of 1.5 orders (f) high pass filter of 1.8 orders

TABLE I
MATLAB SIMULATION RESULTS OF NON INTEGER ORDER HIGH AND LOW PASS
FILTER OF $(1+\alpha)$ ORDER FOR DIFFERENT VALUES OF α

Order of the Filter		Value of α	-3 dB(rad/sec)	Slope(dB/dec)
1.1	LPF	0.1	6.31×10^4	-20.95
1.5	LPF	0.5	6.73×10^4	-30.8
1.8	LPF	0.8	5.37×10^4	-31.81
1.1	HPF	0.1	6.26×10^4	-20.6
1.5	HPF	0.5	4.12×10^4	-27.6
1.8	HPF	0.8	6.86×10^4	-24.3

Matlab simulation results of non integer order high and low pass filter of order $(1+\alpha)$ for different values has been give in table I. In which order, value of α , cut off frequency and slope are mentioned.

III. STABILITY

This method is applicable only for rational powers. Let $\alpha = \frac{K}{m}$, where m and K are positive integers. The stability of the non integer order filters can be analyzed by transforming the S – plane into W planes. So the s plane is mapped into the W – plane. The $\pm j\omega$ axis is mapped on the line $|\theta_\omega| = \frac{\pi}{2m}$ and the negative axis is mapped onto the line $\theta_\omega = \frac{\pi}{m}$. The region $|\theta_\omega| = \frac{\pi}{m}$ is not physical and to become a stable system all roots of the system in the W plane must lie in $|\theta_\omega| = \frac{\pi}{2m}$ as shown in Fig. 2. To oscillate the system, at least one root of the system lies on the $|\theta_\omega| = \frac{\pi}{2m}$ and remaining roots will be in stable region. For the single fractal element in the W plane, if $K < m$ i.e. ($\alpha < 1$), the roots will not lie in the physical S- plane and the system is stable[17-19].

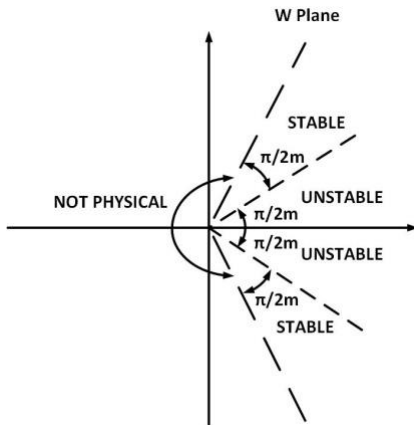


Fig 2. Region of stability in W plane

The above method is applied to the characteristics equation of equation (4) and (8) then the roots of the characteristic equation of the transformed function and minimum root angle $|\theta_\omega|$ of fractional order high and low pass filter has been calculated which is shown in the Table II.

TABLE II
MINIMUM ROOT ANGLE FOR THE $(1+\alpha)$ ORDER FOR DIFFERENT VALUES OF α

Fractional term (α)	Minimum root angle $ \theta_\omega $	Stability
0.1	9°	Stable
0.5	45°	Stable
0.8	18°	Stable

IV. DESIGN AND SIMULATIONS OF PROPOSED CIRCUIT

The proposed work, design of fractional order filters using VDTAs has been shown in this part.

A. Circuit representation

The symbolic representation of the circuit of active cell, VDTA [20] is shown in Fig. 3. It has five terminals, out of five, P, N are input and Z, X+ and X- are output terminals respectively. All I/O terminals are at high impedance level. The mathematical relation between input and output terminals of an ideal VDTA is represented by hybrid matrix given as:

$$\begin{bmatrix} I_z \\ I_{X+} \\ I_{X-} \end{bmatrix} = \begin{bmatrix} g_{m1} & -g_{m1} & 0 \\ 0 & 0 & g_{m2} \\ 0 & 0 & -g_{m2} \end{bmatrix} \begin{bmatrix} V_P \\ V_N \\ V_Z \end{bmatrix} \quad (12)$$

As discuss above, Fig. 3 shows the symbolic representation of circuit of VDTA. Where all input output terminals have been shown. Fig. 4 shows the CMOS implementation of VDTA.

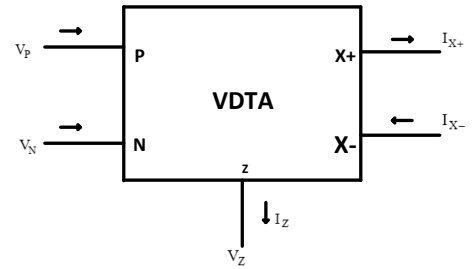


Fig. 3. Symbolic representation of VDTA

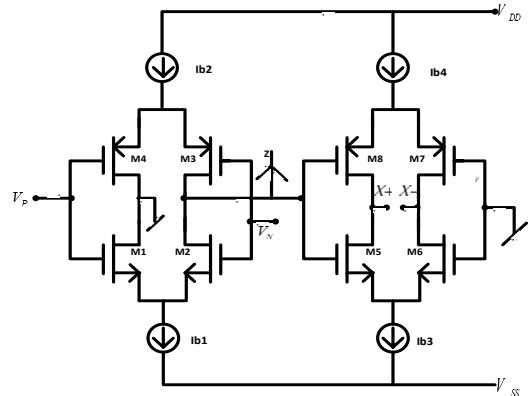


Fig. 4. CMOS representation of VDTA

g_{m1} and g_{m2} , the trans-conductance of the CMOS circuit of VDTA shown in Fig. 4 are given as:

$$g_{m1} = \frac{(g_3 + g_4)}{2} \quad (13)$$

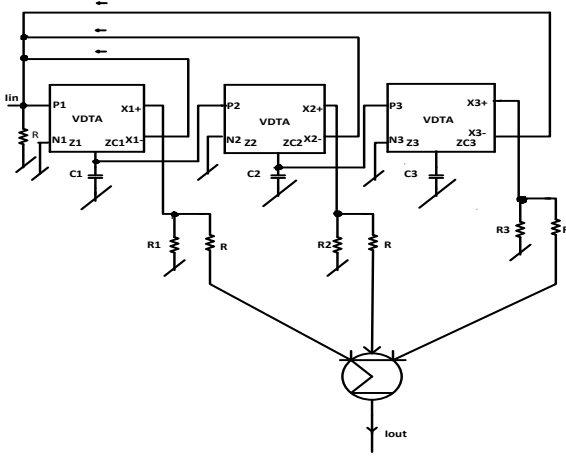
$$g_{m1} = \frac{(g_5 + g_8)}{2} \quad \text{or} \quad g_{m2} = \frac{(g_6 + g_7)}{2} \quad (14)$$

The trans-conductance of nth CMOS is given as:

$$g_{mn} = \sqrt{I_{Bn} \mu_n C_{ox} \left(\frac{w}{l}\right)_n} \quad (15)$$

Where the μ_n denotes the mobility of the carriers, C_{ox} denotes the gate oxide capacitance / unit area; l is length and w is width of the channel.

VDTA found application in analog signal generation and signal processing circuits [21-24] but we didn't find its application in fractional order signal processing and signal generation application. So this work has tried to fill this



vacancy.

Fig. 5. Proposed design of non integer order Low Pass filter using VDTAs

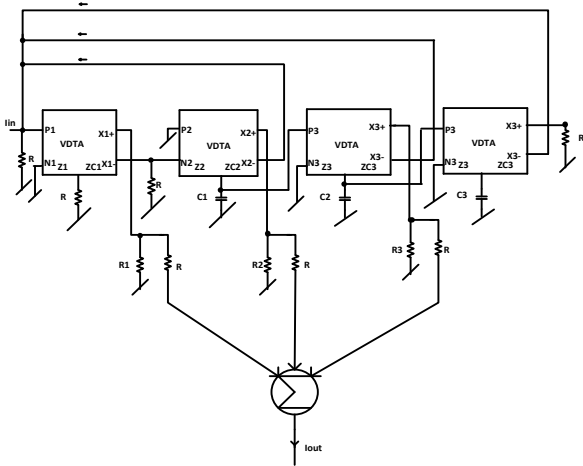


Fig. 6. Proposed design of non integer order High Pass filter using VDTAs

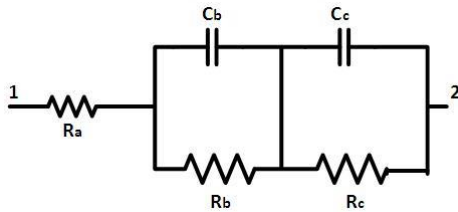


Fig. 7 approximation of fractal capacitor

An approximate transfer function of the non integer order high and low pass filters in equations (17) and (16) respectively have been realized by using the following leader feedback topology as given below [15]:

$$T_{FLPF}(s) = \frac{A_1 s^2 + A_2 s + A_3}{s^3 + \frac{1}{\tau_1} s^2 + \frac{1}{\tau_1 \tau_2} s + \frac{1}{\tau_1 \tau_2 \tau_3}} \quad (16)$$

$$T_{FHPF}(s) = \frac{A_1 s^3 + A_2 s^2 + A_3 s}{s^3 + \frac{1}{\tau_1} s^2 + \frac{1}{\tau_1 \tau_2} s + \frac{1}{\tau_1 \tau_2 \tau_3}} \quad (17)$$

Where, time constant and the gain factor are denoted by τ and A respectively.

In Fig. 5 and 6, the proposed design of non integer order low and high pass filter using VDTA respectively has been shown. The calculated transfer function for proposed design is given in equation (18) and (19) respectively.

$$\frac{I_{out}}{I_{in}} = \frac{S^2 g_m g_m R \left(\frac{R_1}{R_1+R}\right) + S g_m g_m g_m R \left(\frac{R_2}{R_2+R}\right) + \frac{g_m g_m g_m R}{C_1 C_2 C_3} \left(\frac{R_3}{R_3+R}\right)}{S^3 + S^2 \frac{g_m R}{C_1} + S \frac{g_m g_m R}{C_1 C_2} + \frac{g_m g_m g_m R}{C_1 C_2 C_3}} \quad (18)$$

$$\frac{I_{out}}{I_{in}} = \frac{S^3 g_m R \left(\frac{R_1}{R_1+R}\right) + S^2 \frac{g_m g_m R}{C_1} \left(\frac{R_2}{R_2+R}\right) + S \frac{g_m g_m g_m R}{C_1 C_2} \left(\frac{R_3}{R_3+R}\right)}{S^3 + S^2 \frac{g_m g_m R}{C_1} + S \frac{g_m g_m g_m R}{C_1 C_2} + \frac{g_m g_m g_m R}{C_1 C_2 C_3}} \quad (19)$$

In the designed filter circuit the time constant as well as the gain is given by $\tau_i = C_i / g_m$ and $A_i = R_i / (R_i + R)$ respectively. By using the formula and expression given in (16) and (17), the non integer fractional order low and high pass filter is designed. In the proposed work three grounded capacitor are used. The trans-conductance of each VDTA block is considered in the deriving the expression of both the filters.

The proposed design is compared with existing work [27] which comprises the fractional capacitors that are not grounded. While the process of fabrication for the grounded capacitor is easier when compared with non grounded capacitors. This is the advantage of our proposed design and the simulation results shows better performance compared with existing work at high frequency.

B. Simulation results

The presented work has been simulated by using PSPICE with TSMC CMOS 180 nm technology parameters. In Table III the w/l ratio of the transistors is mentioned. In the used active block, the DC voltages, $V_{DD} = -V_{SS} = 0.9V$ has been used [19] and the biasing currents $I_{B1} = I_{B2} = I_{B3} = I_{B4} = 150 \mu A$ are used. According to these supply voltage and biasing current values, the simulation results of VDTA shows that the value of trans-conductance is $g_{m1} = g_{m2} = 636.6 \mu A/V$. Fig. 7 shows the transfer characteristics graph drawn between I_{X-} and I_{X+} Vs V_s .

Table III
TRANSISTOR ASPECT RATIO FOR VDTA

Transistors	Width(μm)	Length(μm)
M1	3.6	0.36
M2	3.6	0.36
M3	16.64	0.36
M4	16.64	0.36
M5	3.6	0.36
M6	3.6	0.36
M7	16.64	0.36
M8	16.64	0.36

Table IV
VALUES OF THE PASSIVE COMPONENT USED IN PROPOSED DESIGN

Element	L P F			H P F		
Order	1.1	1.5	1.8	1.1	1.5	1.8
R (k Ω)	1.57	1.57	1.57	1.57	1.57	1.57
R1 (k Ω)	0.321	0.118	0.85	155.67	14.43	155.67
R2 (k Ω)	5.06	2.45	1.42	6.20	0.2	0.2
R3 (k Ω)	327	461.76	14.27	0.320	2.56	1.284
C1 (nF)	2.32	3.55	3.87	2.326	3.55	3.87
C2 (nF)	9.72	8.79	9.05	9.72	8.79	9.05
C3 (nF)	45.67	33.37	26.73	45.67	33.37	26.73

Table IV
VALUES OF PASSIVE COMPONENTS TO REALIZE NON INTEGER CAPACITOR

Non integer capacitor	Non integer order ($1+\alpha$)	Value of resistors and capacitors				
		Ra(K Ω)	Rb(K Ω)	Rc(K Ω)	Cb(nF)	Cc(nF)
2.32x10 ⁻⁹	1.1	319.60x10 ³	131.01x10 ³	136.36x10 ³	2.58	29.25
9.72x10 ⁻⁹	1.1	76.16x10 ³	31.38x10 ³	58.8x10 ³	10.79	67.77
45.67x10 ⁻⁹	1.1	16.20x10 ³	6.70x10 ³	7.04x10 ³	50.71	563.98
3.55x10 ⁻⁹	1.5	56x10 ³	32.15x10 ³	1050x10 ³	2.21	6.78
8.79x10 ⁻⁹	1.5	23.4x10 ³	128.9x10 ³	388x10 ³	5.49	17.09
33.37x10 ⁻⁹	1.5	5.99x10 ³	34.2x10 ³	109.88x10 ³	20.85	63.71
3.87x10 ⁻⁹	1.8	3.69x10 ³	591.1x10 ³	1217x10 ³	2.94	3.306
9.05x10 ⁻⁹	1.8	15.78x10 ³	264.3x10 ³	492.9x10 ³	7.90	6.78
26.73x10 ⁻⁹	1.8	5.34x10 ³	89.78x10 ³	166.9x10 ³	23.38	19.94

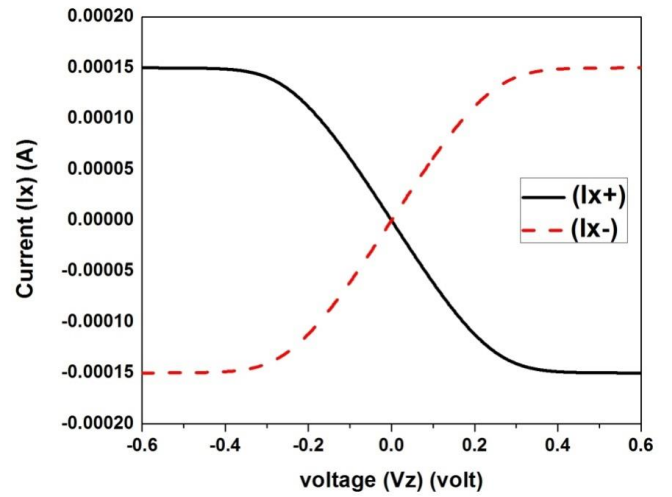


Fig. 8.. DC transfer characteristics of VDTA

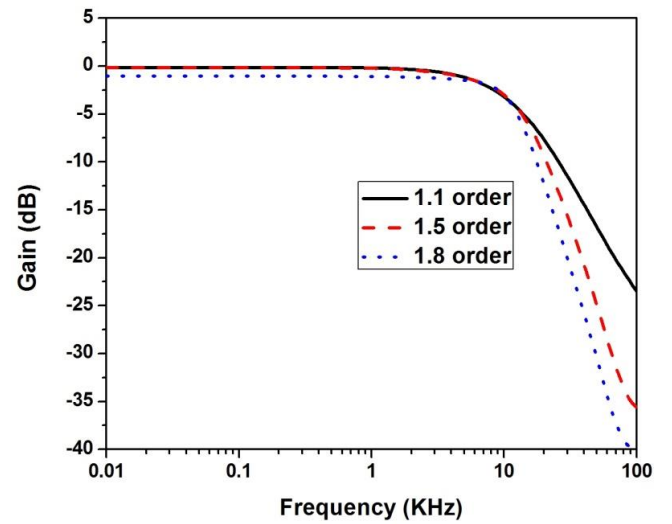


Fig. 9. Frequency Vs Gain plot of the low pass filter of three non integer orders

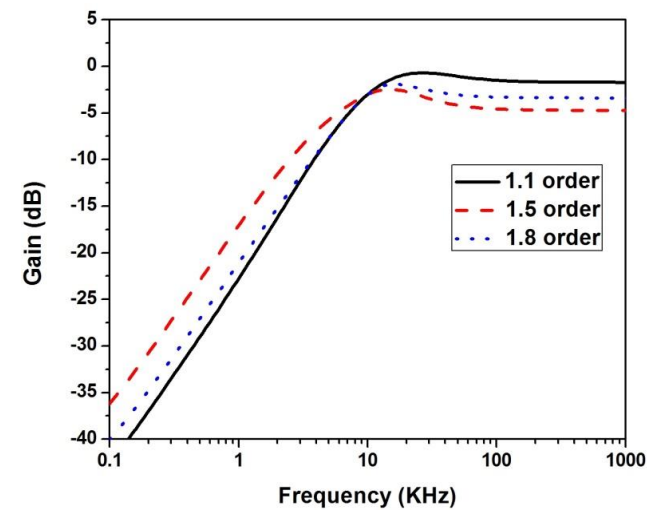


Fig. 10. Frequency Vs Gain plot of the high pass filter of three non integer orders

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