

# Mathematical Model of Calculating Metric Tensor and GNSS-observations Errors Taking into Account Relativistic Effects

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**Abstract**— Study of the trajectories of the motion of satellites remains an urgent task for modern science. This is especially true for GNSS systems and for satellites intended for Earth remote sensing. The basis of their operation is to accurately determine the position of the satellite, and the parameters of signal propagation. Considering the great distances and speeds of both satellites and the Earth in calculating these parameters, it is necessary to take into account the special and general theory of relativity. In the article formulas have been derived for calculating additional corrections for relativistic effects. A mathematical model for calculating the metric tensor was created. A sequence of correction was also proposed.

**Keywords**— GNSS, relativistic effects

## I. INTRODUCTION

ARTIFICIAL Earth satellites are increasingly entering the human life. For most people, this happens unnoticed. They watch television, use satellite communications, GNSS (Global Navigation Satellite System) navigators in cars, Google maps and other services, without much thought about the origin of the information or the signal. The level of satellite use is indicated by the fact that in 2017 a record was set for the simultaneous release of satellites into orbit – 104 satellites. Therefore, the study of the trajectories of the motion of satellites remains an urgent task for modern science. This is especially true for GNSS systems and for satellites intended for Earth remote sensing. After all, the basis of their work is to accurately determine the position of the satellite, and the parameters of signal propagation (GNSS), or information flow from the Earth's surface. Considering the great distances and speeds of both satellites and the Earth in calculating these parameters, it is necessary to take into account the special and general theory of relativity.

## II. ANALYSIS OF PREVIOUS STUDIES

Global navigation satellite systems and Earth remote sensing satellite systems are quite complex and contain many components that are located at great distances [1–6]. Therefore, the accuracy of observations is affected by many corrections for relativistic effects [7–12]. For example, in the work of Puchkov and Sherbaevych [13], the relativistic effects, that affect the corrections are divided into four groups:

1. The Doppler effect of the second order.
2. Gravitational frequency offset.
3. Effects associated with the rotational motion of reference frames.
4. Weak relativistic effects. This is primarily the effect of the delay of the electromagnetic signal in the gravitational field; Dynamic effects that affect the motion in the gravitational fields of various objects, including satellites; effects of the involvement of inertial reference frames by gravitational fields of mobile, including rotating objects; relativistic gravitational effects that affect the value of mutual distances and velocities of objects and some others.

A similar, but somewhat different classification is given by Geyling and Westerman [14]:

1. Three main relativistic effects in GNSS: the effect of constant frequency shift; effect of rotating time delay; Sagnac's delay.
2. Two effects observed in mobile receivers: in receivers on mobile objects; in receivers high above sea level.
3. Four secondary effects: the solar-lunar potential; non-spherical gravity; Shapiro's delay; Lance-Ziering effect.

Thus, there are about ten different relativistic effects affecting the accuracy of GNSS observations. However, the primary cause of all effects is the non-inertiality of reference frames and the influence of the gravitational fields of space objects present in the given system [15, 16]. So, the analysis of the influence of relativistic effects on determining the location under GNSS observations at the present stage has shown that there are some shortcomings, in particular: the rotational motion of the Earth around the Sun is not taken into account, although it affects all the above-mentioned corrections; most corrections take into account only one of the relativistic effects (rotation of the Earth around the axis, the relative motion of the satellite and the Earth, the gravitational fields of the Earth, the Sun, and the Moon), although they can affect the numerical values of each other when combined; for many corrections, simplified relationships are used, by which they are determined.

## III. RESEARCH RESULTS

Let us single out the main corrections for relativistic effects, in which we take into account the influence of these effects as fully as possible.

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First, when the satellite's atomic clock is synchronized with the ground station clock, electromagnetic waves are exchanged between them. Since during synchronization, the satellite and ground station have different spatial coordinates and are in different non-inertial frames of reference, a time correction arises due to the propagation of the synchronization signal in a space with a curved metric [17, 18].

Secondly, when a terrestrial receiver receives a GNSS signal, the satellite is at a different point in space, compared to the synchronization point with other metric parameters, in which the satellite's atomic clock slows down. The same happens when information is received by Earth remote sensing satellites. In this case, a temporal correction occurs, associated with a relativistic change in the atomic clock rate of the satellite.

Thirdly, the Lorentz length is shortened in determining the distance between the ground control station and the satellite.

And fourth, there is a shortening of the Lorentz length in determining the distance between the GNSS receiver and the satellite and between the Earth's surface and the Earth remote sensing satellite.

The first three of these corrections affect the accuracy of determining the satellite's coordinates, and therefore indirectly affect the accuracy of determining the location of the observed object. The fourth of these corrections directly affects the accuracy of observations.

The third and fourth correction is determined by the same mathematical relationship. The difference is that in calculating the third one, the coordinates of the ground control station and the satellite are taken into account, and in calculating the fourth correction, the coordinates of the satellite and the receiver or the surface of the Earth. If we assume that the receiver, like the observation station, is located on the surface of the Earth, then these corrections will be close in magnitude.

Since the reference frame in which the GNSS system is located is non-inertial (moves with acceleration, rotates) and is in a gravitational field, we will determine these corrections within the framework of the general theory of relativity.

In the general theory of relativity, the time interval between any two events at the same point in space is determined by the following formula [17, 19]:

$$\tau = \frac{1}{c} \int \sqrt{q_{00}} dx^0, \quad (1)$$

where  $q_{00}$  is the component of the metric tensor in the Galileo's reference frame,  $x^0$  is the time coordinate,  $c$  is the speed of light in vacuum.

To take into account the time correction of the synchronization of the satellite's atomic clock and the one of the ground control station, we use the following formula [20]:

$$\Delta\tau_{(synch.corr)} = -\frac{1}{c} \sum_{\alpha=1}^3 \int_{[x^{\alpha}(a)]}^{[x^{\alpha}(b)]} \frac{g(8)_{0\alpha}}{g(8)_{00}} d[x(8)^{\alpha}], \quad (2)$$

where  $[x^{\alpha}(a)]$  are the spatial coordinates of the satellite at the point  $a$  at the moment of synchronization;  $[x^{\alpha}(b)]$  are the spatial coordinates of the ground-based control station in the point  $b$ .

We choose the following notation: the coordinates are taken in square brackets, in them the reference frame is denoted in parentheses, the superscript indicates the type of the coordinate. For example,  $a = 0$  corresponds to time coordinate  $ct$ ;  $a = 1, 2, 3$  corresponds to spatial coordinates in the cylindrical coordinate

system,  $R$  is cylindrical radius,  $\varphi$  is cylindrical longitude,  $z$  is axis cylindrical coordinate system respectively;  $g(8)_{00}$ ,  $g(8)_{0\alpha}$  are the components of the metric tensor in the geocentric reference frame  $\{x(8)^{\alpha}\}$  (the four-dimensional metric tensor will be denoted by  $g$ ; the subscripts indicate a specific component of the tensor, the reference frame for which the given tensor is found is indicated in parentheses, the frames of reference will be taken in curly brackets).

The time interval that passed from the moment of synchronization to the transmission of the signal from the satellite to the receiver, according to the satellite clock, is determined by the following formula [17–19]:

$$\tau = \frac{1}{c} \int_{x^0(a)}^{x^0(c)} \sqrt{g(8)_{00}} dx(8)^0 \quad (3)$$

where  $x^0(a)$  is the time coordinate of the satellite at the moment of synchronization,  $x^0(c)$  is the satellite's time coordinate at the time the receiver receives the GNSS signal.

The time correction of the relativistic change in the atomic clock of the satellite can be determined by the formula [18, 21]:

$$\Delta\tau_{(corr)} = \frac{1}{c} \left\{ \int_{[x^0(a)]}^{[x^0(c)]} \left( \sqrt{g(7)_{00}} \right) d[x(7)^0] - \int_{[x^0(a)]}^{[x^0(c)]} \left( \sqrt{g(8)_{00}} \right) d[x(8)^0] \right\}, \quad (4)$$

where  $[x^0(a)]$  is the time coordinate of the satellite at the point  $a$  at the time of synchronization of the atomic clocks,  $[x^0(c)]$  is the satellite's time coordinate at the point  $c$  at the time of observation, in the geocentric reference frame  $\{x(7)^{\alpha}\}$  and in the geocentric satellite reference system  $\{x(8)^{\alpha}\}$ ,  $g(7)_{00}$  is the component of the metric tensor in the geocentric coordinate system, which is connected with the plane of the orbit of the satellite.

The elementary distance between two infinitely near points of space may be calculated by formula [18]:

$$dl^2 = \gamma_{\alpha\beta} dx^{\alpha} dx^{\beta}, \quad (5)$$

where  $\gamma_{\alpha\beta}$  is the three-dimensional metric tensor, which is equal to:

$$\gamma_{\alpha\beta} = -g_{\alpha\beta} + \frac{g_{0\alpha}g_{0\beta}}{g_{00}}, \quad (6)$$

where  $g_{\alpha\beta}$ ,  $g_{00}$ ,  $g_{0\alpha}$ ,  $g_{0\beta}$  are components of four-dimensional metric tensor.  $\alpha$  and  $\beta = 0, 1, 2, 3$  are indices that describe spatial coordinates,  $x^{\alpha}$ ,  $x^{\beta}$  – spatial coordinates.

Having made mathematical transformations and integrating expression (5), and also comparing the magnitude of the pseudo-distance with the analogous value calculated in the reference frame  $\{x(7)^i\}$ , we find the pseudo-distance correction for the relativistic effects:

$$\Delta R = \sum_{\alpha=1}^3 \sum_{\beta=1}^3 \left\{ \int_{[x^{\alpha}(d)]}^{[x^{\alpha}(c)]} \sqrt{\gamma(7)_{\alpha\beta}} \frac{d[x(7)^{\beta}]}{\partial[x(7)^{\alpha}]} d[x(7)^{\alpha}] - \int_{[x^{\alpha}(d)]}^{[x^{\alpha}(c)]} \sqrt{\gamma(8)_{\alpha\beta}} \frac{d[x(8)^{\beta}]}{\partial[x(8)^{\alpha}]} d[x(8)^{\alpha}] \right\} \quad (7)$$

Thus, to find the corrections for relativistic effects, we need to find the components of the metric tensor in the geocentric reference frame and in the reference frame associated with the satellite.

As an inertial Galilean reference frame, we select the Galactic reference frame [21, 22]. We will assume that in this frame of reference the pseudo-Euclidean space-time is realized. Gradual transition from the galactic inertial frame of reference to the geocentric rotating, associated with the satellite, we find the metric tensor needed to determine the corrections for relativistic effects.

The frames of reference will be considered in this sequence [21]: The first frame of reference is the Galactic inertial reference frame, which we assume to be fixed relative to the distant stars. The coordinates of this reference frame are denoted by the symbols  $x(1)^i$ , and the metric tensor  $g(1)_{lm}$  respectively; Next will be the Galactic reference frame, which rotates with the Sun. It will be non-inertial, its coordinates will be denoted by the symbols  $x(2)^i$ , and metric tensor  $g(2)_{ik}$  respectively. In the third stage, we consider the transition to a heliocentric rotating frame of reference related to the Earth. We denote the coordinates of this system by  $x(3)^i$ , and the metrical tensor correspondingly  $g(3)_{np}$ . At the same stage we also take into account the gravitational field of the Sun with the help of the tensor  $g(pc)_{np}$ . The next reference system is the geocentric ecliptic system. It is fixed relative to the previous one. We denote the coordinates by  $x(4)^i$ , and corresponding metric tensor  $g(4)_{rs}$ . In the next step, we turn to the geocentric equatorial reference system. We denote the coordinates by  $x(5)^i$ , and metric tensor by  $g(5)_{lu}$ . The sixth will be a rotating geocentric equatorial reference frame that rotates with the Earth. The coordinates will be denoted by  $x(6)^i$  and the metric tensor by  $g(6)_{vw}$ . At the same stage, the gravitational field of the Earth is also taken into account with the help of the tensor  $g(pz)_{vw}$ . The next frame of reference is geocentric, connected with the plane of the satellite's orbit, fixed relative to the Earth. The coordinates are denoted by  $x(7)^i$ , and metric tensor by  $g(7)_{df}$ . And the last frame of reference is a geocentric rotating system that rotates with the satellite. The coordinates will be denoted by  $x(8)^i$ , and the metric tensor  $g(8)_{gh}$ , respectively.

Since the heliocentric, geocentric and other coordinate systems are rotating, it is convenient to go over to the cylindrical coordinate system. As coordinates of the four-dimensional space-time of the galactic inertial reference frame, we choose the following coordinates:

$$\begin{cases} x(1)^0 = ct(1) \\ x(1)^1 = R(1) \\ x(1)^2 = \varphi(1) \\ x(1)^3 = z(1) \end{cases} \quad (8)$$

where  $t(l)$  is the galactic time and coordinates in the cylindrical coordinate system:  $R(l)$  is the cylindrical radius that lies in the plane of the galactic equator,  $\varphi(l)$  is the galactic longitude,  $z(l)$  is the coordinate axis, to the north pole of the world.

In contrast to ordinary Euclidean space, the square of the length differential in four-dimensional space is determined by the formula:

$$dS^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2, \quad (9)$$

where  $ct$  is the time coordinate,  $x, y, z$  are the spatial coordinates in the rectangular coordinate system.

The quantity  $dS$  is called the interval and is invariant with respect to the transformation of inertial coordinate systems. A consequence of the invariance of the interval is the transformation of the coordinates and the Lorentz time in the transition from the fixed to the mobile reference system.

If the reference frame is non-inertial (moves with acceleration, rotates), and also in the presence of a gravitational field in it, the four-dimensional space-time is distorted and the interval ceases to be an invariant. In this case, the interval can be represented as:

$$dS^2 = \sum_{i=1}^4 \sum_{k=1}^4 g_{ik} dx^i dx^k = g_{ik} dx^i dx^k, \quad (10)$$

where  $g_{ik}$  is a metric second-rank tensor whose components are functions of spatial and temporal coordinates and  $i, k$  are indices that take the values 0, 1, 2, 3.

The metric tensor in a rotating frame of reference is determined by the formula:

$$g(b)_{ik} = \sum_{j=1}^4 \sum_{k=1}^4 \frac{\partial x(a)^j}{\partial x(b)^i} \frac{\partial x(a)^m}{\partial x(b)^k} g(a)_{jm} = \frac{\partial x(a)^j}{\partial x(b)^i} \frac{\partial x(a)^m}{\partial x(b)^k} g(a)_{jm} \quad (11)$$

In expressions (10), (11) and in all subsequent ones, where two identical indices are found, as is customary in tensor analysis, summation over these indices is assumed. In this case, the signs of sums are omitted.

In the case of an inertial frame of reference, the metric tensor has the following components:

$$g_{ik}^{(0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad (12)$$

We now turn to the cylindrical coordinate system, where the relationship between the coordinates is determined by the formulas:

$$\begin{cases} ct' = ct \\ x' = R \cos \varphi \\ y' = R \sin \varphi \\ z' = z \end{cases} \quad (13)$$

We substitute (13) into (9) and find the square of the interval for the cylindrical coordinate system:

$$dS^2 = c^2 dt^2 - \left[ \frac{\partial(R \cos \varphi)}{\partial R} dR + \frac{\partial(R \cos \varphi)}{\partial \varphi} d\varphi \right]^2 \quad (14)$$

$$- \left[ \frac{\partial(R \sin \varphi)}{\partial R} dR + \frac{\partial(R \sin \varphi)}{\partial \varphi} d\varphi \right]^2 - dz^2$$

Taking derivatives and simplifying the expression we get

$$dS^2 = c^2 dt^2 - dR^2 - R^2 d\varphi^2 - dz^2. \quad (15)$$

Thus, the metric tensor of a cylindrical coordinate system has the following components:

$$g^{(1)}_{ik} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -R^2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad (16)$$

Let us now turn to a rotating frame of reference, connected with the standard or basic motion of the Sun [23]. The beginning of such a frame of reference is in the Galactic Center. The rotation axis  $z(2)$  coincides with the axis  $z(1)$ . The cylindrical radius  $R(2)$  located in the plane of the galactic equator, determines the position of the Sun and depends on the longitude  $\varphi(2)$ . Then in the frame of reference, which rotates with the Sun, the coordinates of the four-dimensional space-time are the same:

$$\begin{cases} x(2)^0 = ct(2) \\ x(2)^1 = R(2) \\ x(2)^2 = \varphi(2) \\ x(2)^3 = z(2) \end{cases} \quad (17)$$

From the point of view of the observer located in the galactic coordinate system, and also from the invariance of the interval  $dS^2 = inv$  for the Galilean galactic reference frame and the locally incident inertial reference frame for a given world point of the rotating reference frame, and taking into account that the radius  $R(1)$  and the coordinate  $z(1)$  do not change when moving to a rotating reference frame, since the linear velocities are perpendicular to these coordinates, we obtain the time  $t(1)$ , longitude  $\varphi(1)$ , radius  $R(1)$  and coordinate  $z(1)$ :

From the point of view of the observer located in the galactic coordinate system, and also from the invariance of the interval  $dS^2 = inv$  for the Galilean galactic reference frame and the locally incident inertial reference frame for a given world point of the rotating reference frame, and taking into account that the radius  $R(1)$  and the coordinate  $z(1)$  do not change when moving to a rotating reference frame, since the linear velocities are perpendicular to these coordinates, we obtain the time  $t(1)$ , longitude  $\varphi(1)$ , radius  $R(1)$  and coordinate  $z(1)$ :

$$\begin{cases} t(1) = \frac{t(2)}{\sqrt{1 - \frac{\omega(1)^2 R(1)^2}{c^2}}} \\ R(1) = R(2) \\ \varphi(1) = \varphi(1) + \varphi(2) + \omega(1) \frac{t(2)}{\sqrt{1 - \frac{\omega(1)^2 R(1)^2}{c^2}}} \\ z(1) = z(2) \end{cases} \quad (18)$$

The angular velocity of rotation of the Sun around the center of the Galaxy is equal to  $\omega(1) = 0''0053/h$ , the linear velocity 250 km/s, the period of the cycle  $2,5 \cdot 10^8$  years, the distance from the Sun to the centre of Galaxy  $R(2) = (10.0 \pm 0.8)$  kiloparsec [10, 11].

To determine the components of the metric tensor, it is necessary to establish a relationship between the coordinates of the Galilean reference frame  $x(1)^i$  and the coordinates of the non-inertial rotating reference frame  $x(2)^i$ . For this, we should consider these frames of reference within the framework of the general theory of relativity. Substituting (18) into (8), we find the coupling functions between  $x(1)$  and  $x(2)$ :

$$\begin{cases} x(1)^0 = cA[x(2)^0] \\ x(1)^1 = [x(2)^1] \\ x(1)^2 = \varphi c(1) + [x(2)^2] + A\omega(1)[x(2)^0] \\ x(1)^3 = [x(2)^3] \end{cases}, \quad (19)$$

where

$$A = \frac{1}{c\sqrt{1 - \frac{\omega(1)^2 [x(2)^1]^2}{c^2}}}. \quad (20)$$

We find the metric tensor in the system (2) by the formula:

$$g^{(2)}_{ik} = \frac{\partial x(1)^l}{\partial x(2)^i} \frac{\partial x(1)^m}{\partial x(2)^k} g^{(1)}_{lm} \quad (21)$$

To do this, we find the partial derivatives:

$$\begin{cases} \frac{\partial x(1)^0}{\partial x(2)^0} = cA \\ \frac{\partial x(1)^0}{\partial x(2)^1} = cA^3\omega(1)^2 [x(2)^0][x(2)^1] \\ \frac{\partial x(1)^1}{\partial x(2)^1} = 1 \\ \frac{\partial x(1)^2}{\partial x(2)^0} = A\omega(1) \\ \frac{\partial x(1)^2}{\partial x(2)^1} = A^3\omega(1)^3 [x(2)^0][x(2)^1] \\ \frac{\partial x(1)^2}{\partial x(2)^2} = 1 \\ \frac{\partial x(1)^3}{\partial x(2)^3} = 1 \end{cases} \quad (22)$$

The remaining partial derivatives are zero. Thus, we have completed the first step of finding the components of the required metric tensor.

Similarly, we define a metric tensor in a geocentric frame of reference  $g(3)_{np}$ .

The presence of the gravitational field of the sun leads to a change in the metric tensor by some value of  $g(c)_{np}$ . Therefore, the resulting tensor, which takes into account the rotational motion of the Sun around the center of the Galaxy and the gravitational field of the Sun, can be represented as:

$$g^{(c)}_{np} = g(3)_{np} + g(c)_{np}, \quad (23)$$

where the components of the tensor  $g(c)_{np}$ , taking into account the gravitational field of the Sun, which are determined from the Schwarzschild equation, are:

$$g^{(c)}_{np} = \begin{bmatrix} -\frac{R_g(c)}{\sqrt{R(3)^2 + z(3)^2}} & 0 & 0 & 0 \\ 0 & -\frac{R_g(c)R(3)^2}{(R(3)^2 + z(3)^2)^{\frac{3}{2}}} & 0 & -\frac{R_g(c)R(3)z(3)}{(R(3)^2 + z(3)^2)^{\frac{3}{2}}} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{R_g(c)R(3)z(3)}{(R(3)^2 + z(3)^2)^{\frac{3}{2}}} & 0 & -\frac{R_g(c)z(3)^2}{(R(3)^2 + z(3)^2)^{\frac{3}{2}}} \end{bmatrix} \quad (24)$$

where

$$R_g = \frac{2GM}{c^2}, \quad (25)$$

$R_g$  is the gravitational radius of the Sun,  $G$  is the gravitational constant,  $M$  is the mass of the Sun and  $R, \theta, \varphi$ , are spherical coordinates.

Similarly, we define metric tensors in a geocentric frame of reference  $g(4)_{rs}, g(5)_{vw}, g(6)_{lm}, g(7)_{vw}$ . In this case, the influence of the Earth's gravitational field is taken into account in the metric tensor  $g(6)_{lm}$ .

The last metric tensor in the geocentric satellite frame is determined by the formula:

$$g(8)_{gh} = \frac{\partial x(7)^g}{\partial x(8)^s} \frac{\partial x(7)^e}{\partial x(8)^h} g(7)_{qe} \quad (26)$$

The partial derivatives of the functions will have the form:

$$\left\{ \begin{aligned} \frac{\partial x(7)^0}{\partial x(8)^0} &= \frac{1}{\sqrt{1 - \frac{\omega(7)^2 [x(8)^1]^2}{c^2}}} \\ \frac{\partial x(7)^0}{\partial x(8)^1} &= \frac{\omega(7)^2 [x(8)^0] [x(8)^1]}{c^2 \left( 1 - \frac{\omega(7)^2 [x(8)^1]^2}{c^2} \right)^{\frac{3}{2}}} \\ \frac{\partial x(7)^1}{\partial x(8)^1} &= 1 \\ \frac{\partial x(7)^2}{\partial x(8)^0} &= \frac{\omega(7)}{c^2 \sqrt{1 - \frac{\omega(7)^2 [x(8)^1]^2}{c^2}}} \\ \frac{\partial x(7)^2}{\partial x(8)^1} &= \frac{\omega(7)^3 [x(8)^0] [x(8)^1]}{c^3 \left( 1 - \frac{\omega(7)^2 [x(8)^1]^2}{c^2} \right)^{\frac{3}{2}}} \\ \frac{\partial x(7)^2}{\partial x(8)^2} &= 1 \\ \frac{\partial x(7)^3}{\partial x(8)^3} &= 1 \end{aligned} \right. , \quad (27)$$

where  $\omega(7)$  is angular velocity of satellite rotation around the Earth. The other derivatives are equal to zero.

We note that the metric tensor  $g(8)_{gh}$  in the geocentric satellite reference system takes into account the influence of the Sun's motion in the Galaxy, the rotation of the Earth around the Sun and around its axis, the motion of the satellite around the Earth, as well as the influence of the gravitational field of the Sun and the gravitational field of the Earth. Thus, we have completed the development of a mathematical model for calculating the metric tensor, the components of which are the main components of the corrections for relativistic effects in determining the coordinates of the object in space.

#### IV. CONCLUSIONS

1. The influence on the accuracy of GNSS observations of relativistic effects associated with the non-inertiality of rotating frames of reference is analysed, the influence of gravitational fields, the consequence of which is the curvature of four-dimensional space-time and the change of its metric. This leads to a slowdown of the atomic clock, a relativistic shortening of distances and time delays in the propagation of electromagnetic

waves during the GNSS observations and during clock synchronization.

2. We have derived formulas for calculating three corrections for relativistic effects: the time correction of the synchronization of the atomic clock of the satellite and the ground control station; time correction of the relativistic change in the atomic clock of the satellite; correction of the pseudo-distance for relativistic effects. It is established that the basis of these corrections are the components of the metric tensor.

3. A mathematical model for calculating the metric tensor is created. It is suggested to consider step by step relativistic effects in this order: the rotational motion of the Sun around the centre of the Galaxy – the gravitational field of the Sun – the rotational motion of the Earth around the Sun – the Earth's rotational motion around its own axis – the gravitational field of the Earth – the rotational motion of the satellite around the Earth. A mathematical model has been developed that takes into account the effects of all the above relativistic effects on the corrections to the maximum.

In future work, we will create a program for calculating the corrections for relativistic effects: the temporal correction of the clock synchronization of the satellite and the ground control station; time correction of the relativistic change in the atomic clock of the satellite; correction of the pseudo-distance from the satellite to the receiver or the point of the earth's surface and carry out a comprehensive analysis of the calculation data.

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