

# Modulation-Mode Assignment for SVD-assisted Downlink Multiuser MIMO Transmission Schemes with Iterative Detection

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**Abstract**—Since the capacity of multiple-input multiple-output (MIMO) systems increases linearly with the minimum number of antennas at both, the transmitter as well as the receiver side, MIMO systems have attracted a lot of attention for both frequency and non-frequency selective channels and reached a state of maturity. By contrast, MIMO-aided multiple-user systems require substantial further research. In comparison to zero-forcing (ZF) multiuser transmission techniques, where all users are treated jointly, the investigated singular value decomposition (SVD) assisted DL multiuser MIMO solution takes the individual user's channel characteristics into account. In analogy to bit-interleaved coded irregular modulation, we introduce a MIMO-BICM scheme, where different user-specific signal constellations and mapping arrangement were used within a single codeword. Extrinsic information transfer (EXIT) charts are used for analyzing and optimizing the convergence behaviour of the iterative demapping and decoding. Our results show that in order to achieve the best bit-error rate, not necessarily all user-specific MIMO layers have to be activated.

**Keywords**—Multiple-input multiple-output system, singular-value decomposition, bit allocation, power allocation, wireless transmission, multiuser transmission.

## I. INTRODUCTION

THE never ending desire for increasing the available transmission capacities has attracted a lot of research since Shannon's pioneering work in 1948. A possible solution was presented by Teletar and Foschini in the mid 90's, which revived the MIMO (multiple input multiple output) transmission philosophy introduced by van Etten in the mid 70's [1]–[3]. Since the capacity of multiple-input multiple-output (MIMO) systems increases linearly with the minimum number of antennas at both, the transmitter as well as the receiver side, MIMO-BICM schemes have attracted substantial attention [4], [5] and can be considered as an essential part of increasing both the achievable capacity and integrity of future generations of wireless systems [6], [7]. However, their parameters have to be carefully optimized, especially in conjunction with adaptive modulation [8]. The well-known water-filling technique is virtually synonymous with adaptive modulation and it is used for maximizing the overall data rate [9]–[11]. However, delay-critical applications, such as voice or video transmission schemes, may require a certain fixed data

rate. For these fixed-rate applications it is desirable to design algorithms, which minimize the bit-error rate (BER) at a given fixed data rate.

In this field, coding plays an important role to achieve the required quality criteria. Bit-interleaved coded modulation (BICM) was designed for bandwidth efficient transmission over fading channels [12], [13] and extended to bit-interleaved coded irregular modulation (BICIM) schemes by using different signal constellations and mapping arrangements within a single codeword, offering an improved link adaptation capability and an increased design freedom [14].

Single-user MIMO-BICM transmission schemes for both non-frequency and frequency selective MIMO channels have attracted a lot of attention and reached a state of maturity [6], [15]. By contrast, MIMO-aided multiple-user systems require substantial further research where both multiuser as well as multi-antenna interferences have to be taken into account. Considering the entirety of the antennas of all mobile terminals at one end and the antennas of the base station at the other end of the communication link, state of the art interference cancellation is based on a central signal processing unit, e. g. a central unit at the base station, where joint detection can be applied in the uplink (UL) and joint transmission in the downlink (DL), respectively [16]–[18]. Widely used linear preprocessing techniques such as Minimum Mean Square Error or Zero Forcing (ZF) have attracted a lot of research and have reached a state of maturity, too [19]. Therefore, in this work a SVD-assisted downlink (DL) multiuser MIMO-BICM system is considered, which takes the individual user's channel characteristics into account rather than treating all users channels jointly as in ZF multiuser transmission techniques [20]. Besides the signal processing needed to separate the users within a multiuser system, multiple antenna systems are affected by signal correlation among antennas, which produces a degradation of the link performance, depending on the physical relative position of the antennas and the mobility of the front-ends [21], [22].

In this context, the choice of the number of bits per symbol and the number of activated MIMO layers combined with powerful error correcting codes offer a certain degree of design freedom [23]. In addition to bit loading algorithms, in this contribution the benefits of channel coding are also investigated. The proposed iterative decoder structures employ symbol-by-symbol soft-output decoding based on the Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm and are analyzed under the constraint of a fixed data throughput [24].

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Against this background, the novel contribution of this paper is that we jointly optimize the number of activated user-specific MIMO layers and the number of bits per symbol combined with powerful error correcting codes under the constraint of a given fixed data throughput and integrity. Since the "design-space" is large, a two-stage optimization technique is considered. Firstly, the uncoded spatial division multiplexing (SDM) MIMO scheme is analyzed, investigating the allocation of both the number of bits per modulated symbol and the number of activated MIMO layers at a fixed data rate. Secondly, the optimized uncoded system is extended by incorporating bit-interleaved coded modulation using iterative detection (BICM-ID), whereby both the uncoded as well as the coded systems are required to support the same user data rate within the same bandwidth.

The remaining part of this contribution is organized as follows: Section II introduces our system model, while the proposed uncoded solutions are discussed in section III. In section IV the channel encoded MIMO system is introduced. The associated performance results are presented and interpreted in section V. Finally, section VI provides our concluding remarks.

## II. MULTIUSER SYSTEM MODEL

The system model considered in this work consists of a single base station (BS) supporting  $K$  mobile stations (MSs). The BS is equipped with  $n_T$  transmit antennas, while the  $k$ th (with  $k = 1, \dots, K$ ) MS has  $n_{Rk}$  receive antennas, i. e. the total number of receive antennas including all  $K$  MSs is given by  $n_R = \sum_{k=1}^K n_{Rk}$ . The  $(n_{Rk} \times 1)$  user specific symbol vector  $\mathbf{c}_k$  to be transmitted by the BS is given by

$$\mathbf{c}_k = (c_{k,1}, c_{k,2}, \dots, c_{k,n_{Rk}})^T . \quad (1)$$

The vector  $\mathbf{c}_k$  is preprocessed before its transmission by multiplying it with the  $(n_T \times n_{Rk})$  DL preprocessing matrix  $\mathbf{R}_k$  and results in the  $(n_T \times 1)$  user-specific transmit vector

$$\mathbf{s}_k = \mathbf{R}_k \mathbf{c}_k . \quad (2)$$

After DL transmitter preprocessing, the  $n_T$ -component signal  $\mathbf{s}$  transmitted by the BS to the  $K$  MSs results in

$$\mathbf{s} = \sum_{k=1}^K \mathbf{s}_k = \mathbf{R} \mathbf{c} , \quad (3)$$

with the  $(n_T \times n_R)$  preprocessing matrix

$$\mathbf{R} = (\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_K) . \quad (4)$$

In (3), the overall  $(n_R \times 1)$  transmitted DL data vector  $\mathbf{c}$  combines all  $K$  DL transmit vectors  $\mathbf{c}_k$  (with  $k = 1, 2, \dots, K$ ) and is given by

$$\mathbf{c} = (\mathbf{c}_1^T, \mathbf{c}_2^T, \dots, \mathbf{c}_K^T)^T . \quad (5)$$

At the receiver side, the  $(n_{Rk} \times 1)$  vector  $\mathbf{u}_k$  of the  $k$ th MS results in

$$\mathbf{u}_k = \mathbf{H}_k \mathbf{s} + \mathbf{n}_k = \mathbf{H}_k \mathbf{R} \mathbf{c} + \mathbf{n}_k \quad (6)$$

and can be expressed by

$$\mathbf{u}_k = \mathbf{H}_k \mathbf{R}_k \mathbf{c}_k + \sum_{i=1, i \neq k}^K \mathbf{H}_k \mathbf{R}_i \mathbf{c}_i + \mathbf{n}_k , \quad (7)$$

where the MSs received signals experience both multi-user and multi-antenna interferences. In (6), the  $(n_{Rk} \times n_T)$  channel matrix  $\mathbf{H}_k$  connects the  $n_T$  BS specific transmit antennas with the  $n_{Rk}$  receive antennas of the  $k$ th MS.

It is quite common to assume that the coefficients of the  $(n_{Rk} \times n_T)$  channel matrix  $\mathbf{H}_k$  are independent and Rayleigh distributed with equal variance. However, in many cases correlations between the transmit antennas as well as between the receive antennas can't be neglected. There are several methods to model and characterize antenna signal correlation into the MIMO channel model for Rayleigh flat-fading channels. In this work it is assumed that the correlation among receive antennas is independent of the correlation between transmit antennas. The way to include the antenna signal correlation into the MIMO channel model for Rayleigh flat-fading like channels, is given by [21], [22] and results in

$$\mathbf{H}_{\text{corr } k} = \mathbf{H}_{\text{Rx}}^{1/2} \cdot \mathbf{H}_k \cdot \mathbf{H}_{\text{Tx}}^{1/2} , \quad (8)$$

where  $\mathbf{H}_k$  is a  $(n_{Rk} \times n_T)$  uncorrelated channel matrix with independent, identically distributed complex Gaussian zero-mean unit variance elements and where  $(\cdot)^{1/2}$  stands for the square root of a matrix. The  $(n_{Rk} \times n_{Rk})$  matrix  $\mathbf{H}_{\text{Rx}}$  is used to model the correlation between the  $k$ th MS receive antennas. Moreover, the  $(n_T \times n_T)$  transmit correlation matrix  $\mathbf{H}_{\text{Tx}}$  models the correlation between the transmit antennas.

The interference, which is introduced by the off-diagonal elements of the channel matrix  $\mathbf{H}_k$ , requires appropriate signal processing strategies. A popular technique is based on the SVD of the system matrix  $\mathbf{H}_k$ . Upon carrying out the SVD of  $\mathbf{H}_k$  with  $n_T \geq n_R$  and assuming that the rank of the matrix  $\mathbf{H}_k$  equals  $n_{Rk}$ , i. e.,  $\text{rank}(\mathbf{H}_k) = n_{Rk}$ , we get

$$\mathbf{H}_k = \mathbf{U}_k \cdot \mathbf{V}_k \cdot \mathbf{D}_k^H , \quad (9)$$

with the  $(n_{Rk} \times n_{Rk})$  unitary matrix  $\mathbf{U}_k$  and the  $(n_T \times n_T)$  unitary matrix  $\mathbf{D}_k^H$ , respectively<sup>1</sup>. The  $(n_{Rk} \times n_T)$  diagonal matrix  $\mathbf{V}_k$  can be decomposed into a  $(n_{Rk} \times n_{Rk})$  matrix  $\mathbf{V}_{k \text{ u}}$  containing the non-zero square roots of the eigenvalues of  $\mathbf{H}_k^H \mathbf{H}_k$ , i. e.,

$$\mathbf{V}_{k \text{ u}} = \begin{bmatrix} \sqrt{\xi_{k,1}} & 0 & \cdots & 0 \\ 0 & \sqrt{\xi_{k,2}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{\xi_{k,n_{Rk}}} \end{bmatrix} , \quad (10)$$

and a  $(n_{Rk} \times (n_T - n_{Rk}))$  zero-matrix  $\mathbf{V}_{k \text{ n}}$  according to

$$\mathbf{V}_k = (\mathbf{V}_{k \text{ u}} \mathbf{V}_{k \text{ n}}) = (\mathbf{V}_{k \text{ u}} \mathbf{0}) . \quad (11)$$

Additionally, the  $(n_T \times n_T)$  unitary matrix  $\mathbf{D}_k$  can be decomposed into a  $(n_T \times n_{Rk})$  matrix  $\mathbf{D}_{k \text{ u}}$  constituted by the eigenvectors corresponding to the non-zero eigenvalues of

<sup>1</sup>The transpose and conjugate transpose (Hermitian) of  $\mathbf{D}_k$  are denoted by  $\mathbf{D}_k^T$  and  $\mathbf{D}_k^H$ , respectively.

$\mathbf{H}_k^H \mathbf{H}_k$  and a  $(n_T \times (n_T - n_{R_k}))$  matrix  $\mathbf{D}_{k_n}$  constituted by the eigenvectors corresponding to the zero eigenvalues of  $\mathbf{H}_k^H \mathbf{H}_k$ . The decomposition of the matrix  $\mathbf{D}_k^H$  results in

$$\mathbf{D}_k^H = \begin{pmatrix} \mathbf{D}_{k_u}^H \\ \mathbf{D}_{k_n}^H \end{pmatrix}. \quad (12)$$

Finally, the received downlink signal  $\mathbf{u}_k$  of the  $k$ th MS may be expressed as

$$\mathbf{u}_k = \mathbf{U}_k \mathbf{V}_{k_u} \mathbf{D}_{k_u}^H \mathbf{R} \mathbf{c} + \mathbf{n}_k, \quad (13)$$

with the vector  $\mathbf{n}_k$  of the additive, white Gaussian noise (AWGN). Taking all MSs received DL signals  $\mathbf{u}_k$  into account, the  $(n_R \times 1)$  receive vector results in

$$\mathbf{u} = (\mathbf{u}_1^T, \mathbf{u}_2^T, \dots, \mathbf{u}_K^T)^T. \quad (14)$$

Then, the overall DL signal vector  $\mathbf{u}$  including the received signals of all  $K$  MSs can be expressed by

$$\mathbf{u} = \mathbf{U} \mathbf{V}_u \mathbf{D}_u^H \mathbf{R} \mathbf{c} + \mathbf{n}, \quad (15)$$

with the overall  $(n_R \times 1)$  noise vector

$$\mathbf{n} = (\mathbf{n}_1^T, \mathbf{n}_2^T, \dots, \mathbf{n}_K^T)^T, \quad (16)$$

the  $(n_R \times n_R)$  block diagonal matrix  $\mathbf{U}$

$$\mathbf{U} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{U}_K \end{bmatrix}, \quad (17)$$

the  $(n_R \times n_R)$  block diagonal matrix  $\mathbf{V}_u$

$$\mathbf{V}_u = \begin{bmatrix} \mathbf{V}_{1u} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_{2u} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{V}_{Ku} \end{bmatrix}, \quad (18)$$

and the  $(n_T \times n_R)$  matrix  $\mathbf{D}_u$  which is given by

$$\mathbf{D}_u = (\mathbf{D}_{1u}, \mathbf{D}_{2u}, \dots, \mathbf{D}_{Ku}). \quad (19)$$

In order to suppress the DL multi-user interferences (MUI) perfectly, the DL preprocessing matrix  $\mathbf{R}$  has to be designed to satisfy the following condition

$$\mathbf{D}_u^H \mathbf{R} = \mathbf{P}, \quad (20)$$

with the real-valued  $(n_R \times n_R)$  diagonal matrix  $\mathbf{P}$  taking the transmit-power constraint into account. In order to satisfy (20),  $\mathbf{R}$  can be defined as follows

$$\mathbf{R} = \mathbf{D}_u (\mathbf{D}_u^H \mathbf{D}_u)^{-1} \mathbf{P}. \quad (21)$$

Taking the ZF design criterion for the DL preprocessing matrix into account, the matrix  $\mathbf{P}$  simplifies to an  $(n_R \times n_R)$  diagonal matrix, i. e.  $\mathbf{P} = \sqrt{\beta} \mathbf{I}_{n_R \times n_R}$ , with the parameter  $\sqrt{\beta}$  taking the transmit-power constraint into account. When taking the DL preprocessing matrix, defined in (21), into account, the overall received vector of all  $K$  MSs, defined in (15), can be simplified to

$$\mathbf{u} = \mathbf{U} \mathbf{V}_u \mathbf{P} \mathbf{c} + \mathbf{n}. \quad (22)$$

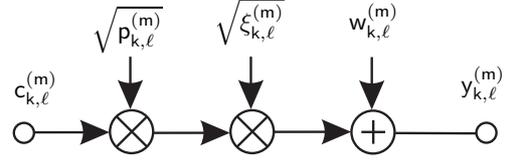


Fig. 1. Resulting  $k$ th user-specific system model per MIMO layer  $\ell$  (with  $\ell = 1, 2, \dots, n_{R_k}$ ) and per transmitted symbol block  $m$ .

Therein, the  $(n_R \times n_R)$  block diagonal matrix  $\mathbf{P}$  is given by

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{P}_K \end{bmatrix}. \quad (23)$$

In (22), the user-specific  $(n_{R_k} \times 1)$  vector  $\mathbf{u}_k$  can be expressed as

$$\mathbf{u}_k = \mathbf{U}_k \mathbf{V}_{k_u} \mathbf{P}_k \mathbf{c}_k + \mathbf{n}_k, \quad (24)$$

with the user-specific  $(n_{R_k} \times n_{R_k})$  power allocation matrix

$$\mathbf{P}_k = \begin{bmatrix} \sqrt{p_{k,1}} & 0 & \cdots & 0 \\ 0 & \sqrt{p_{k,2}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{p_{k,n_{R_k}}} \end{bmatrix}. \quad (25)$$

As long as the transmit power is uniformly distributed over the number of activated MIMO layers, the matrix  $\mathbf{P}_k$  simplifies to  $\mathbf{P}_k = \sqrt{\beta} \mathbf{I}_{n_{R_k} \times n_{R_k}}$ . After postprocessing of the received signal vectors  $\mathbf{u}_k$  with the corresponding unitary matrix  $\mathbf{U}_k^H$ , the user-specific decision variables result with  $\mathbf{U}_k^H \mathbf{n}_k = \mathbf{w}_k$  in

$$\mathbf{y}_k = \mathbf{U}_k^H \mathbf{u}_k = \mathbf{V}_{k_u} \mathbf{P}_k \mathbf{c}_k + \mathbf{w}_k, \quad (26)$$

or alternatively with  $\mathbf{U}^H \mathbf{n} = \mathbf{w}$  in

$$\mathbf{y} = \mathbf{U}^H \mathbf{u} = \mathbf{V}_u \mathbf{P} \mathbf{c} + \mathbf{w}, \quad (27)$$

where interferences between the different antenna data streams as well as MUI imposed by the other users are avoided. The resulting system model is depicted in Fig. 1

### III. QUALITY CRITERIA

In general, the user-specific quality of data transmission can be informally assessed by using the signal-to-noise ratio (SNR) at the detector's input defined by the half vertical eye opening and the noise power per quadrature component according to

$$\varrho = \frac{(\text{Half vertical eye opening})^2}{\text{Noise Power}} = \frac{(U_A)^2}{(U_R)^2}, \quad (28)$$

which is often used as a quality parameter [23]. The relationship between the signal-to-noise ratio  $\varrho = U_A^2/U_R^2$  and the bit-error probability evaluated for AWGN channels and  $M$ -ary Quadrature Amplitude Modulation (QAM) is given by [25]

$$P_{\text{BER}} = \frac{2}{\log_2(M)} \left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc} \left(\sqrt{\frac{\varrho}{2}}\right). \quad (29)$$

When applying the proposed system structure for the  $k$ th user, depicted in Fig. 1, the applied signal processing leads to different eye openings per activated MIMO layer  $\ell$  (with  $\ell = 1, 2, \dots, L$  and  $L \leq n_{Rk}$  describing the number of activated user-specific MIMO layers) and per transmitted symbol block  $m$  according to

$$U_{Ak}^{(\ell,m)} = \sqrt{p_{k,\ell}^{(m)}} \cdot \sqrt{\xi_{k,\ell}^{(m)}} \cdot U_{sk}^{(\ell)}, \quad (30)$$

where  $U_{sk}^{(\ell)}$  denotes the half-level transmit amplitude assuming  $M_\ell$ -ary QAM,  $\sqrt{\xi_{k,\ell}^{(m)}}$  represents the corresponding positive square roots of the eigenvalues of the matrix  $\mathbf{H}_k^H \mathbf{H}_k$  and  $\sqrt{p_{k,\ell}^{(m)}}$  represents the corresponding power allocation weighting parameters (Fig. 1). Together with the noise power per quadrature component, introduced by the additive, white Gaussian noise (AWGN) vector  $\mathbf{w}_k = \mathbf{U}_k^H \mathbf{n}_k$  in (26), the  $k$ th user-specific SNR per MIMO layer  $\ell$  at the time  $m$  becomes

$$\varrho_k^{(\ell,m)} = \frac{\left(U_{Ak}^{(\ell,m)}\right)^2}{U_R^2}. \quad (31)$$

Using the parallel transmission over  $L$  MIMO layers, the overall mean user-specific transmit power becomes  $P_{sk} = \sum_{\ell=1}^L P_{sk}^{(\ell)}$ . Considering QAM constellations, the average user-specific transmit power  $P_{sk}^{(\ell)}$  per MIMO layer  $\ell$  may be expressed as [25]

$$P_{sk}^{(\ell)} = \frac{2}{3} \left(U_{sk}^{(\ell)}\right)^2 (M_{k\ell} - 1). \quad (32)$$

Combining (31) and (32) together with (30), the layer-specific SNR at the time  $m$  results in

$$\varrho_k^{(\ell,m)} = p_{k,\ell}^{(m)} \xi_{k,\ell}^{(m)} \frac{3}{2(M_{k\ell} - 1)} \frac{P_{sk}^{(\ell)}}{U_R^2}. \quad (33)$$

Assuming that the user-specific transmit power is uniformly distributed over the number of activated MIMO layers, i.e.,  $P_{sk}^{(\ell)} = P_{sk}/L$ , the layer-specific signal-to-noise ratio at the time  $m$ , defined in (33), results with the ratio of symbol energy to noise power spectral density  $E_s/N_0 = P_{sk}/(2U_R^2)$  in

$$\varrho_k^{(\ell,m)} = p_{k,\ell}^{(m)} \xi_{k,\ell}^{(m)} \frac{3}{L(M_{k\ell} - 1)} \frac{E_s}{N_0}. \quad (34)$$

In order to transmit at a fixed data rate while maintaining the best possible integrity, i.e., bit-error rate, an appropriate number of user-specific MIMO layers has to be used, which depends on the specific transmission mode, as detailed in Table I for the exemplarily investigated two-user multiuser-system ( $n_{Rk} = 4$  (with  $k = 1, 2$ ),  $K = 2$ ,  $n_R = n_T = 8$ ).

TABLE I  
Investigated User-specific Transmission Modes

throughput	layer 1	layer 2	layer 3	layer 4
8 bit/s/Hz	256	0	0	0
8 bit/s/Hz	64	4	0	0
<b>8 bit/s/Hz</b>	<b>16</b>	<b>16</b>	<b>0</b>	<b>0</b>
<b>8 bit/s/Hz</b>	<b>16</b>	<b>4</b>	<b>4</b>	<b>0</b>
8 bit/s/Hz	4	4	4	4

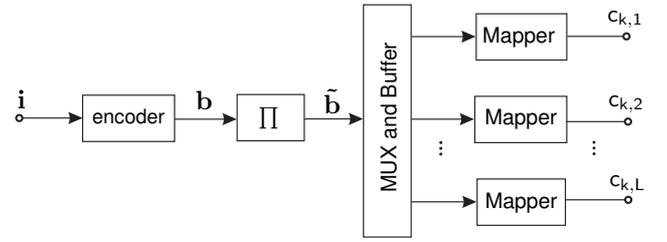


Fig. 2. The channel-encoded  $k$ th user-specific MIMO transmitter structure.

An optimized adaptive scheme would now use the particular transmission modes, e.g., by using bit auction procedures [26], that results in the lowest BER for each SDM MIMO data vector. However, this would lead to a high signaling overhead. Therefore, in order to avoid any signalling overhead, fixed transmission modes are used in this contribution regardless of the channel quality.

#### IV. CHANNEL-ENCODED MIMO SYSTEM

The channel encoded user-specific transmitter structure is depicted in Fig. 2. The encoder employs a half-rate nonrecursive, non-systematic convolutional (NSC) code using the generator polynomials (7, 5) in octal notation. The uncoded information is organized in blocks of  $N_i$  bits, consisting of at least 3000 bits, depending on the specific QAM constellation used. Each data block  $\mathbf{i}$  is encoded and results in the block  $\mathbf{b}$  consisting of  $N_b = 2N_i + 4$  encoded bits, including 2 termination bits. The encoded bits are interleaved using a random interleaver and stored in the vector  $\tilde{\mathbf{b}}$ . The encoded and interleaved bits are then mapped to the MIMO layers. The task of the multiplexer and buffer block of Fig. 2 is to divide the user-specific vector of encoded and interleaved information bits  $\tilde{\mathbf{b}}$  into subvectors according to the chosen transmission mode (Table I). The individual user-specific binary data vectors are then mapped to the QAM symbols  $c_{k,\ell}$  according to the specific mapper used. The iterative demodulator structure is shown in Fig. 3 [27]. When using the iteration index  $\nu$ , the first iteration of  $\nu = 1$  commences with the soft-demapper delivering the  $N_b$  log-likelihood ratios (LLRs)  $L_2^{(\nu=1)}(\tilde{\mathbf{b}})$  of the encoded and interleaved information bits, whose de-interleaved version  $L_{a,1}^{(\nu=1)}(\mathbf{b})$  represents the input of the convolutional decoder as depicted in Fig. 3 [6], [24]. This channel decoder provides the estimates  $L_1^{(\nu=1)}(\mathbf{i})$  of the original uncoded information bits as well as the LLRs of the  $N_b$  NSC-encoded bits in the

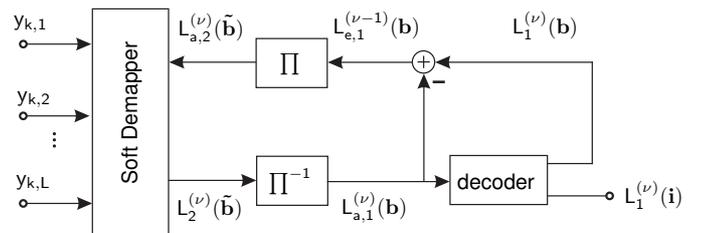


Fig. 3. Iterative demodulator structure.

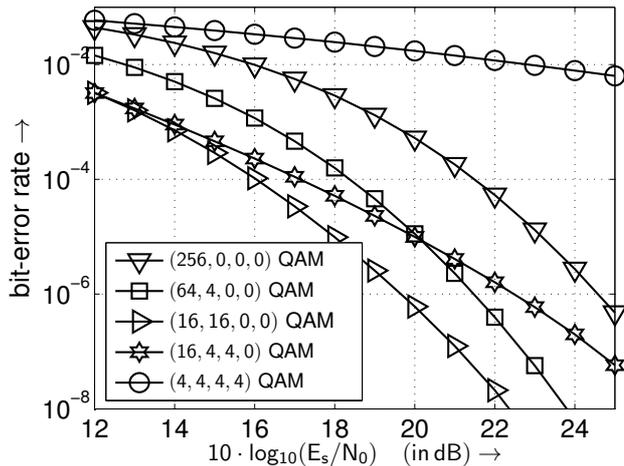


Fig. 4. Uncoded BERs when using the MIMO configurations introduced in Table I and transmitting 8 bit/s/Hz over non-frequency selective uncorrelated Rayleigh channels.

form of

$$L_1^{(\nu=1)}(\mathbf{b}) = L_{a,1}^{(\nu=1)}(\mathbf{b}) + L_{e,1}^{(\nu=1)}(\mathbf{b}). \quad (35)$$

As seen in Figure 3 and (35), the LLRs of the NSC-encoded bits consist of the receiver's input signal itself plus the extrinsic information  $L_{e,1}^{(\nu=1)}(\mathbf{b})$ , which is generated by subtracting  $L_{a,1}^{(\nu=1)}(\mathbf{b})$  from  $L_1^{(\nu=1)}(\mathbf{b})$ . The appropriately ordered, i. e. interleaved extrinsic LLRs are fed back as *a priori* information  $L_{a,2}^{(\nu=2)}(\tilde{\mathbf{b}})$  to the soft demapper of Fig. 3 for the second iteration.

## V. RESULTS

Assuming predefined QAM constellation sizes, a fixed total throughput can be guaranteed for each SDM MIMO block regardless of the channel quality.

### A. Single-User System

Considering a non-frequency selective single-user SDM MIMO link ( $K = 1$ ) composed of  $n_T = 4$  transmit and  $n_R = 4$  receive antennas, the corresponding calculated BER curves are depicted in Fig. 4 for the different QAM constellation sizes and MIMO configurations of Table I, when transmitting at a bandwidth efficiency of 8 bit/s/Hz, assuming a Nyquist roll-off factor of 0.5.

Assuming a uniform distribution of the transmit power over the number of activated MIMO layers, it turns out that not all MIMO layers have to be activated in order to achieve the best BERs. However, it is worth noting that the lowest BERs can only be achieved by using bit auction procedures leading to a high signalling overhead [26]. Analyzing the probability of choosing a specific transmission mode by using optimal bitloading, as depicted in Table II, it turns out that only an appropriate number of MIMO layers has to be activated, e. g., the (16, 4, 4, 0) QAM configuration. The results, obtained by using bit auction procedures justify the choice of fixed transmission modes regardless of the channel quality as investigated in the contribution.

TABLE II

Probability of Choosing Specific Transmission Modes ( $K = 1$ ) at a Fixed Data Rate by Using Optimal Bitloading ( $10 \cdot \log_{10}(E_s/N_0) = 10$  dB, Non-frequency Selective Uncorrelated Rayleigh Channels)

mode	(64, 4, 0, 0)	(16, 16, 0, 0)	(16, 4, 4, 0)	(4, 4, 4, 4)
pdf	0.0116	0.2504	0.7373	0.0008

Besides this, analyzing the distribution of the singular values, as depicted in Fig. 5 for an uncorrelated ( $4 \times 4$ ) MIMO channel and comparing them with the correlated MIMO channel, as highlighted in Fig. 6, it can be stated correlation shifts the pdf (probability density function) of the largest singular value to higher values at the cost of the remaining layers. For the theoretical analysis a transmit antenna separation of 10-wavelength and a receive antenna separation of 4-wavelength at a carrier frequency of 2.4 GHz was assumed.

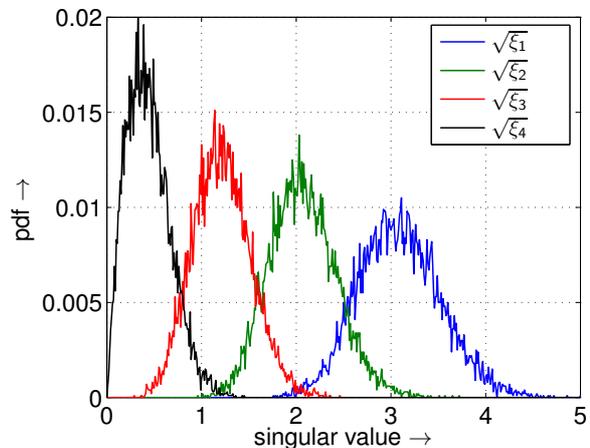


Fig. 5. PDF (probability density function) of the layer-specific amplitudes  $\sqrt{\xi_\ell}$  for uncorrelated ( $4 \times 4$ ) MIMO channels.

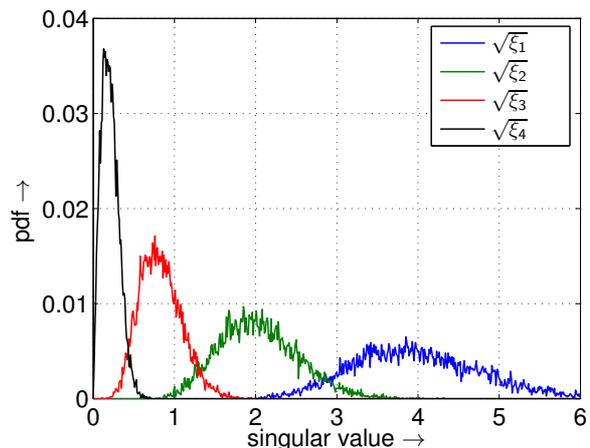


Fig. 6. PDF (probability density function) of the layer-specific amplitudes  $\sqrt{\xi_\ell}$  for correlated ( $4 \times 4$ ) MIMO channels.

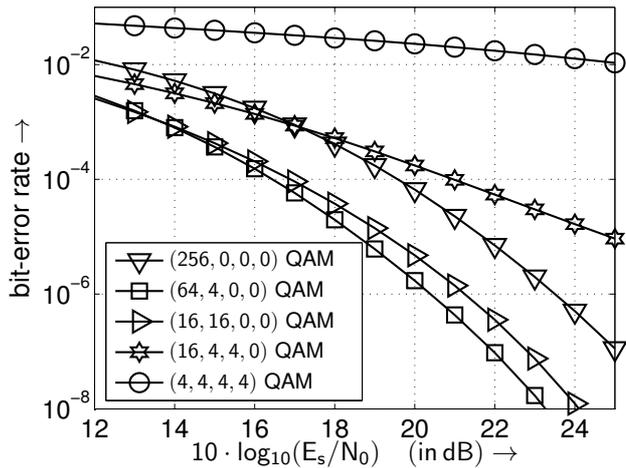


Fig. 7. BER when using the transmission modes introduced in Tab. I and transmitting 8 bit/s/Hz over correlated non-frequency selective channels.

Thus, taking the correlated MIMO channel instead of the uncorrelated one into consideration, we observe that the influence of the layer with the largest weighting factor increases. Since the performance of a MIMO transmission is strongly affected by the smallest singular values of the channel matrix, the statistical distribution of the smallest singular values is of great importance for the characterization of a MIMO transmission scheme [28]. In consequence, as the ratio between the largest and smaller singular-value increases as the correlation between antennas increases, it is expected that the resulting BER increases with respect to the uncorrelated case. Thus, since the pdf dispersion of the singular values changes, the best uncoded solution for the uncorrelated MIMO channel doesn't necessarily lead to the best solution for the correlated one. The obtained BER curves are depicted in Fig. 7 for the different QAM constellation sizes and MIMO configurations of Tab. I, when transmitting at a bandwidth efficiency of 8 bit/s/Hz. Here, the joint optimization of the number of activated MIMO layers along with the number of bits per symbol allows us to minimize the overall BER under the constraint of a given fixed data throughput efficiently.

Besides, the joint optimization of the number of activated MIMO layers as well as the number of bits per symbol was found to be effective at high SNRs. However, iterative receivers are able to work in a much lower SNR region. Therefore it would be interesting to see how the design criteria change when coding is added to the transmission system.

Using the half-rate, constraint-length  $K_{cl} = 3$  NSC code with the generator polynomials of  $(7, 5)$  in octal notation, the BER performance is analyzed for an effective throughput of 4 bit/s/Hz based on the best uncoded schemes of Table I. In addition to the number of bits per symbol and the number of activated MIMO layers, the achievable performance of the iterative decoder is substantially affected by the specific mapping of the bits to both the QAM symbols as well as to the MIMO layers. While the employment of the classic Gray-mapping is appropriate in the absence of *a priori* information, the availability of *a priori* information in iterative receivers

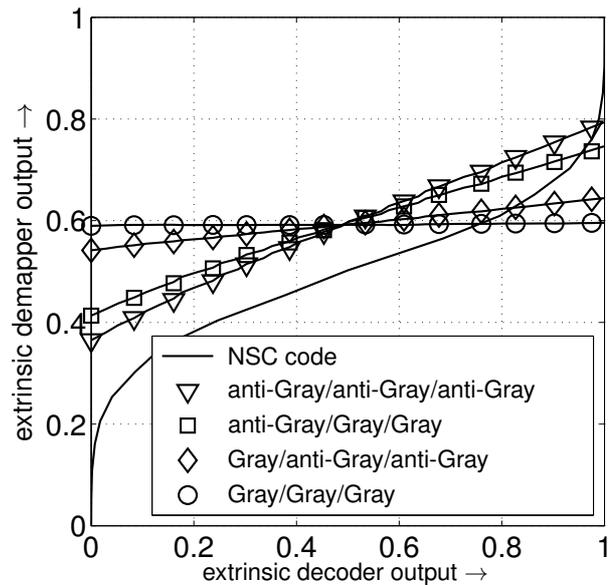


Fig. 8. EXIT chart for an effective throughput of 4 bit/s/Hz and the  $(16, 4, 4, 0)$  QAM constellation at  $10 \log_{10}(E_s/N_0) = 2$  dB (non-frequency selective uncorrelated Rayleigh channels).

requires an exhaustive search for finding the best non-Gray – synonymously also referred to anti-Gray – mapping scheme [13].

A mapping scheme optimized for perfect *a priori* information has usually a poor performance, when there is no *a priori* information. However, when applying iterative demapping and decoding, large gains can be achieved as long as the reliability of the *a priori* information increases upon increasing the number of iterations. As depicted in Fig. 8, the maximum iteration gain can only be guaranteed, if anti-Gray mapping is used on all activated MIMO layers. At the first iteration, using anti-Gray mapping on all MIMO layers results in a lower extrinsic demapper output, compared with layer-specific or Gray mapping schemes (e. g. layer 1: anti-Gray, layer 2 and 3: Gray). However, anti-Gray mapping on all MIMO layers outperforms layer-specific mapping strategies for high *a priori* information. Furthermore, observed by comparing the EXIT chart results of Fig. 8, the overall performance is strongly influenced by the most susceptible MIMO layer, which is here the MIMO layer transmitting 4 bit/s/Hz. Finally, the BER performance is characterized in Fig. 9 based on the best uncoded schemes of Table I. The information word length is 3000 bits and a random interleaver is applied. The influence of the Gray versus anti-Gray mapping is clearly visible in Fig. 9.

Further improvements in terms of the BER are possible by using unequal power allocation. However, as shown in [23] and [15], unequal power allocation in combination with the joint optimization of the number of activated MIMO layers as well as the number of bits per symbol was found to be effective at high SNRs. However, iterative receivers are able to work in a much lower SNR region, where a power allocation scheme was found to be inefficient.

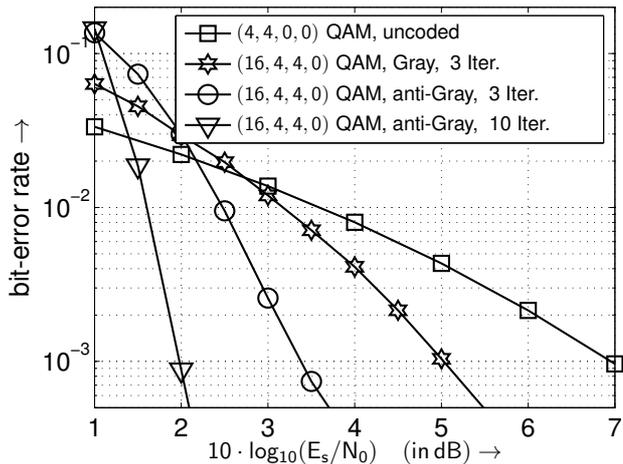


Fig. 9. BERs assuming Gray or anti-Gray mapping schemes on all activated MIMO layers for an effective user throughput of 4 bit/s/Hz (non-frequency selective uncorrelated Rayleigh channels).

TABLE III

Probability of Choosing User-specific Transmission Modes ( $K = 2$ ) at a Fixed Data Rate by Using Optimal Bitloading ( $10 \cdot \log_{10}(E_s/N_0) = 10$  dB, Non-frequency Selective Uncorrelated Rayleigh Channels)

mode	(64, 4, 0, 0)	(16, 16, 0, 0)	(16, 4, 4, 0)	(4, 4, 4, 4)
pdf	0	0.0102	0.9524	0.0374

### B. Multiuser System

The parameters of the analyzed two-users MIMO system are chosen as follows<sup>2</sup>:  $P_{s,k} = 1 V^2$ ,  $n_{R,k} = 4$  (with  $k = 1, 2$ ),  $K = 2$ ,  $n_R = n_T = 8$ . The obtained user-specific BER curves are depicted in Fig. 10 for the different QAM constellation sizes and MIMO configurations of Table I. Assuming a uniform distribution of the transmit power over the number of activated MIMO layers, it still turns out that not all MIMO layers have to be activated in order to achieve the best BERs. This can still be confirmed by analyzing the probability of choosing user-specific transmission modes within the multiuser DL MIMO system by using optimal bitloading [26], as depicted in Table III. However, based on the higher total throughput within the given bandwidth compared to the single-user system, the gap between the different transmission modes becomes smaller.

Using the half-rate, constraint-length  $K_{cl} = 3$  NSC code and comparing the EXIT chart results of Fig. 11, the overall performance is still strongly influenced by the number of activated MIMO layers, suggesting that at low SNR not all MIMO layers has to be activated in order to guarantee an efficient information exchange between the soft-demapper and the corresponding decoder. The user-specific BER performance is given in Fig. 12 and underlines that in order to minimize the overall BER not necessarily all user-specific MIMO layers has to be activated.

<sup>2</sup>In this contribution a power with the dimension (voltage)<sup>2</sup> (in  $V^2$ ) is used. At a real constant resistor this value is proportional to the physical power (in W).

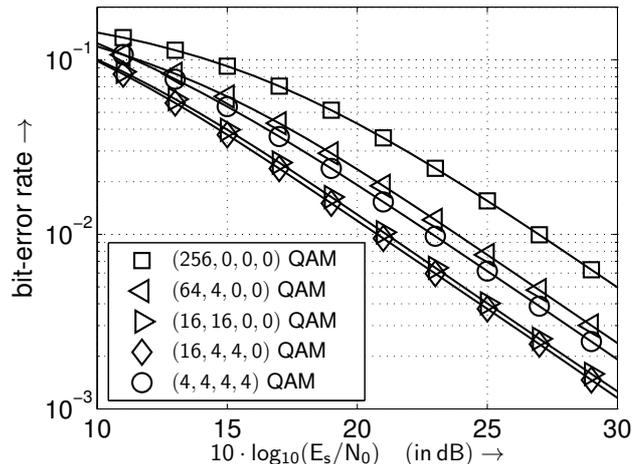


Fig. 10. User-specific BERs when using the transmission modes introduced in Table I and transmitting 8 bit/s/Hz over non-frequency selective uncorrelated Rayleigh channels.

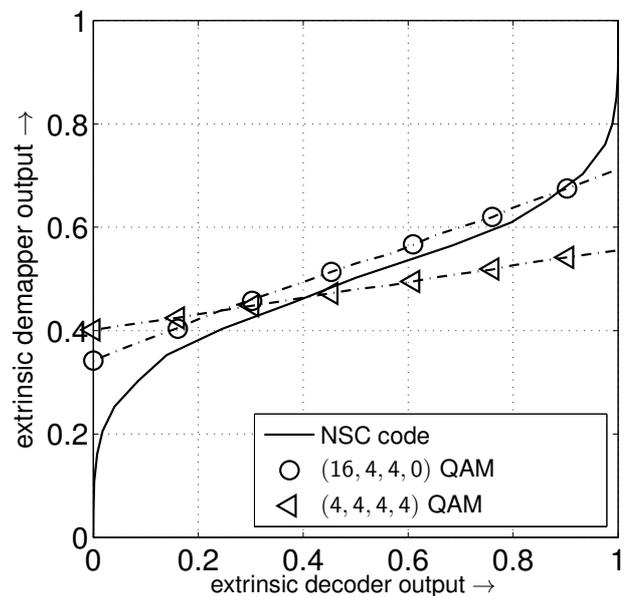


Fig. 11. User-specific EXIT chart for an effective throughput of 4 bit/s/Hz when using anti-Gray mapping on all activated MIMO layers ( $10 \log_{10}(E_s/N_0) = 7$  dB) and the half-rate NSC code with the generator polynomials of (7, 5) in octal notation (non-frequency selective uncorrelated Rayleigh channels).

## VI. CONCLUSION

In analogy to BICIM, we introduced a multi-user MIMO-BICIM scheme, where different user-specific signal constellations and mappings were used within a single codeword. The proposed system includes an adaptation of the transmit parameters. EXIT charts are used for analysing and optimizing the convergence behaviour of iterative demapping and decoding.

The choice of the number of bits per symbol and the number of MIMO layers combined with powerful error correcting codes substantially affects the performance of a MIMO system, suggesting that not all MIMO layers have to be activated

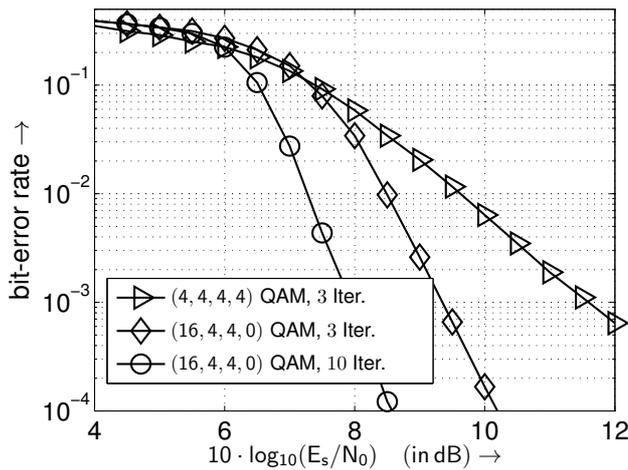


Fig. 12. User-specific BERs for an effective throughput of 4 bit/s/Hz and anti-Gray mapping in combination with different transmission modes and the half-rate NSC code with the generator polynomials of (7, 5) in octal notation (non-frequency selective uncorrelated Rayleigh channels).

in order to achieve the best BERs. Here, anti-Gray mapping on all activated MIMO layers seems to be a promising solution for minimizing the overall BER characteristic.

#### REFERENCES

- [1] E. Telatar, "Capacity of multi-antenna gaussian channels," *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585–595, November/Dezember 1999.
- [2] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multiple antennas," *Bell Labs Technical Journal*, vol. 1, no. 2, pp. 41–59, 1996.
- [3] W. van Etten, "An Optimum Linear Receiver for Multiple Channel Digital Transmission Systems," *IEEE Transactions on Communications*, vol. 23, no. 8, pp. 828–834, 1975.
- [4] M. R. McKay and I. B. Collings, "Capacity and Performance of MIMO-BICM with Zero-Forcing Receivers," *IEEE Transactions on Communications*, vol. 53, no. 1, pp. 74–83, January 2005.
- [5] S. H. Müller-Weinfurter, "Coding Approaches for Multiple Antenna Transmission in Fast Fading and OFDM," *IEEE Transactions on Signal Processing*, vol. 50, no. 10, pp. 2442–2450, October 2002.
- [6] V. Kühn, *Wireless Communications over MIMO Channels – Applications to CDMA and Multiple Antenna Systems*. Chichester: Wiley, 2006.
- [7] L. Zheng and D. N. T. Tse, "Diversity and Multiplexing: A Fundamental Tradeoff in Multiple-Antenna Channels," *IEEE Transactions on Information Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.
- [8] Z. Zhou, B. Vucetic, M. Dohler, and Y. Li, "MIMO Systems with Adaptive Modulation," *IEEE Transactions on Vehicular Technology*, vol. 54, no. 5, pp. 1073–1096, September 2005.
- [9] B. S. Krongold, K. Ramchandran, and D. L. Jones, "Computationally Efficient Optimal Power Allocation Algorithms for Multicarrier Communications Systems," *IEEE Transactions on Communications*, vol. 48, no. 1, pp. 23–27, 2000.
- [10] R. F. H. Fischer and J. B. Huber, "A New Loading Algorithm for Discrete Multitone Modulation," in *IEEE Global Telecommunications Conference (GLOBECOM)*, London, 1996, pp. 724–728.
- [11] C. S. Park and K. B. Lee, "Transmit Power Allocation for BER Performance Improvement in Multicarrier Systems," *IEEE Transactions on Communications*, vol. 52, no. 10, pp. 1658–1663, 2004.
- [12] G. Caire, G. Taricco, and E. Biglieri, "Bit-Interleaved Coded Modulation," *IEEE Transactions on Information Theory*, vol. 44, no. 3, pp. 927–946, March 1998.
- [13] J. A. Chindapol, A. Ritcey, "Design, Analysis, and Performance Evaluation for BICM-ID with square QAM Constellations in Rayleigh Fading Channels," *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 5, pp. 944–957, May 2001.
- [14] F. Schreckenbach and G. Bauch, "Bit-Interleaved Coded Irregular Modulation," *European Transactions on Telecommunications*, vol. 17, no. 2, pp. 269–282, March 2006.
- [15] A. Ahrens and C. Benavente-Peces, "Modulation-Mode and Power Assignment for Broadband MIMO-BICM Schemes," in *IEEE 20th Personal, Indoor and Mobile Radio Communications Symposium (PIMRC)*, Tokyo (Japan), 13–16. September 2009.
- [16] M. Meurer, P. W. Baier, T. Weber, Y. Lu, and A. Papathanassiou, "Joint Transmission: An Advantageous Downlink Concept for CDMA Mobile Radio Systems using Time Division Duplexing," *Electronics Letters*, vol. 36, no. 10, pp. 900–901, May 2000.
- [17] R. L. Choi and R. D. Murch, "A Transmit Preprocessing Technique for Multiuser MIMO Systems using a Decomposition Approach," *IEEE Transactions on Wireless Communications*, vol. 3, no. 1, pp. 20–24, January 2004.
- [18] M. Joham, W. Utschick, and J. A. Nossek, "Linear Transmit Processing in MIMO Communications Systems," *IEEE Transactions on Signal Processing*, vol. 53, no. 8, pp. 2700–2712, August 2005.
- [19] R. L. Choi and R. D. Murch, "New Transmit Schemes and Simplified Receivers for MIMO Wireless Communication Systems," *IEEE Transactions on Wireless Communications*, vol. 2, no. 6, pp. 1217–1230, November 2003.
- [20] W. Liu, L. L. Yang, and L. Hanzo, "SVD Assisted Joint Transmitter and Receiver Design for the Downlink of MIMO Systems," in *IEEE 68th Vehicular Technology Conference (VTC)*, Calgary, 2008, pp. 1–5.
- [21] G. D. Durgin and T. S. Rappaport, "Effects of Multipath Angular Spread on the Spatial Cross-Correlation of Received Voltage Envelopes," in *IEEE Vehicular Technology Conference (VTC)*, Houston, Texas, USA, 16–20 May 1999, pp. 996–1000.
- [22] A. v. Zelst and J. S. Hammerschmidt, "A Single Coefficient Spatial Correlation Model for Multiple-Input Multiple-Output (MIMO) Radio Channels," in *27th General Assembly of the International Union of Radio Science*, Maastricht, 02. August 2002.
- [23] A. Ahrens and C. Lange, "Modulation-Mode and Power Assignment in SVD-equalized MIMO Systems," *Facta Universitatis (Series Electronics and Energetics)*, vol. 21, no. 2, pp. 167–181, August 2008.
- [24] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal Decoding of Linear Codes for Minimizing Symbol Error Rate," *IEEE Transactions on Information Theory*, vol. 20, no. 3, pp. 284–287, March 1974.
- [25] J. G. Proakis, *Digital Communications*. Boston: McGraw-Hill, 2000.
- [26] C. Y. Wong, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser OFDM with Adaptive Subcarrier, Bit, and Power Allocation," *IEEE Journal on Selected Areas in Communications*, vol. 17, no. 10, pp. 1747–1758, October 1999.
- [27] A. Ahrens, S. X. Ng, V. Kühn, and L. Hanzo, "Modulation-Mode Assignment for SVD-Aided and BICM-Assisted Spatial Division Multiplexing," *Physical Communications (PHYCOM)*, vol. 1, no. 1, pp. 60–66, 2008.
- [28] G. Burel, "Statistical Analysis of the Smallest Singular Value in MIMO Transmission Systems," in *Int. Conf. on Signal, Speech and Image Processing (ICOSSIP)*, Skiathos Island, Greece, 25–28 September 2002.
- [29] S. Aust, A. Ahrens, and C. Benavente-Peces, "Modulation-Mode and Power Assignment for SVD- and GMD-assisted Downlink MIMO Systems," in *7th IEEE International Conference on Signals and Electronic Systems (ICSES)*, Gliwice (Poland), 7–10. September 2010, pp. 367–370.
- [30] A. Ahrens and C. Benavente-Peces, "Modulation-Mode and Power Assignment for SVD-assisted and Iteratively Detected Downlink Multiuser MIMO Systems," in *International Conference on Wireless Information Networks and Systems (WINSYS)*, Athens (Greece), 26–28. July 2010, pp. 107–114.
- [31] —, "Modulation-Mode and Power Assignment for SVD-assisted Downlink Multiuser MIMO Systems," in *18th International Conference on Microwave, Radar and Wireless Communications (MIKON)*, Vilnius (Lithuania), 14–16. June 2010, pp. 415–418.