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Two Approaches to Network Design Problem for Overlay Multicast with Limited Tree Delay – Model and Optimal Results

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***Abstract*—Live multimedia streaming and on-demand streaming applications (such as Internet radio or Internet TV) have been gaining more popularity in recent years. They require significant amount of bandwidth from media streaming servers and can easily saturate network infrastructure when the number of participant or bit rate of streaming content increases. Overlay multicast is an effective approach to the problem of streaming distribution. It combines flexibility of application layer multicast with efficiency of network layer multicast. Since overlay networks are built on the top of existing infrastructure, the cost of maintenance and deployment of this solution is relatively low compared to traditional Content Distribution Networks (CDN). Based on our previous works, we focus on solving the overlay network design problem to economically distribute content among the participants using overlay multicast. The optimization goal is to minimize the overlay network cost expressed by the cost of access links. Additionally, we assume that the maximum total delay of a streaming tree is upper bounded to provide QoS (Quality of Service) guarantees. We present two approaches to this problem and construct model using Levels and Flow Conservation Constraints. We show how various constraints following from real overlay systems influence the behavior of the distributing system. In numerical experiments we use real ISPs' price lists. To illustrate our approach we present optimal results obtained from the CPLEX solver.**

***Keywords*—Overlay network, multicasting, optimization, network design.**

I. Introduction

THE content to be disseminated through the overlay system can be divided into two categories: elastic content (e.g. data files) and streaming content with specific bit rate requirements (e.g. media streaming). In this paper we focus on the latter case. Streaming network services have nowadays a significant role in the Internet. Not only aren't they flouting the artists' copyright but also have a definite advantage over Internet's major sharing mechanism in which user can access file only when it has been fully downloaded. This is the reason why different approaches to distributing streaming content for users have been developed. Overlays are built on the top of the existing infrastructure, and therefore they do not require any special transmission infrastructure to be installed. Consequently, the investments and maintenance costs are significantly lower than those of more traditional data distribution systems like IP multicast or Content Distribution Networks (CDNs). Another advantage of overlay systems is a very high service reliability achieved by the lack of single point of failure and the fact that every user acts as both client and server, i.e., each user connected to the overlay can both download and upload the content. Thanks to that system is very scalable unlike in the case of the streaming servers, when for the typical streaming rate of 800 kbps for near-DVD video quality, OC-12 link with 622 Mbps upload is saturated with only 800 users [1]. Note that overlay multicast is named also application-layer multicast or end system multicast [2], [3].

Since content distribution is not only considered at the network layer, the multicast approach can be deployed also in upper layers. This fact has led to the development of end-system approaches, as well as a wide variety of related schemes relevant to peer-to-peer content delivery architectures [4], [5]. Many of these approaches overcome the deployment difficulties faced by IP multicast as they do not require any changes to existing infrastructure, but construct overlay topologies consisting of unicast connecting nodes, and map these topologies onto the underlying physical network.

There are several approaches to implement overlay systems based mostly on the type of distribution graph they use. Tree based overlays implement a tree distribution graph. The root of the tree is a source of content, and each node receives data from its parent. Unlike tree-based overlays, which require a little overhead as packets are forwarded from node to node without extra messages, nodes in a mesh based systems must know, which chunks are owned by its peers, and therefore involves much overhead [6]. An example of the mesh-based system is PPLive, which is the most popular live streaming client. It operates similarly to BitTorent with extra requirement for the delivery time. Unlike delay tolerant applications such as on-demand streaming or file download clients, where a peer storing the content does not have any restriction on providing the capacity at a specific time, in live streaming the delay is very critical, and the content quickly loses relevance after a few seconds of delay. For more information on various aspects of P2P and multicasting refer to [1]–[22].

There is a growing need for applications that will both stream real time content and retrieve on-demand content. For this reason, there have been many surveys on application layer multicasting [6]. Overlay multicast seems to be the best solution to meet all the requirements without violation of underlying physical core [8]. Among overlay solutions for multicasting, some have taken into consideration P2P streaming applications [3], [7], [9], [14], [18], [19], [22]. This involves delay-efficient, bandwidth-efficient overlay tree

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construction combined with optimal resource allocation approaches and leads to optimizing the overall receiving rate in the multicast group as objective. Wu and Li in [3] and Akbari, Rabiee and Ghanabari in [7] take into consideration bandwidth allocation algorithms, which recognize bandwidth on each link, therefore they guarantee meeting the streaming bandwidth demand. They assume that this constraint is always satisfied and do not allow creating your own access link by combining links offered by different ISPs. Most of approaches focus on the problem how to construct a overlay multicast topology and assign streaming rates to created trees when overlay network is given, i.e. links connecting peers to overlay networks are given. Authors of [12] and [15] take into account also dimensioning of access links, however the proposed method is based only on simulations and concerns fixed trees, i.e., multicast trees are computer based on the given routing algorithm for all the trees, then by summing up the volume on each tree, the amount of bandwidth to be leased on every overlay link is determined.

In this paper we present two approaches to the problem of designing network in overlay multicast and investigate impact on the optimal results introduced by constraints and factors following from real systems. We create two models, one using the *level* approach introduced by Walkowiak in [19], and the other one using *flow conservation constraints*. We compare number of variables necessary in both models and time required to compute optimal solutions. We assume that the overlay multicast is applied for relatively static applications with low membership change rate, e.g., videoconferencing, personal video broadcast in small groups, distance learning, collaborated workgroup, delivery of important messages (stocks, weather forecast, emergency alerts) [2]. This means that participants of the overlay system are stable connected and there are not dynamic changes of the structure as in file-sharing P2P systems. Similarly to [19], we propose a model that involves creation of multiple trees to satisfy fairness, utility and performance requirement. Same as in ChunkySpread [17], fairness means that each peer should transmit at least the same volume it receives, utility involves nodes with more available capacity to transmit more content, whilst performance requires that any node should not be a bottleneck. For the given streaming rate we want to determine how much resource capacity is needed for each peer and how to economically distribute the streaming content in the overlay network using overlay multicast. The former goal consists in selection of one access link type among options proposed by the ISP conducted by a given peer. The latter goal is to construct the P2P multicast trees in the overlay topology, subject to capacity and streaming cost constraints. The overall objective of the proposed problem is to minimize the cost of the network, i.e., the sum of all access link costs expressed e.g. in Euro/month. Comparing to our previous work [19], we add to the model a new constraint on the maximum limit of the streaming delay. It should be noted that since overlay multicast networks are built on top of a general Internet unicast infrastructure rather than point-to-point links, the problem of overlay network design is somewhat different than in networks that do have their own links [15].

The main contributions of this paper are as follows: (1) MIP formulations of P2P multicasting systems with the following performance metrics: streaming cost, maximum delay and throughput. (2) Numerical experiments on MIP models showing the impact of real system constraints on P2P multicasting. (3) Simulation evaluation of how additional real system constraints influence considered objectives.

This paper is an extended version of the paper [16] presented at IB2Com 2010: Fifth International Conference on Broadband and Biomedical Communications, held in Malaga, Spain, on December 15-17, 2010. This extended paper contains the following new results. (1) A MIP formulation of a Network Design Problem for Overlay Multicast with Limited Tree Delay with the streaming cost objective and a detailed comparison against the formulation given in [16]. (2) A MIP formulation and results (offline optimization and simulations) of the P2P multicasting problem with the goal to minimize the cost of building the network.

The rest of the paper is as follows. Section II presents mathematical formulation of two approaches to the overlay network design problem with limited tree delay. In Section III we present and discuss optimal results we obtained from CPLEX solver and compare two approaches. Finally, last section concludes this work.

II. Overlay Network Design Problem

In this section we present two mathematical models of the overlay network design problem with limited P2P streaming cost. As the underlying core network is usually overprovisioned and the only bottlenecks are access links [7], [20], our objective is to select the access link among link types offered by Internet Service Providers. Our model is an overlay tree distribution graph rooted at the source of the content, in which we assume division of the main stream into substreams. Multiple delivery trees are used, each tree carries a different substream. This also prevents establishment of leaf nodes, which do not contribute to the overall distribution. Each node must be connected to all trees to satisfy the requirement of high quality of the downloaded stream. However, the model can be modified to consider a scenario where some nodes receive the streaming of lower quality, i.e., nodes are connected only a subset of substream trees.

Model 1 (*level*)

Let y_{vk} denote a binary decision variable equal to 1, if node v is connected to overlay network by a link of type k; 0, otherwise. For each access link type offered by a given ISP we know the download capacity (denoted as d_{vk}), upload capacity (denoted as u_{vk}) and cost (denoted by ξ_{vk}). The second type of decision variables is necessary to construct multicast trees. Let x_{wvtl} denote a binary decision variable equal to 1 if there is a link from node (peer) w to node v in the multicast tree t, and node w is located on level l of the tree t; 0 otherwise. Index t is associated with multiple multicast trees, but if there is only one tree in the network we can ignore this index. We assume that the root node of the tree is located on level 1. All the children of the root (peers that have direct link from the root) are located on the level 2, etc. The proposed notation enables us to set the value of L as a limit of the maximal depth of the tree. Following from real systems and having in mind qualities of overlay network, we denote c_{wv} as a delay introduced by a link between the nodes w and v. For each peer we are given constants a_v and b_v denoting download and upload traffic respectively, as besides participating in overlay trees they can also use other network services and resources.

indices

$v, w = 1, 2, \ldots, V$	overlay nodes
$k = 1, 2, \ldots, K_v$	access link types for node v
$t = 1, 2, \ldots, T$	multicast trees
$l = 1, 2, \ldots, L$	levels of links (uploading nodes) in the multicast tree

constants

a_v	download background transfer of node v (kbps)
b_v	upload background transfer of node v (kbps)
ξ_{vk}	cost of link type k for node v (Euro/Month)
d_{vk}	download capacity of link type k for node v (kbps)
u_{vk}	upload capacity of link type k for node v (kbps)
q_t	streaming rate of the tree t (kbps)
r_v	$= 1,$ if node v is the root of the tree; 0, otherwise
c_{wv}	delay introduced by link between nodes w and v (ms)
M	large number
D	maximum total delay of the tree (ms)

variables

y_{vk}	$= 1,$ if node v is connected to the overlay network by a link of type k; 0, otherwise
x_{wvtl}	$= 1,$ if in multicast tree t there is a link from the node w to the node v and w is located on the level l of the multicast tree t; 0, otherwise

objective
It is to minimize the cost of access links of the overlay P2P multicast network:

$$\min F = \Sigma_v \Sigma_k y_{vk} \xi_{vk} \quad (1)$$

constraints

a) Each node $v = 1, 2, \ldots, V$ – except for the source node of the tree t ($r_v = 1$), for each multicast tree $t = 1, 2, \ldots, T$, must have exactly one parent node:

$$\Sigma_{w \neq v} \Sigma_l x_{wvtl} = (1 - r_v) \quad v = 1, \ldots, V \quad t = 1, \ldots, T \quad (2)$$

b) Node w can be a parent on the first level, only if it is the root node:

$$\Sigma_{v \neq w} \Sigma_t x_{wvt1} \leq M r_w \quad w = 1, \ldots, V \quad (3)$$

c) Each node $w = 1, 2, \ldots, V$ cannot be a parent on the level $(l + 1)$ if it is not a child on the level l:

$$\Sigma_{v \neq w} x_{wvt(l+1)} \leq M \Sigma_{v \neq w} x_{vwtl} \quad (4)$$

$$w = 1, 2, \ldots, V \quad t = 1, 2, \ldots, T \quad l = 1, 2, \ldots, L - 1$$

d) Only one access link is selected for each node $v = 1, 2, \ldots, V$:

$$\Sigma_k y_{vk} = 1 \qquad v = 1, \ldots, V \quad (5)$$

e) Download capacity constraint – background traffic of node v and streaming rates of all the multicast trees cannot exceed download capacity of node $v = 1, 2, \ldots, V$:

$$a_v + \Sigma_t q_t \leq \Sigma_k y_{vk} d_{vk} \quad v = 1, \ldots, V \quad (6)$$

f) Upload capacity constraint – the summary upload transfer of w which follows from the number of children nodes, the streaming rate and the background traffic cannot exceed upload capacity of node $v = 1, 2, \ldots, V$:

$$b_w + \Sigma_{v \neq w} \Sigma_l \Sigma_t x_{wvtl} q_t \leq \Sigma_k y_{wk} u_{wk} \quad w = 1, \ldots, V \quad (7)$$

g) Constraint following from real systems – total delay of each tree cannot be greater than given:

$$\Sigma_w \Sigma_{v \neq w} \Sigma_l x_{wvtl} c_{wv} \leq D \quad t = 1, 2, \ldots, T \quad (8)$$

Model 2 (flow)

The following types of decision variables are necessary to construct multicast trees: x_{wvt}, z_{wvet}, z_{wvt}. Let x_{wvt} denote continuous decision variable representing streaming rate on a link between nodes w and v in multicast tree t. Let z_{wvt} denote binary decision variable equal to 1, if there is a link from node (peer) w to node v (no other nodes in between) in the multicast tree t; 0 otherwise. Variable z_{wvet} equals to 1, if there is a path from the root node to node e, and it traverses through the link between nodes w and v in the tree t; 0, otherwise.

In order to limit the depth of the constructed trees let L represent maximal number of hops from root node to every node in the tree.

indices (additional)

$v, w, e = 1, 2, \ldots, V$ overlay nodes

constants (additional)

L	maximal number of hops from root node to every node in the tree

variables (additional)

z_{wvet}	$= 1,$ if there is a path from the root node to node $e,$ and it traverses through the link between nodes w and v in the tree t; 0, otherwise
z_{wvt}	1, if link from node w to node v (no other peer nodes in between) is in multicast tree t; 0, otherwise

objective

It is to minimize the cost of access links of the overlay P2P multicast network:

$$\min F = \Sigma_v \Sigma_k y_{vk} \xi_{vk} \quad (9)$$

constraints

a) Each node $v = 1, 2, \ldots, V$ – except for the source node of the tree t ($r_v = 1$), for each multicast tree $t = 1, 2, \ldots, T$, must have exactly one parent node:

$$\Sigma_{w \neq v} z_{wvt} = (1 - r_v) \quad v = 1, \ldots, V \quad t = 1, \ldots, T \quad (10)$$

b) There is a path from root node to node e traversing through link between nodes w and v only if this link exists:

$$z_{wvet} \leq z_{wvt} \quad v, w, e = 1, \ldots, V \quad w \neq v \quad t = 1, \ldots, T \quad (11)$$

c) Flow conservation control for the nodes being destination node, traversing node and root, respectively:

$$\Sigma_{w \neq v} z_{wvet} - \Sigma_w z_{vwet} = 1$$

$$v = e \quad e,\ v = 1, \ldots, V \quad t = 1, \ldots, T \quad (12)$$

$$\Sigma_{w \neq v} z_{wvet} - \Sigma_w z_{vwet} = 0$$

$$v \neq e, \quad r_v = 0 \quad e,\ v = 1, \ldots, V \quad t = 1, \ldots, T \quad (13)$$

$$\Sigma_{w \neq v} z_{wvet} - \Sigma_w z_{vwet} = -1$$

$$r_v = 1 \quad e,\ v = 1, \ldots, V \quad t = 1, \ldots, T \quad (14)$$

d) Only one access link is selected for each node $v = 1, 2, \ldots, V$:

$$\Sigma_k y_{vk} = 1 \quad v = 1, \ldots, V \quad (15)$$

e) Download capacity constraint – background traffic of node v and streaming rates of all the multicast trees cannot exceed download capacity of node $v = 1, 2, \ldots, V$:

$$a_v + \Sigma_t q_t \leq \Sigma_k y_{vk} d_{vk} \quad v = 1, \ldots, V \quad (16)$$

f) Upload capacity constraint – the summary upload transfer of w which follows from the number of children nodes, the streaming rate and the background traffic cannot exceed upload capacity of node $v = 1, 2, \ldots, V$:

$$b_w + \Sigma_{v \neq w} \Sigma_t z_{wvt} q_t \leq \Sigma_k y_{wk} u_{wk} \quad w = 1, \ldots, V \quad (17)$$

g) Constraint following from real systems – total delay of each tree cannot be greater than given:

$$\Sigma_w \Sigma_{v \neq w} z_{wvt} c_{wv} \leq D \quad t = 1, \ldots, T \quad (18)$$

h) Total length of hops from root node to every node can't be greater than the given:

$$\Sigma_w \Sigma_{v \neq w} z_{wvet} \leq L \quad t = 1, \ldots, T \quad e = 1, \ldots, V \quad (19)$$

TABLE I
INTERNET SERVICE PROVIDERS PRICE LIST

ISP	PRICE [EURO/ MONTH]	Download Limit [kbps]	Upload Limit [kbps]
Inea	18	8192	640
	18	25600	1536
	23	51200	4096
Dialog	9	2048	512
	10	4096	512
	11	10240	640
	13	20480	1024
UPC	14	5120	512
	17	10240	1024
	20	25600	5120
	25	51200	5120

III. RESULTS

In order to solve the model 1 (1)-(8) and model 2 (9)-(19) in optimal way and obtain computation time we used CPLEX 11.0 solver [23]. Our goal was to obtain optimal results in reasonable time of about 2 hour so we decided to test networks consisting of 5-15 overlay nodes (peers). Introducing time limitation lead to CPLEX yielding feasible solution instead of optimal, for some scenarios. We use DSL price lists of three ISPs operating in Poland (Inea, Dialog and UPC – Table I) with prices in Euro/month . Each node is randomly assigned to one of the ISPs and chooses any option included in the price list. The values of download background transfer were selected at random between 512 kilobits per second and 1024 kilobits per second. Analogously, the values of upload background transfer were selected at random between 64 kilobits per second and 128 kilobits per second. The streaming rate was divided proportionally to 1, 2, or 3 multicast trees. We examined trees consisting of 3 – 9 levels, with varying overall streaming rate (sum over all streaming trees) in range from 360 kbps to 2304 kbps and total delay of a tree in the range of 280 – 400 ms. Streaming rate equal to 360 kbps represents low quality content, streaming rate equal to 2304 kbps represents high definition content.

In this section we compare both *level* and *flow* formulations in terms of complexity and calculation time. In Table II we report the number of variables and constraints for both models. We can notice that level formulation provides significant reduction in the number of variables and constraints comparing to the flow model.

Fig. 1 shows number of variables as a function of the number of levels and approach type. We can notice that introducing more levels doesn't influence complexity of the *flow* model, nevertheless, *level* model complexity is much lower.

Fig. 2 and 3 depict the number of variables and number of constraints respectively. For small number of trees difference between the complexity of two approaches is minimal, however, when the number of trees increases, complexity of *flow* model increases dramatically.

Table III shows computation times of optimal solutions for the streaming rate equal to 1080 kbps, total delay of the tree equal to 400 ms, 11 link types and various scenarios of number of trees (1-3), nodes (5,10,15) and levels (2,8). *Flow* model computation time is much shorter for bigger networks

TABLE II
NUMBER OF VARIABLES AND CONSTRAINTS IN LEVEL AND FLOW MODELS

Nodes	Trees	Levels	Link types	Level model		Flow model	
				Variables	Constraints	Variables	Constraints
V	T	L	K	V^2TL+VK	$4V+VT+T+VT(L-1)$	$V^2T(V-T+1)+VK$	$2VT+V^3T+V^2T+3V+T$
5	1	3	11	130	36	180	176
5	1	8	11	255	61	180	176
5	2	3	11	205	52	255	337
5	2	8	11	455	102	255	337
5	3	3	11	280	68	280	498
5	3	8	11	655	143	280	498
10	1	3	11	410	71	1110	1151
10	1	8	11	910	121	1110	1151
10	2	3	11	710	102	1910	2272
10	2	8	11	1710	202	1910	2272
10	3	3	11	1010	133	2510	3393
10	3	8	11	2510	283	2510	3393
15	1	3	11	840	106	3540	3676
15	1	8	11	1965	181	3540	3676
15	2	3	11	1515	152	6465	7307
15	2	8	11	3765	302	6465	7307
15	3	3	11	2190	198	8940	10938
15	3	8	11	5565	423	8940	10938

TABLE III
COMPUTATION TIMES OF OPTIMAL VALUES IN LEVEL AND FLOW MODELS

Nodes	Trees	Levels	Execution time [s]	
			Level model	Flow
5	1	3	0,01	0,09
5	1	8	0,03	0,06
5	2	3	0,08	0,11
5	2	8	0,09	0,10
5	3	3	0,06	0,15
5	3	8	0,09	0,04
10	1	3	0,25	5,20
10	1	8	0,38	4,29
10	2	3	1,04	15,84
10	2	8	5,80	15,87
10	3	3	1,31	15,48
10	3	8	7,53	36,67
15	1	3	1,70	48,16
15	1	8	8,11	13,52
15	2	3	13,65	7188,11
15	2	8	506,06	251,23
15	3	3	264,19	7187,63
15	3	8	7174,31	385,70

than *level* model, however for small networks *level* models computation time is lower.

As already proved in [16], introducing more levels and more trees reduces the cost of building the network, and increasing the overall streaming rate increases this cost. In this paper we decided to focus on the computation time of getting optimal solution. Fig. 4 shows computation time as a function of the total delay of the tree and the type of approach. Introducing more levels increases computation time, however, as already stated computation time of *flow* model is much shorter that *level* model when the delay increases, and, based on experiments, same thing was observed when introducing more levels and trees. Fig. 5 reports the computation time sa a function of the number of levels. Consistent with theoretical considerations presented above, the *level* model outperforms the *flow* model in the case of small number of levels. However, with the increase of the level number, the *flow* model gains some advantage over the *level* model.

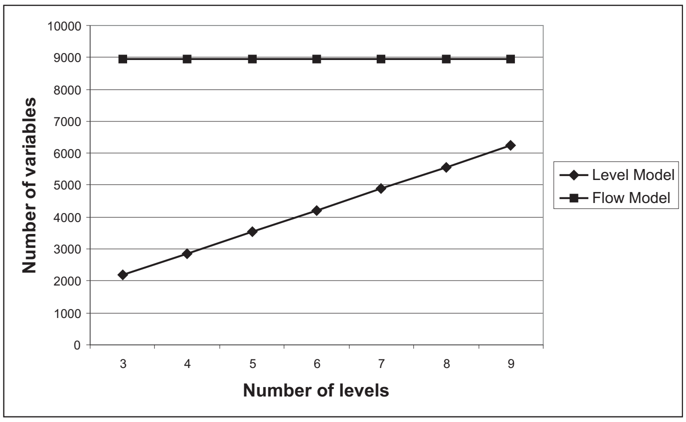

Fig. 1. Number of variables as a function of the number of levels and the type of approach (3 trees, 11 link types).

IV. CONCLUSION

In this paper we have addressed the problem of network design problem for overlay multicast with limited tree delay. The objective was to minimize the cost of building the system. To solve the formulated problem to optimality we have used CPLEX solver. The results of the numerical experiments confirm what we have already proved in [16], namely, introducing more trees and more levels reduces the cost of building the network, and increasing overall streaming rate increases this cost. In this paper we show that *flow* model is more complex than *level* model, but computation time for bigger networks is lower.

In future work we plan to develop effective heuristic algorithms to solve the presented problem for much larger networks in terms of the number of users and trees.

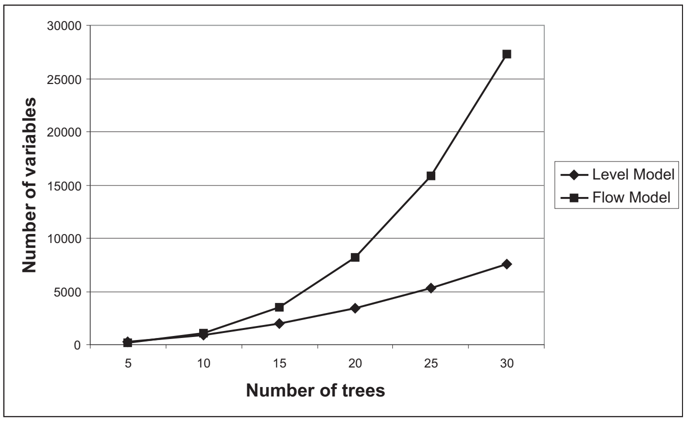

Fig. 2. Number of variables as a function of the number of trees and the type of approach (8 levels, 11 link types).

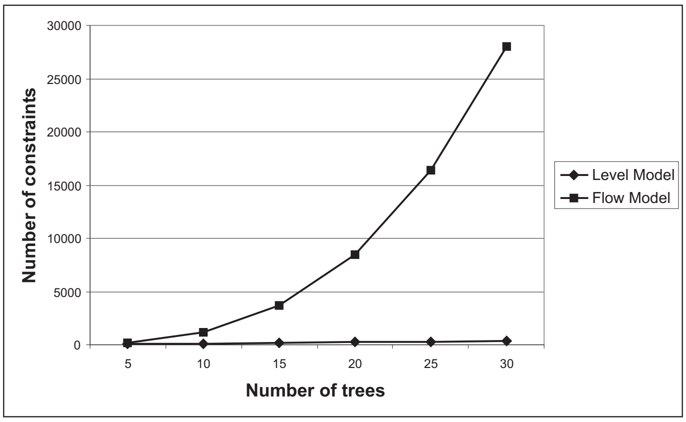

Fig. 3. Number of constraints as a function of the number of trees and the type of approach (8 levels, 11 link types).

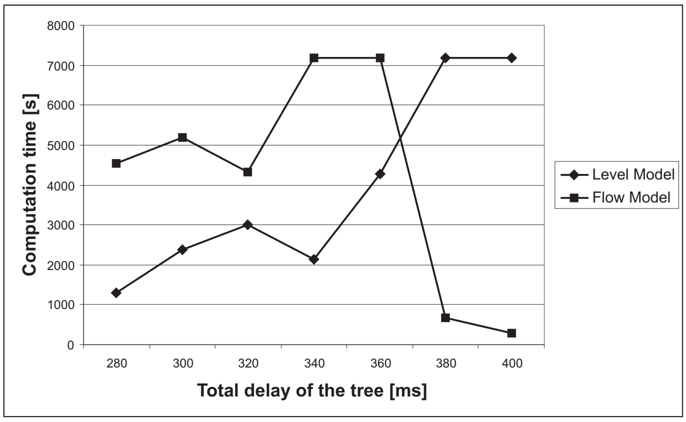

Fig. 4. Computation time as a function of the total delay of the tree and the type of approach (3 trees, 7 levels, streaming rate 2304 kbps).

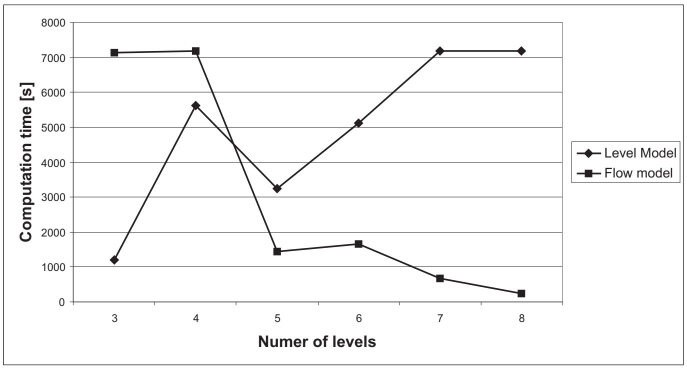

Fig. 5. Computation time as a function of the number of levels (3 trees, streaming rate 2304 kbps, delay limit 380 ms).

References

[1] R. Steinmetz and K. Wehrle, "Peer-to-Peer Systems and Applications," *Lecture Notes in Computer Science*, vol. 3485, 2005.

[2] J. Buford, H. Yu, and E. Lua, *P2P Networking and Applications*. Morgan Kaufmann, 2009.

[3] C. Wu and B. Li, "On Meeting P2P Streaming Bandwidth Demand with Limited Supplies," in *Proceedings of the Fifteenth Annual SPIE/ACM International Conference on Multimedia Computing and Networking*, 2008.

[4] M. Kwon and S. Fahmy, "Path-Aware Overlay Multicast," *Computer Networks*, vol. 47, no. 1, pp. 23–45, January 2005.

[5] S. Ratnasamy, M. Handley, R. Karp, and S. Shenker, "Topologically-Aware Overlay Construction and Server Selection," in *Proceedings of Twenty-First Annual Joint Conference of the IEEE Computer and Communications Societies*, 2002.

[6] T. Small, B. Li, and B. Liang, "Outreach: Peer-to-Peer Topology Construction Towards Minimized Server Bandwidth Costs," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 1, pp. 35–45, 2007.

[7] B. Akbari, H. Rabiee, and M. Ghanabari, "An Optimal Discrete Rate Allocation for Overlay Video Multicasting," *Computer Communications*, no. 31, pp. 551–562, 2008.

[8] S. Banerjee, C. Kommareddy, K. Kar, B. Bhattacharjee, and S. Khuller, "OMNI: An Efficient Overlay Multicast Infrastructure for Real-Time Applications," *Computer Networks: The International Journal of Computer and Telecommunications Networking*, vol. 50, no. 6, pp. 826–841, April 2006.

[9] Y. Cui, Y. Xue, and K. Nahrstedt, "Optimal Resource Allocation in Overlay Multicast," *IEEE Transactions on Parallel and Distributed Systems*, vol. 17, no. 8, pp. 808–823, 2006.

[10] T. Koch and A. Martin, "Solving Steiner Tree Problems in Graphs to Optimality," *Networks*, vol. 32, no. 3, pp. 207–232, 1998.

[11] B. Leuf, *Peer to Peer Collaboration and Sharing over the Internet*. Addison Wesley, 2002.

[12] L. Lao, J. Cui, and M. Gerla, "Multicast Service Overlay Design," *Proceedings of International Symposium on Performance Evaluation of Computer and Telecommunication Systems (SPECTS'05)*, July 2005.

[13] V. Padmanabhan, H. Wang, P. Chou, and K. Sripanidkulchai, "Distributing Streaming Media Content Using Cooperative Networking," Microsoft Research, MSR-TR-2002-37, 2002.

[14] A. Sentinelli, G. Marfia, M. Gerla, L. Kleinrock, and S. Tewari, "Will IPTV Ride the Peer-to-Peer Stream," *IEEE Communications Magazine*, vol. 45, no. 6, pp. 86–92, 2007.

[15] S. Shi and J. Turner, "Multicast Routing and Bandwidth Dimensioning in Overlay Networks," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 8, pp. 1444–1455, 2002.

[16] M. Szostak and K. Walkowiak, "Network Design Problem for Overlay Multicast with Limited Tree Delay – Model and Optimal Results," in *Fifth International Conference on Broadband and Biomedical Communications (IB2Com)*, 2010.

[17] V. Venkatraman and P. Francis, "ChunkySpread Overlay Multicast," *Proceedings of 2nd Symposium on Networked Systems Design and Implementation*, May 2005.

[18] V. Venkataraman, P. Francis, and J. Calandrino, "ChunkySpread: Multitree Unstructured Peer to Peer Multicast," *Proceedings of the Fifth International Workshop on Peer-to-Peer Systems*, 2006.

[19] K. Walkowiak, "Network Design Problem for P2P Multicasting," *International Network Optimization Conference*, 2009.

[20] G. Wu and T. Chiueh, "Peer to Peer File Download and Streaming," *RPE report*, 2005, TR-185.

[21] J. Wu, *Handbook on Theoretical and Algorithmic Aspects of Sensor, Ad Hoc Wireless and Peer-to-Peer Networks*. Aurebach Publications, 2006.

[22] Y. Zhu and B. Li, "Overlay Networks with Linear Capacity Constraints," *IEEE Transactions on Parallel and Distributed Systems*, vol. 19, no. 2, pp. 159–173, 2008.

[23] ILOG CPLEX, "11.0 User's Manual," France, 2007.