Two Approaches to Network Design Problem for Overlay Multicast with Limited Tree Delay – Model and Optimal Results

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Abstract—Live multimedia streaming and on-demand streaming applications (such as Internet radio or Internet TV) have been gaining more popularity in recent years. They require significant amount of bandwidth from media streaming servers and can easily saturate network infrastructure when the number of participant or bit rate of streaming content increases. Overlay multicast is an effective approach to the problem of streaming distribution. It combines flexibility of application layer multicast with efficiency of network layer multicast. Since overlay networks are built on the top of existing infrastructure, the cost of maintenance and deployment of this solution is relatively low compared to traditional Content Distribution Networks (CDN). Based on our previous works, we focus on solving the overlay network design problem to economically distribute content among the participants using overlay multicast. The optimization goal is to minimize the overlay network cost expressed by the cost of access links. Additionally, we assume that the maximum total delay of a streaming tree is upper bounded to provide QoS (Quality of Service) guarantees. We present two approaches to this problem and construct model using Levels and Flow Conservation Constraints. We show how various constraints following from real overlay systems influence the behavior of the distributing system. In numerical experiments we use real ISPs’ price lists. To illustrate our approach we present optimal results obtained from the CPLEX solver.

Keywords—Overlay network, multicasting, optimization, network design.

I. INTRODUCTION

The content to be disseminated through the overlay system can be divided into two categories: elastic content (e.g. data files) and streaming content with specific bit rate requirements (e.g. media streaming). In this paper we focus on the latter case. Streaming network services have nowadays a significant role in the Internet. Not only aren’t they flouting the artists’ copyright but also have a definite advantage over Internet’s major sharing mechanism in which user can access file only when it has been fully downloaded. This is the reason why different approaches to distributing streaming content for users have been developed. Overlays are built on the top of the existing infrastructure, and therefore they do not require any special transmission infrastructure to be installed. Consequently, the investments and maintenance costs are significantly lower than those of more traditional data distribution systems like IP multicast or Content Distribution Networks (CDNs). Another advantage of overlay systems is a very high service reliability achieved by the lack of single point of failure and the fact that every user acts as both client and server, i.e., each user connected to the overlay can both download and upload the content. Thanks to that system is very scalable unlike in the case of the streaming servers, when for the typical streaming rate of 800 kbps for near-DVD video quality, OC-12 link with 622 Mbps upload is saturated with only 800 users [1]. Note that overlay multicast is named also application-layer multicast or end system multicast [2], [3].

Since content distribution is not only considered at the network layer, the multicast approach can be deployed also in upper layers. This fact has led to the development of end-system approaches, as well as a wide variety of related schemes relevant to peer-to-peer content delivery architectures [4], [5]. Many of these approaches overcome the deployment difficulties faced by IP multicast as they do not require any changes to existing infrastructure, but construct overlay topologies consisting of unicast connecting nodes, and map these topologies onto the underlying physical network.

There are several approaches to implement overlay systems based mostly on the type of distribution graph they use. Tree based overlays implement a tree distribution graph. The root of the tree is a source of content, and each node receives data from its parent. Unlike tree-based overlays, which require a little overhead as packets are forwarded from node to node without extra messages, nodes in a mesh based systems must know, which chunks are owned by its peers, and therefore involves much overhead [6]. An example of the mesh-based system is PPLive, which is the most popular live streaming client. It operates similarly to BitTorrent with extra requirement for the delivery time. Unlike delay tolerant applications such as on-demand streaming or file download clients, where a peer storing the content does not have any restriction on providing the capacity at a specific time, in live streaming the delay is very critical, and the content quickly loses relevance after a few seconds of delay. For more information on various aspects of P2P and multicasting refer to [1]–[22].

There is a growing need for applications that will both stream real time content and retrieve on-demand content. For this reason, there have been many surveys on application layer multicasting [6]. Overlay multicast seems to be the best solution to meet all the requirements without violation of underlying physical core [8]. Among overlay solutions for multicasting, some have taken into consideration P2P streaming applications [3], [7], [9], [14], [18], [19], [22]. This involves delay-efficient, bandwidth-efficient overlay tree
The main contributions of this paper are as follows: (1) MIP formulations of P2P multicasting systems with the following performance metrics: streaming cost, maximum delay and throughput. (2) Numerical experiments on MIP models showing the impact of real system constraints on P2P multicasting. (3) Simulation evaluation of how additional real system constraints influence considered objectives.

This paper is an extended version of the paper [16] presented at IB2Com 2010: Fifth International Conference on Broadband and Biomedical Communications, held in Malaga, Spain, on December 15-17, 2010. This extended paper contains the following new results. (1) A MIP formulation of a Network Design Problem for Overlay Multicast with Limited Tree Delay with the streaming cost objective and a detailed comparison against the formulation given in [16]. (2) A MIP formulation and results (offline optimization and simulations) of the P2P multicasting problem with the goal to minimize the cost of building the network.

The rest of the paper is as follows. Section II presents mathematical formulation of two approaches to the overlay network design problem with limited tree delay. In Section III we present and discuss optimal results we obtained from CPLEX solver and compare two approaches. Finally, last section concludes this work.

II. OVERLAY NETWORK DESIGN PROBLEM

In this section we present two mathematical models of the overlay network design problem with limited P2P streaming cost. As the underlying core network is usually overprovisioned and the only bottlenecks are access links [7], [20], our objective is to select the access link among link types offered by Internet Service Providers. Our model is an overlay tree distribution graph rooted at the source of the content, in which we assume division of the main stream into substreams. Multiple delivery trees are used, each tree carries a different substream. This also prevents establishment of leaf nodes, which do not contribute to the overall distribution. Each node must be connected to all trees to satisfy the requirement of high quality of the downloaded stream. However, the model can be modified to consider a scenario where some nodes receive the streaming of lower quality, i.e., nodes are connected only a subset of substream trees.

Model 1 (level)

Let \( y_{v,k} \) denote a binary decision variable equal to 1, if node \( v \) is connected to overlay network by a link of type \( k \); 0, otherwise. For each access link type offered by a given ISP we know the download capacity (denoted as \( d_{v,k} \)), upload capacity (denoted as \( u_{v,k} \)) and cost (denoted by \( \zeta_{v,k} \)). The second type of decision variables is necessary to construct multicast trees. Let \( x_{wvtl} \) denote a binary decision variable equal to 1 if there is a link from node (peer) \( w \) to node \( v \) in the multicast tree \( t \), and node \( w \) is located on level \( l \) of the tree \( t \); 0 otherwise. Index \( l \) is associated with multiple multicast trees, but if there is only one tree in the network we can ignore this index. We assume that the root node of the tree is located on level 1. All the children of the root (peers...
that have direct link from the root) are located on the level 2, etc. The proposed notation enables us to set the value of $L$ as a limit of the maximal depth of the tree. Following from real systems and having in mind qualities of overlay network, we denote $c_{uv}$ as a delay introduced by a link between the nodes $u$ and $v$. For each peer we are given constants $a_v$ and $b_v$ denoting download and upload traffic respectively, as besides participating in overlay trees they can also use other network services and resources.

indices

$v, w = 1, 2, \ldots, V$ overlay nodes
$k = 1, 2, \ldots, K_v$ access link types for node $v$
$l = 1, 2, \ldots, T$ multicast trees
$l = 1, 2, \ldots, L$ levels of links (uploading nodes) in the multicast tree

constants

$a_v$ download background transfer of node $v$ (kbps)
$b_v$ upload background transfer of node $v$ (kbps)
$\xi_{vk}$ cost of link type $k$ for node $v$ (Euro/Month)
$d_{vk}$ download capacity of link type $k$ for node $v$ (kbps)
$u_{vk}$ upload capacity of link type $k$ for node $v$ (kbps)
$q_t$ streaming rate of the tree $t$ (kbps)
$\gamma_v = 1$, if node $v$ is the root of the tree; 0, otherwise
$\varepsilon_{wv}$ delay introduced by link between nodes $w$ and $v$ (ms)
$M$ large number
$D$ maximum total delay of the tree (ms)

variables

$y_{vk} = 1$, if node $v$ is connected to the overlay network by a link of type $k$; 0, otherwise
$x_{wvt} = 1$, if in multicast tree $t$ there is a link from the node $w$ to the node $v$ and $w$ is located on the level $l$ of the multicast tree $t$; 0, otherwise

objective

It is to minimize the cost of access links of the overlay P2P multicast network:

$$\min F = \sum_v \sum_{k} y_{vk} \xi_{vk}$$

constraints

a) Each node $v = 1, 2, \ldots, V$ – except for the source node of the tree $t$ ($\gamma_v = 1$), for each multicast tree $l = 1, 2, \ldots, T$, must have exactly one parent node:

$$\sum_{w\neq v} \sum_{k} x_{wvt} = (1 - \gamma_v) \quad v = 1, \ldots, V \quad t = 1, \ldots, T$$

b) Node $w$ can be a parent on the first level, only if it is the root node:

$$\sum_{v \neq w} \sum_{k} x_{wvt1} \leq Mr_w \quad w = 1, \ldots, V$$

c) Each node $w = 1, 2, \ldots, V$ cannot be a parent on the level $(l + 1)$ if it is not a child on the level $l$:

$$\sum_{v \neq w} x_{wvt(l+1)} \leq M \sum_{v \neq w} x_{wvtl}$$

$w = 1, 2, \ldots, V \quad t = 1, 2, \ldots, T \quad l = 1, 2, \ldots, L - 1$

d) Only one access link is selected for each node $v = 1, 2, \ldots, V$:

$$\sum_k y_{vk} = 1 \quad v = 1, \ldots, V$$

e) Download capacity constraint – background traffic of node $v$ and streaming rates of all the multicast trees cannot exceed download capacity of node $v = 1, 2, \ldots, V$:

$$a_v + \sum_t q_t \leq \sum_k y_{vk} d_{vk} \quad v = 1, \ldots, V$$

f) Upload capacity constraint – the summary upload transfer of $w$ which follows from the number of children nodes, the streaming rate and the background traffic cannot exceed upload capacity of node $v = 1, 2, \ldots, V$:

$$b_w + \sum_{v \neq w} \sum_{t} x_{wvt} q_t \leq \sum_k y_{vk} u_{vk} \quad w = 1, \ldots, V$$

g) Constraint following from real systems – total delay of each tree cannot be greater than given:

$$\sum_{w} \sum_{v \neq w} \sum_{t} x_{wvt} \varepsilon_{wv} \leq D \quad t = 1, 2, \ldots, T$$

Model 2 (flow)

The following types of decision variables are necessary to construct multicast trees: $x_{wvt}$, $z_{wvt}$. Let $x_{wvt}$ denote continuous decision variable representing streaming rate on a link between nodes $w$ and $v$ in multicast tree $t$. Let $z_{wvt}$ denote binary decision variable equal to 1, if there is a link from node (peer) $w$ to node $v$ (no other nodes in between) in the multicast tree $t$; 0 otherwise. Variable $z_{wvt}$ equals to 1, if there is a path from the root node to node $v$, and it traverses through the link between nodes $w$ and $v$ in the tree $t$; 0, otherwise.

In order to limit the depth of the constructed trees let $L$ represent maximal number of hops from root node to every node in the tree.

indices (additional)

$v, w, e = 1, 2, \ldots, V$ overlay nodes

constants (additional)

$L$ maximal number of hops from root node to every node in the tree

variables (additional)

$z_{wvt} = 1$, if there is a path from the root node to node $v$, and it traverses through the link between nodes $w$ and $v$ in the tree $t$; 0, otherwise

$z_{wvt}$ 1, if link from node $w$ to node $v$ (no other peer nodes in between) is in multicast tree $t$; 0, otherwise

objective

It is to minimize the cost of access links of the overlay P2P multicast network:

$$\min F = \sum_v \sum_{k} y_{vk} \xi_{vk}$$
constraints

a) Each node \( v = 1, 2, \ldots, V \) – except for the source node of the tree \( t \) (\( r_v = 1 \)), for each multicast tree \( t = 1, 2, \ldots, T \), must have exactly one parent node:

\[
\sum_{w \neq v} z_{wvt} = (1 - r_v) \quad v = 1, \ldots, V \quad t = 1, \ldots, T \quad (10)
\]

b) There is a path from root node to node \( e \) traversing through link between nodes \( w \) and \( v \) only if this link exists:

\[
z_{wvt} \leq z_{wve} \quad v, w, e = 1, \ldots, V \quad w \neq v \quad t = 1, \ldots, T \quad (11)
\]

c) Flow conservation control for the nodes being destination node, traversing node and root, respectively:

\[
\sum_{w \neq e} z_{wvt} - \sum_{w \neq v} z_{wve} = 1 \\
v = e \quad e, v = 1, \ldots, V \quad t = 1, \ldots, T \quad (12)
\]

\[
\sum_{w \neq e} z_{wvt} - \sum_{w \neq v} z_{wve} = 0 \\
v \neq e \quad r_v = 0 \quad e, v = 1, \ldots, V \quad t = 1, \ldots, T \quad (13)
\]

\[
\sum_{w \neq v} z_{wvt} - \sum_{w \neq v} z_{wve} = -1 \\
r_v = 1 \quad e, v = 1, \ldots, V \quad t = 1, \ldots, T \quad (14)
\]

d) Only one access link is selected for each node \( v = 1, 2, \ldots, V \):

\[
\sum_{k} y_{vck} = 1 \quad v = 1, \ldots, V \quad (15)
\]

e) Download capacity constraint – background traffic of node \( v \) and streaming rates of all the multicast trees cannot exceed download capacity of node \( v = 1, 2, \ldots, V \):

\[
a_v + \sum_{e} q_{e} \leq \sum_{k} y_{vck} d_{vck} \quad v = 1, \ldots, V \quad (16)
\]

f) Upload capacity constraint – the summary upload transfer of \( w \) which follows from the number of children nodes, the streaming rate and the background traffic cannot exceed upload capacity of node \( v = 1, 2, \ldots, V \):

\[
b_w + \sum_{w \neq v} z_{wve} q_{e} \leq \sum_{k} y_{wck} u_{wck} \quad w = 1, \ldots, V \quad (17)
\]

g) Constraint following from real systems – total delay of each tree cannot be greater than given:

\[
\sum_{w \neq v} z_{wvt} C_{wvt} \leq D \quad t = 1, \ldots, T \quad (18)
\]

h) Total length of hops from root node to every node can’t be greater than the given:

\[
\sum_{w \neq v} z_{wvt} \leq L \quad t = 1, \ldots, T \quad e = 1, \ldots, V \quad (19)
\]

III. RESULTS

In order to solve the model 1 (1)-(8) and model 2 (9)-(19) in optimal way and obtain computation time we used CPLEX 11.0 solver [23]. Our goal was to obtain optimal results in reasonable time of about 2 hour so we decided to test networks consisting of 5-15 overlay nodes (peers). Introducing time limitation lead to CPLEX yielding feasible solution instead of optimal, for some scenarios. We use DSL price lists of three ISPs operating in Poland (Inea, Dialog and UPC – Table I) with prices in Euro/month. Each node is randomly assigned to one of the ISPs and chooses any option included in the price list. The values of download background transfer were selected at random between 512 kilobits per second and 1024 kilobits per second. Analogously, the values of upload background transfer were selected at random between 64 kilobits per second and 128 kilobits per second. The streaming rate was divided proportionally to 1, 2, or 3 multicast trees. We examined trees consisting of 3 – 9 levels, with varying overall streaming rate (sum over all streaming trees) in range from 360 kbps to 2304 kbps and total delay of a tree in the range of 280 – 400 ms. Streaming rate equal to 360 kbps represents low quality content, streaming rate equal to 2304 kbps represents high definition content.

In this section we compare both level and flow formulations in terms of complexity and calculation time. In Table II we report the number of variables and constraints for both models. We can notice that level formulation provides significant reduction in the number of variables and constraints comparing to the flow model.

Fig. 1 shows number of variables as a function of the number of levels and approach type. We can notice that introducing more levels doesn’t influence complexity of the flow model, nevertheless, level model complexity is much lower.

Fig. 2 and 3 depict the number of variables and number of constraints respectively. For small number of trees difference between the complexity of two approaches is minimal, however, when the number of trees increases, complexity of flow model increases dramatically.

Table III shows computation times of optimal solutions for the streaming rate equal to 1080 kbps, total delay of the tree equal to 400 ms, 11 link types and various scenarios of number of trees (1-3), nodes (5,10,15) and levels (2,8). Flow model computation time is much shorter for bigger networks.
than level model, however for small networks level models computation time is lower.

As already proved in [16], introducing more levels and more trees reduces the cost of building the network, and increasing the overall streaming rate increases this cost. In this paper we decided to focus on the computation time of getting optimal solution. Fig. 4 shows computation time as a function of the total delay of the tree and the type of approach. Introducing more levels increases computation time, however, as already stated computation time of flow model is much shorter that level model when the delay increases, and, based on experiments, same thing was observed when introducing more levels and trees. Fig. 5 reports the computation time as a function of the number of levels. Consistent with theoretical considerations presented above, the level model outperforms the flow model in the case of small number of levels. However, with the increase of the level number, the flow model gains some advantage over the level model.

### IV. Conclusion

In this paper we have addressed the problem of network design problem for overlay multicast with limited tree delay. The objective was to minimize the cost of building the system. To solve the formulated problem to optimality we have used CPLEX solver. The results of the numerical experiments confirm what we have already proved in [16], namely, introducing more trees and more levels reduces the cost of building the network, and increasing overall streaming rate increases this cost. In this paper we show that flow model is more complex than level model, but computation time for bigger networks is lower.

In future work we plan to develop effective heuristic algorithms to solve the presented problem for much larger networks in terms of the number of users and trees.
REFERENCES