

Using of Entropy at Estimation Business Risks

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Abstract—The possibility and expediency of estimation of risk factors based on fundamental positions of information and entropy are grounded. In accordance with the principle of addition, the possibility of using the H-criterion as an indicator of business uncertainty is shown. The algorithm of risk estimation of these investments is offered.

Keywords—information uncertainty, entropy, risk, indicators of a business project

I. INTRODUCTION

COST-benefit analysis represents the most frequent technique used for a rational allocation of resources. This modality of evaluating the expenditure programs is an attempt to measure the costs and gains of a community as a result of running the evaluated program. It is not a direct decision-making tool, but one that leads to a decision that is better focused, if it is accurate. This paper aims to introduce the methodological issues involved in achieving a cost-benefit analysis for the investment projects financed from public and/or other funds.

We propose to use grade risk methods to provide evidence that regulatory and traditional methods are less efficient than approaches based on fundamental statements of information and entropy theory.

Peculiarities of investment management are that the implementation of investment projects is usually a long process, and the conditions in which the project is implemented may change over time compared to those adopted at the beginning of these projects. For these reasons, the organization, planning and management of investment activities requires anticipation of the consequences of possible changes in the formation of project effectiveness indicators, assessment of the factors that caused them, which will ensure the formation of adequate management decisions. Initially, railroad shippers' forecasts were analyzed using this approach. The NBER has published several volumes on this type of data, for example, Quality and Economic Significance of Anticipations Data (1960). The uncertainty of the investment system in the future imposes certain restrictions on the methods and means of economic evaluation of investment projects. Thus, there is a need to study the possibilities of economic analysis methods for the purposes of scientific forecasting of possible changes in investment processes under the conditions of uncertainty of the future.

However, in the 20th century a new economic investment

approach has been adopted in every state, because it applies to the entire field of fiduciary investing, including pension funds and charitable endowments, and because it has been adopted across the Commonwealth, the rule governs the investment of many trillions of dollars in assets.

Decision-making process by a function of economic systems implicates the elements of the future no prediction. Hereby arises the necessity to forecast internal and external conditions of the function of such a system and also forecasting consequences of the realization of the managerial decisions.

The situation when a person generally or an entrepreneur, in particular, does not has an explicit decision, and the situation certainly requires accepting one of a few possible versions to make a decision in literature called risk situation.

Based on possible interpretations of risk [1, 2, 3], we can identify two approaches to the risk essence study. By the first approach, the risk is estimated as the direction on the special attractive purpose, achievement which is banded with danger. By the second approach – risk means the accomplishment of alternative choice in the ambiguity or situation of uncertainty where the success or failure (setback) depends on the event and fail appears in non-fulfilment of the desirable result. The first approach is more researched in European researches and accents in the risk a danger factor. It is oriented on analysis of cases when the subject chooses more dangerous missions or more dangerous methods for its achievements in comparison with other purposes or approaches where such danger is less or generally does not exist. By the second approach that is more often used in American research, risk question is reviewed in condition with difficulty to make a decision and danger to not achieve purpose because of the fail decision [4, 5, 6]. Here the risk is evaluated as a process to make a decision, as an act of advantage, which subject gives away to hardy achieved purposes despite purposes which achievement is guaranteed. This approach learns general cases of an alternative choice when the subject is considered on the achievement of purposes and when exists different chances on success and fail [7, 8]. Comparing European and American approaches to learning risk, it is also possible to mention as likeness than as difference of these methods. If, for example, according to the European interpretation of risk, the existence of a threat also does not exclude the possibility of accomplishing the desired mission in the appropriate conditions, then, according to the American interpretation, the very same threat simply implies a failure to achieve set objective.

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Considering risk as situational characteristic of activity in the conditions of uncertainty and possible unlucky results it is good to point out at least three characteristic definitions. First of all, risk as a measure of desirable unluckiness in case of unlucky in the activity is defined by combing possibilities of unlucky and measure of the unlucky consequences in this case. Secondly risk as an act, in the one or other correlation that threatens to the subject by losses. In terms of correlation of the expected advantage and foreseeable loss by the realization of certain activity defines justifiable and unjustifiable risk. Thirdly, risk as a situation of choice in conditions of ambiguity between versions: "less attractive but more reliable" and vice-versa. Therefore, the outgoing purpose of analysis and forecasting consists of disclosure and assessment all sides and aspects of the risk situation combined with a financial decision, penetration in the essence of processes and creation of a mechanism to protect the managerial subject from possible financial losses [12,13].

II. METHOD

According to the essence of the above-reviewed approaches, we would like to point out that risk evaluated by the following underline components:

- possibility to not achieve the predictable activity result;
- the dimension of loss of this result.

Firstly, entropy idea had been used into thermodynamics for definition irreversibility of scattering energy. In static physics, entropy is used as a measurement of possibilities accomplishing whichever microscopically phase. In informative theory entropy is measurement ambiguity, which can have different final results. Therefore, considering risk as possibility or probability those or other unexpected results we want to make an assumption about the possibility of usage of information theory for risk assessment in general and investment activity in particular [14,15].

Let us consider the financial-economic situation, which has a row of unspecified conditions. For assessment ambiguity of the final result, we use main statements of static physics and informative theory, namely entropy of the system.

As for [9, 10], the essence of entropy lies in the following. Let any system can have n conditions which are described by dimensions X_1, X_2, \dots, X_n with probabilities of these conditions, accordingly P_1, P_2, \dots, P_n by $P_1 + P_2 + \dots + P_n = 1$, probability of condition which is fully defined.

Concerning informative theory under condition possibility we have to understand digital characteristics of level appearance possibility of any event [10] for discrete separation probabilities P_n entropy system is called dimension:

$$H(a) = -(P_1 \log_q P_1 + P_2 \log_q P_2 + \dots + P_n \log_q P_n) \quad (1)$$

or:

$$H(a) = -\sum_{k=1}^n P_k \log_q P_k \quad (2)$$

where $q > 1$ —system logarithm base.

Expression (1) is fundamental in information theory. On this basis, the unit of information quantity is determined as a measure of the uncertainty of the event.

In order to quantify the entropy of the system according to (1), it is necessary to specify the basis of the logarithm. In

information theory, the basis of the logarithm is equal to 2, as the smallest among integers greater than one. Given that the uncertainty of the economic system is also established on the basis of information about the state of the market or other parameters of the system, to define the entropy of economic system, we also take the number 2 as a basis for the logarithm.

It should be noted that in expression (1) the uncertainty of the state of the system $H(a)$ becomes zero when one of the possible states is reliable, ie $P_k = 1$ and reaches a maximum for equally possible actions P_k :

$$P_k = \frac{1}{n} \quad (3)$$

where: P_k is the probability of k -th action, n is the number of equally probable states of the system.

The expression $(P_k \log_2 P_k)$ in (2) should be considered as the uncertainty of one of the possible states of the final event, and its value with the opposite sign, as the entropy of the k -th state of this event.

With respect to financial and economic systems, the probabilities of states P_1, P_2, \dots, P_n in expressions (1, 2, 3) can be determined by the magnitude of possible standard deviations, variances, or otherwise using statistics [11,13].

III. METHOD

With the purpose to verify the possibility of practical usage of fundamental phrase (1) in the uncertain condition theory financial economical system as an example we calculated the values of entropy for different meanings probabilities P_k that summed up in Table I.

TABLE I
ENTROPY SYSTEM WITH POSSIBILITIES P_k

n	Probability of state P_k		$\log_2 P_k$		Informative uncertainty $P_k \log_2 P_k$		$H(a)$
1	1	1	0	0	0	0	0
2	0,95	0,05	-0,0740	-4,3219	-0,0703	-0,2161	0,0286
2	0,9	0,1	-0,1520	-3,3219	-0,1368	-0,3322	0,4690
2	0,8	0,2	-0,3219	-2,3219	-0,2575	-0,4644	0,7219
2	0,7	0,3	-0,5146	-1,7370	-0,3602	-0,5211	0,8813
2	0,6	0,4	-0,7370	-1,3219	-0,4422	-0,5288	0,9710
2	0,5	0,5	-1,0000	-1,0000	-0,5000	-0,5000	1,0000
	0	0	-0,0740	-4,3219	-0,0703	-0,2161	0,0286
3	0,3333		-1,5850		-0,5283		1,5850
4	0,2500		-2,0000		-0,5000		2,0000
5	0,2000		-2,3219		-0,4644		2,3219
6	0,1667		-2,5850		-0,4308		2,5850
7	0,1429		-2,8074		-0,4011		2,8074
8	0,1250		-3,0000		-0,3750		3,0000
9	0,1111		-3,1699		-0,3522		3,1699
10	0,1000		-3,3219		-0,3322		3,3219
12	0,0833		-3,5850		-0,2987		3,5850
14	0,0714		-3,8074		-0,2720		3,8074
16	0,0625		-4,0000		-0,2500		4,0000
18	0,0556		-4,1699		-0,2317		4,1699
20	0,0500		-4,3219		-0,2161		4,3219

The diagram of entropy values of various systems which can occupy from 1 to 20 probable states with the corresponding probabilities P_k is constructed (Fig. 1).

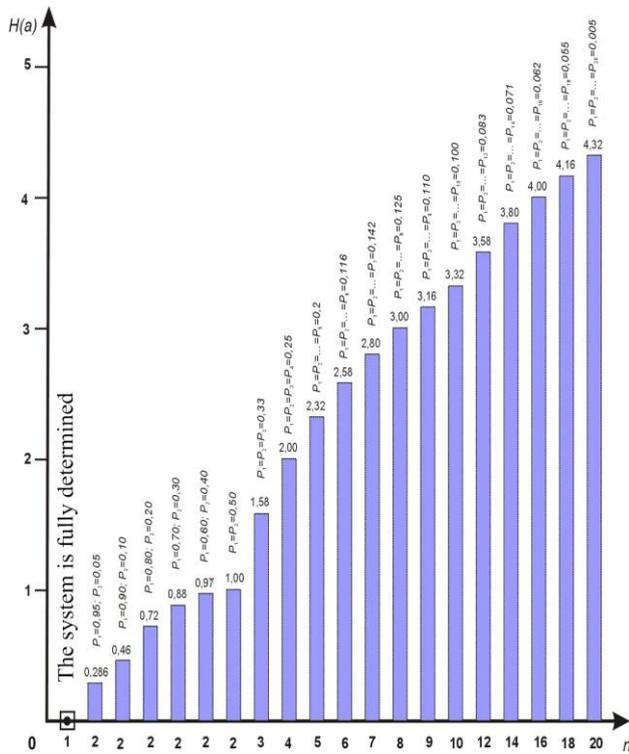


Fig. 1. Entropy of the system for different number of probable states n.

Let us note that for the system with two probable conditions (n=2) considered only six possible versions.

In practice better to define the meaning of entropy system $H(a)$ by different meaning of possibilities P_k . This diagram is developed and showed on Fig. 2.

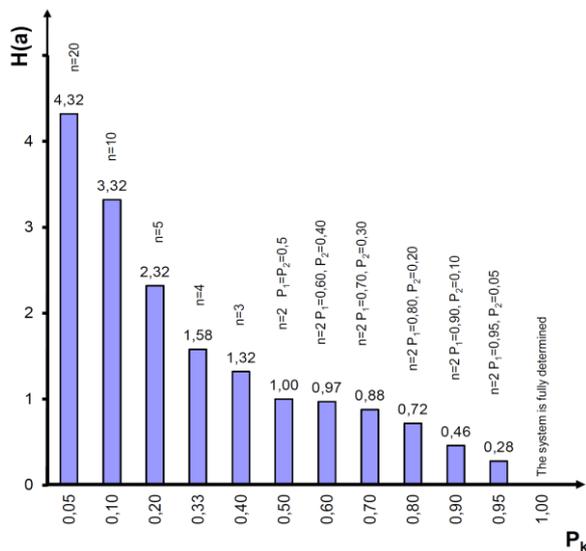


Fig. 2. Entropy system by different meaning possibilities P_k .

In turn, the information uncertainty of each individual state of the event depending on the different values of the probabilities P_k is shown in Fig. 3.

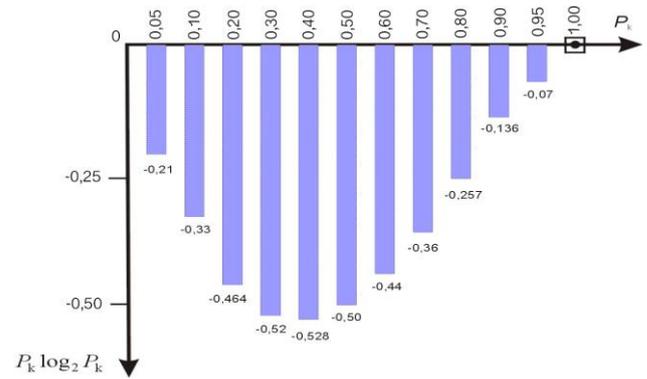


Fig. 3. Information uncertainty k -th state of the event P_k .

This confirms that information uncertainty is nothing more than the entropy of information, which is described by a set of quantities X_1, X_2, \dots, X_n with probabilities P_1, P_2, \dots, P_n of occurrence of these quantities in this information.

Obtained diagrams can be used for economic assessment of the investment system. As known a determinant that defines result of investment activity, first of all is necessary to define aspects that influences on this determinant. For each of these aspects we define probabilities of deviation P_k for condition that sum of these probabilities determine action and equals one. For evaluation P_k can be used as static dates, as the dates of analytical accounting.

By measurement of probabilities condition P_k defines informative uncertain each of the conditions of the action

$P_k \log_2 P_k$. Summed up the informative uncertain of all conditions accordingly phrase (2) we get entropy of the final action $H(a)$. By value $H(a)$ from diagram 1 designate probability of the final action P_k which assign possibility achievement or non-achievement of the final result.

For ensuring the enough discrete probabilities of parameters action, the table of informative uncertainty was computed (Table II).

TABLE II
DETERMINANTS OF THE INVESTMENT PROJECT

Determinants		Project Company A	Project Company B
K_{ge}	General capital of a company, currency	4 000 000	4 000 000
K_{in}	Capital, that investing currency	4 000 000	800 000
n	Quantity of different financial instruments (types of royalties) where the capital is invested	2	2
T	Period, duration of the project	1	1
i	Project yield of the investing capital, %	30	30

To check up the possibility of usage of the proposed method regarding investment systems let us consider the next example. It is necessary to identify probability and risk measure of two different conditional investment projects with the following determinants [8,14,15].

On the base of data in the Table III let us calculate informative ambiguity of each of the determinants which characterize mentioned investing projects.

First of all, we must define the validity or significance of these determinants in the project.

TABLE III
INFORMATIVE UNCERTAINTY OF THE PROJECTS

Parameters	Weight coefficients K_k		Probability of state P_k		Informative uncertainty $P_k \log_2 P_k$	
	Company A	Company B	Company A	Company B	Company A	Company B
k	1,0	0,2	0,469	0,194	-0,512	-0,455
n	0,5	0,5	0,235	0,194	-0,488	-0,455
T	0,33	0,33	0,155	0,320	-0,411	-0,527
i	0,3	0,3	0,141	0,291	-0,397	-0,529
Sum	2,13	1,03	1	1	-1,808	-1,966
Informative uncertainty by data Fig.1					P_i^A 0,28÷0,29	P_i^B 0,25÷0,26

Validity coefficient of the investment capital, for the example, defines as correlation - investment capital to its general size:

$$K_K = \frac{K_{in}}{K_{gen}} \quad (4a)$$

then,

$$\text{in project A: } K_K^A = \frac{4000000}{4000000} = 1,0; \quad (4b)$$

$$\text{in project B: } K_K^B = \frac{800000}{4000000} = 0,2. \quad (4c)$$

The validity of the quantity of the financial instruments we can determine from correlation the unit to its quantity

$$K_n = \frac{1}{n} \quad (5a)$$

$$\text{for project A: } K_n^A = \frac{1}{2} = 0,5 \quad (5b)$$

$$\text{for project B: } K_n^B = \frac{1}{5} = 0,2 \quad (5c)$$

The validity of such parameter as project duration we can define from the average statistical duration of changes the instrument market price. For example, in the accepted accounted period triple price fluctuation of the financial instruments took place, so $t_n = 3$. The validity of the fluctuation we can find as duration correlation to quantity of fluctuations

$$K_m = \frac{1}{3} = 0,33 \text{ as for company A and company B.}$$

Valid coefficients of the project yield we take equally to the parameter value and compressed in the parts of unit. So $K_i = 0,3$ for company A and B.

Let us define the sum value for companies mentioned above

$$K_{sum} = K_k + K_n + K_m + K_i. \quad (4)$$

For company A: $K_{sum}^A = 1,0 + 0,5 + 0,33 + 0,3 = 2,13$.

For company B: $K_{sum}^B = 0,2 + 0,2 + 0,33 + 0,3 = 1,03$.

That fact in our example the sum of the validity coefficients is more than one does not allow identifying the valid coefficients with a probability of uncertain determinants that cause the problem to find the method of quantitative appraisal of the probabilities of the information ambiguity. In our example the

possibility to identify informative ambiguity as a correlation to each of valid parameters to its sum value. The results of the accounts described in Table IV.

TABLE IV
TABLE INFORMATIVE UNCERTAINTY OF THE PROJECTS

Parameters x	Weight coefficients K_k^y		Probability of state $P_k^y = K_k^y/2,13$		Informative uncertainty $P_k \log_2 P_k$	
	Comp. A	Comp. B	Comp. A	Comp. B	Company A	Company B
K_{in}	1,0	0,2	0,469	0,194	-0,512	-0,455
n	0,5	0,2	0,235	0,194	-0,488	-0,455
T	0,33	0,33	0,155	0,320	-0,411	-0,527
i	0,3	0,3	0,141	0,291	-0,397	-0,529
Σ	2,13	1,03	1,0	1,0	-1,808	-1,966
Informative uncertainty by data Fig. 1.					P_i^A 0,28÷0,29	P_i^B 0,25÷0,26

Received summed values of the informative ambiguity of the separate parameters calculated by $P_k \log_2 P_k$ with sign minus is the entropy of the final action $H(a)$. Its meaning for company A is 1,808 and for the company, B is 1,966. Including entropy meaning company with diagram 1 define the probability of the final action P_k .

As a result, we want to define in our example project company A has ambiguity in measures 0,28 – 0,29 in that time as for company B ambiguity consists 0,25÷0,26. So the failure probability of the final result in company A is more than in company B. Accordingly investment project company A is more risky.

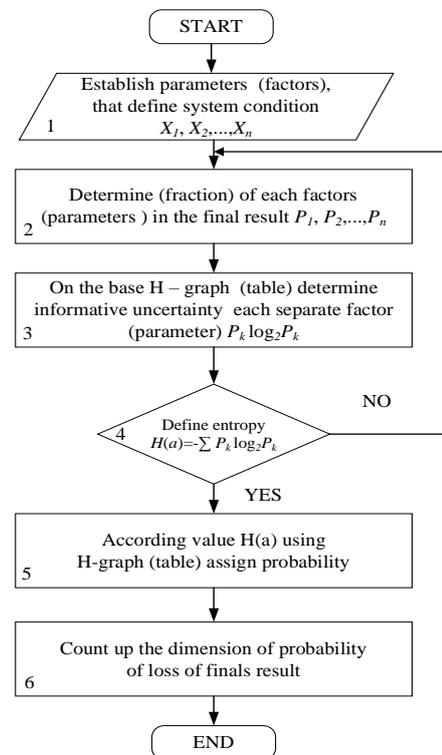


Fig. 4. Process of investment risk assessment based on H-criterion.

Received results of the investment ambiguity define with diversification concept. So, in our example company B invested not the whole capital, but only 20% in the sum of 800 000

monetary units and diversify it in more quantity of the financial instruments. In addition, we need to mention that reviewed method of the appraisal probabilities parameter condition system through the valid coefficients that are shown as one of the simplest. Moreover, form the problem of appraisal and forecast of the effectiveness of investments in the first approximation.

For spreading the possibility of using the received results and calculation with EOM is proposed in the following algorithm (see Fig. 4).

The assessment of informative ambiguity by the well-known meaning of entropy can be accomplished in the phrase (2). If the quantity of possible n is known, then let's build up the graph of dependence $H(a)$ from the biggest from possibilities P_k , for example, $P_1 = P$. Here with consider that other possibilities $P_k, k = \overline{2, n}$ are divided equally, so $P_k = (1 - P) \times (n - 1)^{-1}, k = \overline{2, n}$. So, we have the following graph (Fig. 5).

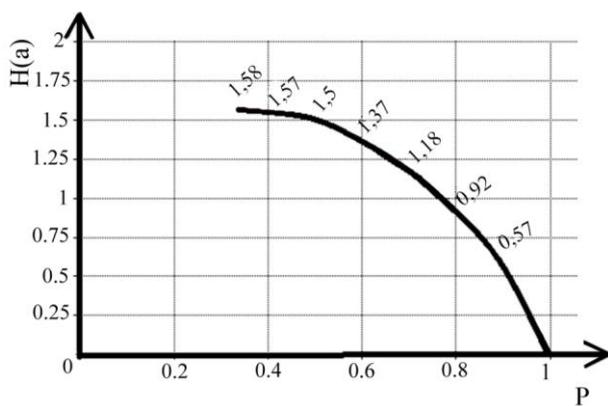


Fig. 5. Graph of the dependence of $H(a)$ on P .

Then form this graph we are looking for possibility P , which corresponds the specified value entropy $H(a)$.

For this by approximate methods we can find the solution of the following nonlinear equation:

$$H(a) = -P \log_2 P - (1 - P) \cdot \log_2 \frac{1 - P}{n - 1} \quad (6)$$

Received probability P we will consider as uncertainty grade. In the case where $H(a)$ is bigger than maximum, which can be reached by $P = \frac{1}{n}$, then the program will assume this as a mistake in incoming data. If the quantity of possible conditions is unknown, will consider, that they are even, $P_k = P = n^{-1}, k = \overline{1, n}$. In the case formula (2), which is discrete, regardless of the quantity of conditions n will be:

$$H(a) = -\sum_{k=1}^n P_k \log_2 P_k = -n \cdot P \cdot \log_2 P = \log_2 P \quad (7)$$

Namely, generalizing this formula for uninterrupted even, we can count up the risk probability by the inverse formula $P = 2^{H(a)}$.

In order to minimize the time spent on calculating the probability of risk, a program for its calculation is proposed. The working window of the program and an example of its work are shown in Fig. 6.

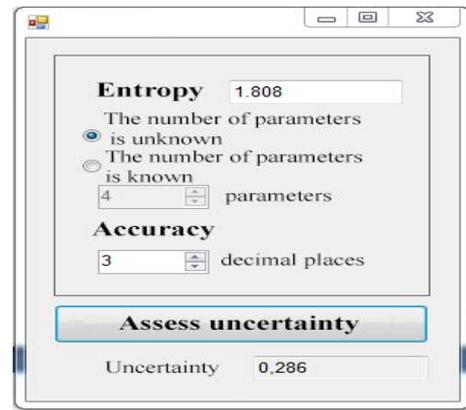


Fig. 6. The working window of the program.

Thus, risk assessment has a multifaceted nature and ends with the identification of possible consequences of the implementation of risky decisions and the corresponding probabilities of their occurrence. In the context of creating a methodological framework for making risky financial decisions, risk assessment technology covers the following stages:

- definition and selection of financial indicator X (or group of indicators) as an effective indicator of assessing the consequences of decisions made in terms of risk. Such indicators may be income, profit margin, etc.;
- definition and description of the probabilities distribution law and certain characteristics of the random variable X ;
- determining the significance of the impact of the parameters of the investment system on the generalized indicator X .

Estimation of information uncertainty of indicators of the concrete investment project, according to the resulted algorithm (Fig. 4), it is expedient to carry out as follows.

1. Based on the method of relative deviations, the relative deviation of the generalized indicator is recorded due to the values of deviations of parameters and their coefficients of influence.
2. The values of the coefficients of influence of each of the parameters taking as the initials known values of these parameters are calculated.
3. According to the analytical dependence, which describes the relative deviation of the generalized indicator, the total, modulo, value of the coefficients of influence is determined.
4. The information probability of the state of each of the parameters, provided that the total information probability of the event is equal to one is determined.
5. According to the value of the obtained probability values P_i , the entropy of each individual parameter is set numerically.
6. The total entropy of the generalized indicator or event is calculated.
7. By the magnitude of the obtained value of the entropy of the event, $H(a)$ the probability of the state P_{i_x} corresponding to this entropy is determined.

The resulting probability value of the condition should be considered as the probability of uncertainty of the final event, or as a possible risk caused by deviations of the parameters describing the event.

Risk management is the development and implementation of targeted management actions on the object of management and includes the development, adoption and implementation of decisions, its adjustment and control over implementation. The purposefulness of actions means the presence of a criterion (or group of criteria) that helps to assess the alternative and the multivariate problem statement to choose the best (optimal) solution.

CONCLUSION

Therefore, based on the proposed method in the entropy shows possibility of assessment of the quantitative type ambiguity condition in the investment system for the exact investing projects. Herewith the complexity of such calculations is enough less, then in other traditional, famous methods of grade risk. The usage of grade risk method of the conditions of investment projects on the base of entropy with using *H*-criteria is possible for efficiency assessment the financial and real investments. Our data expectation analysis submits two main outcomes. It was found that such used investment indicators as NVP, IRR, PI and methods using them allow estimating the net present value, internal rate of return, profitability index of investment projects, and they should be considered as a necessary but insufficient condition to forecast deviations of these indicators in the conditions of factors. Therefore, the improvement of methods for evaluating and forecasting the effectiveness of investments is an urgent problem that needs a scientific solution.

On the other hand, the critical question is whether expectations play a significant role in macroeconomic modeling.

However, a completely different perspective of perceiving the problem was adopted. Considering risk as a probabilistic characteristic of the quantitative uncertainty of the economic system, the possibility and expediency of assessing risk factors based on the fundamental provisions of information and entropy are substantiated. Expectations have a reasonably precise extrapolation structure and do not randomly vibrate; it is a systematic error pattern. Furthermore, market participants make widespread prediction errors.

It is worth considering whether such errors can explain macroeconomic fluctuations. Examples include aggregated overbuilding in essential sectors such as the housing market or prolonged recessions related to the lack of corporate investment and hiring. Moreover, common error effects accumulated investment perversion, which should also be considered in further analysis.

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