

# Shape Independent Coding Using NURBS and Bézier Approximation and Interpolation Curves

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**Abstract**—In this paper a scheme of utilizing shape independent basis functions for the hierarchical multiresolution image compression is shown. For a given image texture region segmentation method is used. Following polygonal approximation of created segments causes a degradation of their boundaries. Using NURBS and Bézier interpolation and approximation segments' boundaries are created, thus achieving an image mask. As an input of the three-level hierarchical encoder this image mask and image are used. The image mask and image are subsampled by a factor of 2 on each level. The hierarchical encoder encodes them shape independently. Especially for a very low bit rate image coding gives better results for objective criteria (PSNR). For segment approximation the 2D shape independent orthogonal transform (DCT II) is used. Splines encoding and decoding is very efficient, because only their control points need to be stored. The segment is coded with a modified code similar to the JPEG code.

**Keywords**—NURBS, Bézier, image compression, segmentation, shape independent transform.

## I. INTRODUCTION

IN recent years, modern communication technologies develop very fast. Transmitted information needs to be displayed to the user. In general, different compression modes can be used – sequential, progressive, and hierarchical. They define the way the most important information from the data is chosen and the order in which it is processed.

Different users use different display devices, ranging from the cell phone displays to the high resolution LCDs, to portray image information. Thus we recognize the need for multiresolution processing which would ensure different resolutions in one bit stream that does not have to be fully decoded to receive lower resolution image. Multiresolution output bit stream is achieved by hierarchical schemes, e.g. [1] used in JPEG. In the hierarchical scheme, an image is coded as a sequence of layers in pyramid. Decoder then processes only pyramid layers necessary to decode to specific resolution. Hence one bit stream contains all the data needed to decode to multiple resolutions.

### A. Prior Work

The scheme is based on region-oriented image coding algorithm employing orthonormal basis functions [2] improved on by segment-oriented shape independent orthogonal transform texture approximation in sequential mode. Progressive and

This work was supported by the Slovak Research Grant Agency (VEGA) under Grant No. 1/0602/11.

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hierarchical modes should be also of our interest. Progressive compression mode processes spectral coefficients from each region in order of their significance. It would require additional information about coefficient's position to be coded and also add complexity to the processing. Thus, we focus only on hierarchical mode. The crucial prerequisite for hierarchical compression mode is the ability to perform interpolation of respective region with chosen basis functions. No significant additional data reduction is expected, but hierarchical compression and transfer mode may be of importance, for specific applications as we outlined above.

## II. NURBS CURVES

A  $p$  th-degree NonUniform Rational B-Spline (NURBS) curve is defined by [3]:

$$C(u) = \frac{\sum_{i=0}^n \omega_i P_i N_{i,p}(u)}{\sum_{i=0}^n \omega_i N_{i,p}(u)}, \quad a \leq u \leq b \quad (1)$$

where the  $P_i$  are the control points, the  $\omega_i$  are the weights and the  $N_{i,p}(u)$  are the  $p$  th-degree B-spline functions defined on the nonperiodic knot vector [3]:

$$U = \{a, \dots, a, u_{p+1}, \dots, u_{m-p-1}, b, \dots, b\} \quad (2)$$

## III. BÉZIER CURVES

A  $n$  th-degree Bézier curve is defined by [3]:

$$C(u) = \sum_{i=0}^n B_{i,n}(u) P_i, \quad 0 \leq u \leq 1 \quad (3)$$

where the  $P_i$  are the control points and the  $B_{i,p}(u)$  are the  $n$  th-degree Bernstein polynomials given by [3]:

$$B_{i,p}(u) = \frac{n!}{i!(n-i)!} u^i (1-u)^{n-i} \quad (4)$$

## IV. PREPROCESSING

Segment-oriented approach has many advantages and therefore the scheme is based on it. Raw image is used as the input of the scheme. It is segmented and segments' boundaries are approximated before it enters hierarchical processing where level coders use shape independent transforms (see Fig. 1).

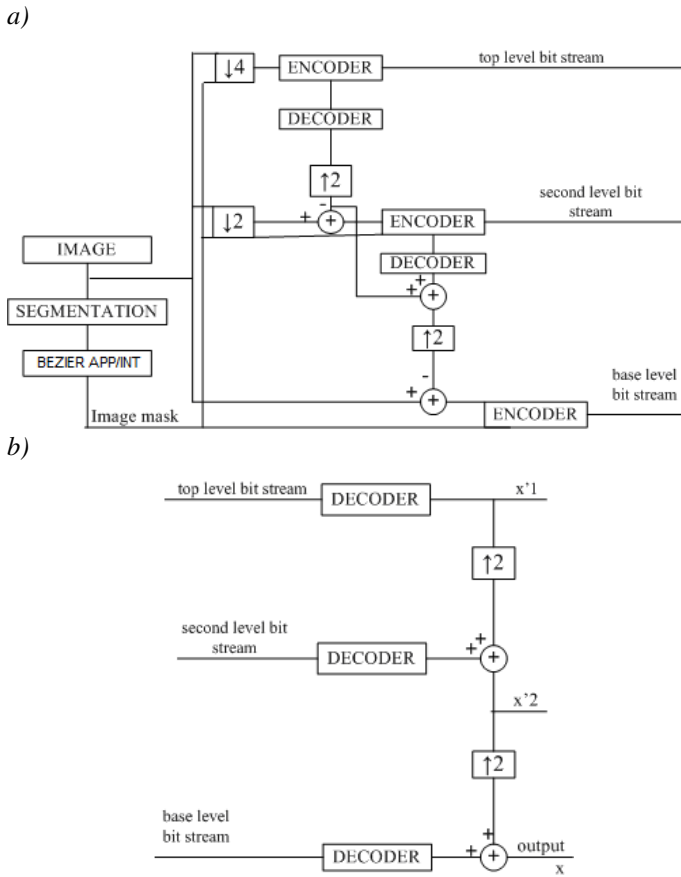


Fig. 1. Block diagram of the three-level hierarchical a) encoder; b) decoder.

A. Segmentation

To partition the image an unsupervised segmentation method for colour – texture regions [4] is used. Segments are found in few steps [5]. First, colours in the image are quantized to several representative classes and image pixels are then replaced by their corresponding colour class labels. In this way a class map is obtained. Class map is solved for so called J-values from a local neighborhood of a pixel. The J-values correspond to the minimum variation of image values in segments. The larger the J-value is the more likely the corresponding pixel is near a segment boundary. Finally, a region is growing and merging. Block diagram of proposed scheme method is applied to the image of J-values to obtain the final segmentation (Fig. 2). The result of segmentation can be an image where each region is defined by its unique gray

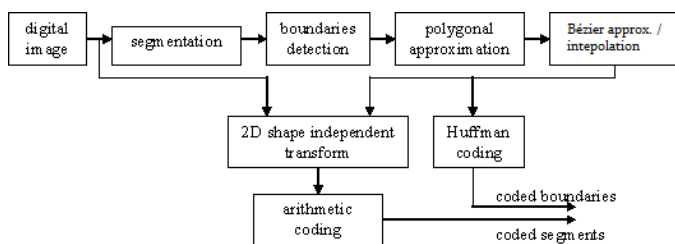


Fig. 2. Block diagram of proposed scheme.

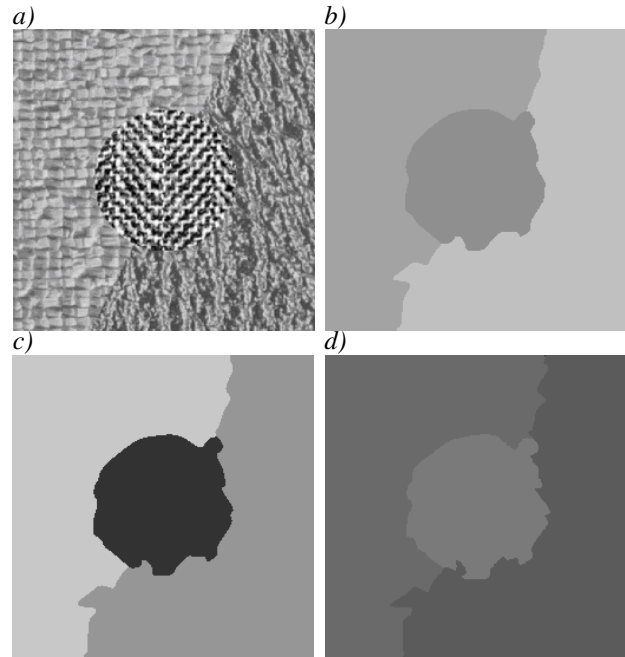


Fig. 3. Image Ring 256x256 pixels, 256 gray levels: a) original, b) image mask (segmented image with NURBS approximation applied); c) image mask (segmented image with Bézier interpolation applied), d) image mask (segmented image with Bézier approximation applied).

value or colour.

The result of unsupervised segmentation [2] is an image where each segment is defined by its unique gray or color value. Segment’s textures and boundaries are processed separately. Boundaries are then approximated or interpolated [6], [7] and thus we get the image mask. Fig.3 shows original image and its masks.

V. HIERARCHICAL ENCODER

As is shown in Fig.1 image and its mask are the input into the hierarchical encoder. Both are subsampled by the factor of 2 and 4 in each dimension on the second and the top level, respectively. Each lower size (higher level) image provides prediction for the next level. Except for the top

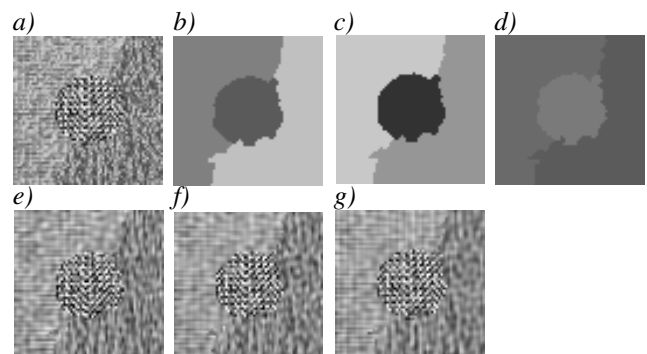


Fig. 4. Top level processing: a) subsampled original image (Fig.3a) by the factor of 4 in each dimension (64x64 pixels), b-d) subsampled image masks (Fig.3b-d) by the factor of 4 in each dimension, e) decoded subsampled image using mask b) (0.071 bpp of original size); f) decoded subsampled image using mask c) (0.071 bpp of original size); g) decoded subsampled image using mask d) (0.069 bpp of original size).

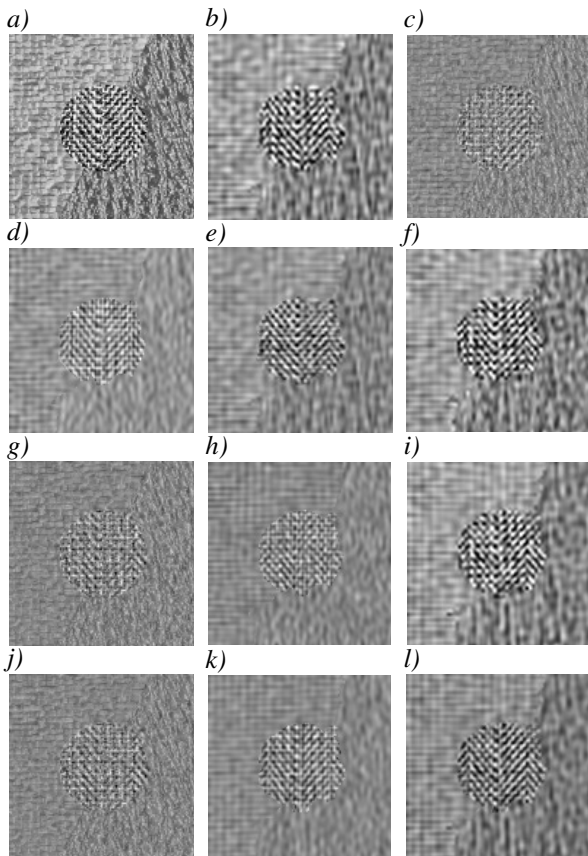


Fig. 5. Second level processing: a) subsampled original image (Fig.3a) by the factor of 2 in each dimension (128 x 128 pixels); NURBS: b) upsampled decoded image from top level (Fig.4e); c) difference between a) and b); d) decoded difference c) (0.064 bpp of original size); e) final result of second level processing – sum of b) and e); Bézier approximation: f) upsampled decoded image from top level (Fig.4f); g) difference between a) and f); h) decoded difference f) (0.058 bpp of original size); i) final result of second level processing – sum of b) and h); Bézier interpolation: j) upsampled decoded image from top level (Fig.4g); k) difference between a) and j); l) decoded difference j) (0.066 bpp of original size); m) final result of second level processing – sum of b) and k).

level of the pyramid, the difference between the source image and the reference reconstructed image is coded. On the top level, subsampled image is coded. The level texture coding uses basis functions of 2D shape independent orthogonal discrete cosine transform II [8]. Shape independent transforms with different basis functions are also possible [9]. Decoder operates in reversed order.

Performance of the proposed scheme with NURBS, Bézier approximation and Bézier interpolation is shown on Fig. 4-8. All methods show the processing on the top level (Fig. 4), the second level (Fig. 5) and the base level (Fig. 6-8).

## VI. RESULTS

To test the proposed method a number of experiments were performed, some of which are presented here. Besides numerical results, examples showing the visual outcome are shown. In order to quantify the performance of the proposed method, we will use objective criteria PSNR. We will compare our method to the JPEG. Finally, reconstructed pictures will be illustrated for subject evaluation (Fig.9.).

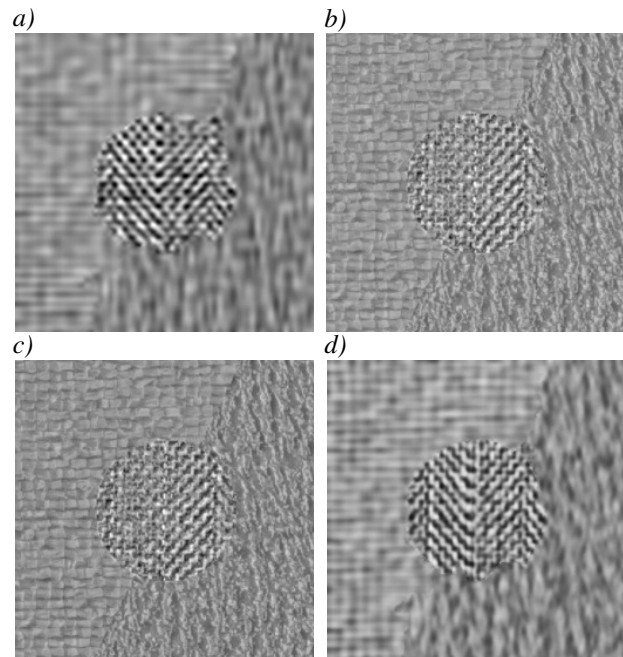


Fig. 6. Base level processing: a) upsampled decoded image from second level (Fig. 5e) using NURBS; b) difference between original (Fig.3a) and a); c) decoded difference b); d) sum of a) and c) – final product of decoding process (0.151 bpp; PSNR 21,056 dB).

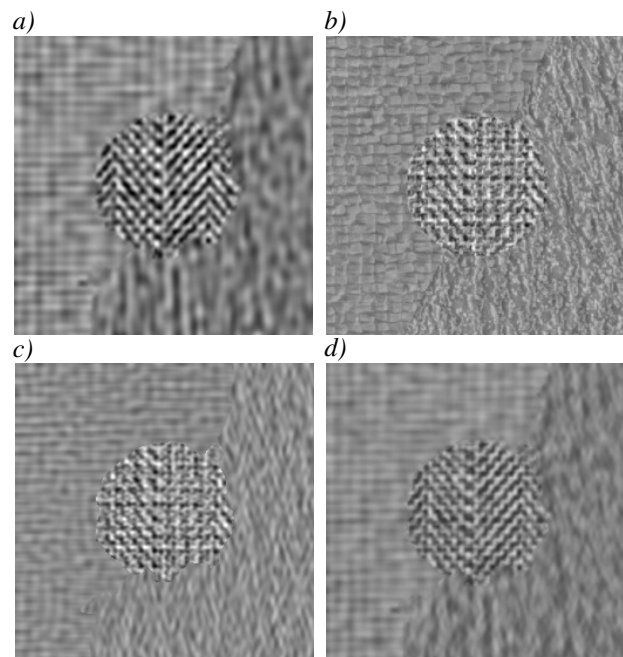


Fig. 7. Base level processing: a) upsampled decoded image from second level (Fig. 5i) using Bézier approximation; b) difference between original (Fig.3a) and a); c) decoded difference b); d) sum of a) and c) – final product of decoding process (0.14 bpp; PSNR 20,556 dB).

In our experiment, various types of pictures (artificial, photographic, and inhomogeneous) were used. In this paper we present only inhomogeneous. Fig.9 depicts the final base level results of NURBS approximation (a), Bézier approximation (b), Bézier interpolation (c) and JPEG (d). Comparison of the PSNR performance of different image coding approaches is shown in Fig.10. Hierarchical Bézier interpolation, approx-

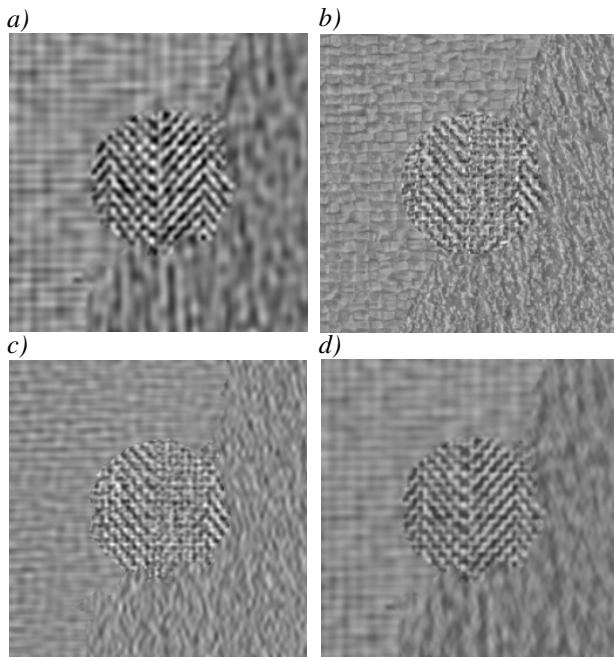


Fig. 8. Base level processing: a) upsampled decoded image from second level (Fig. 51) using Bézier interpolation; b) difference between original (Fig.3a) and a); c) decoded difference b); d) sum of a) and c) – final product of decoding process (0.153 bpp; PSNR 20,267 dB).

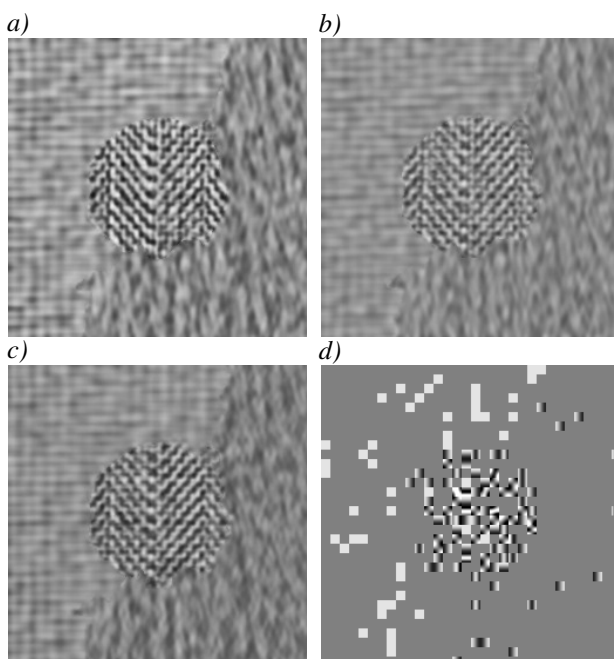


Fig. 9. Base level processing: a) NURBS (0.151 bpp; PSNR 21,056 dB) b) Bézier approximation (0.14 bpp; PSNR 20,556 dB); c) Bézier interpolation (0.153 bpp, PSNR ); d) JPEG (0.153 bpp; PSNR 20,267 dB).

imation and NURBS approximation of segment boundaries have similar results. Visually (Fig.9), there is no perceivable difference between these three models.

## VII. CONCLUSION

Motivated by curves' ability to model natural forms successfully, we decided to use NURBS and Bézier curves in our encoder. The proposed method is very sensitive to the

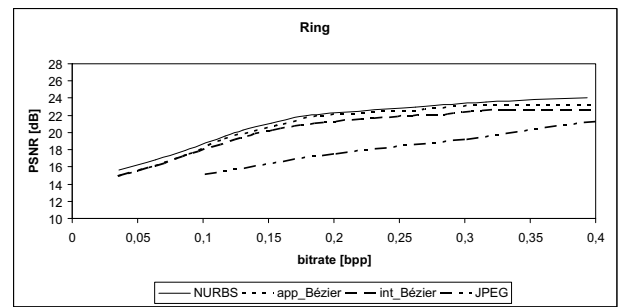


Fig. 10. Diagram of PSNR performance.

quality of segmentation, number of segments, their sizes and degree (depth) of the approximation. Especially, big segments increase the coding time rapidly. For higher number of regions the method is not efficient with respect to the bit rate. An increase of code efficiency is achieved by segment boundary degradation. The scheme introduces additional compression mode of hierarchical coding for shape independent transform. Our goal was to show that this mode is feasible and implementable without loss of efficiency. Initial segmentation of the test image of Ring produced 3 arbitrary shapes. They were processed at different resolutions and overall results were comparable to the sequential mode results of JPEG. Hence, functionality of our scheme was demonstrated.

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