INTL JOURNAL OF ELECTRONICS AND TELECOMMUNICATIONS, 2016, VOL. 62, NO. 1, PP. 61-64
Manuscript received December 30, 2015; revised March, 2016. DOI: 10.1515/eletel-2016-0008

Mathematical Model of Communication Delays in Wireless Networks

Beata Krupanek and Ryszard Bogacz

***Abstract*—The paper presents a new conception of building probabilistic models of communication delays in wireless networks that basis on using a delta function sequence to describe retransmissions between a transmitter and a receiver. It is assumed that the access time of the transmitter is described by a probability density function and the communication channel established in the wireless medium is disturbed by passive or active factors which cause that the transmission can be not correct and the sent data have to be retransmitted. Theoretical considerations have been verified by measurement results obtained by using the experimental system developed for investigating delays caused by external disturbances influencing the wireless transmission. A method of identification of the proposed model parameters and verification of the identified values has been presented.**

***Keywords*—wireless networks, delays, mathematical modeling, delta function**

I. Introduction

NOWADAYS, in times of common usage of wireless networks for data transmission in measuring and control systems, determination of quality of services provided by such networks is an important problem because the knowledge of transmission channel quality parameters allows adequate design of the system components [1], [2], [3]. From measurement and functional points of view, the essential quality parameters of data transmission in the system are connected with delays which arise in a communication channel during sending a message from a transmitter to a receiver [4].

The method described in the paper represents a probabilistic approach to modelling properties of wireless networks. Its novelty consists in treating the Dirac's delta function as a probability density function describing delays which occur when the wireless transmission is disturbed by external factors [4] and it is necessary to retransmit data. Such mathematical means allow obtaining relatively simple probabilistic descriptions of delays in networks composed of many nodes. The delay model obtained in this way is useful in simulative analysis of wireless networks [5].

Described method of modeling a telecommunication delays in wireless networks can be easily adopted for wired networks. Additionally it can be used to determine the maximum transmission time and consequently for choosing the best path of routing messages.

First Author is from Silesian University of Technology, Faculty of Electrical Engineering, Institute of Measurement Science, Electronics and Control, Akademicka 10, 44-100 Gliwice, Poland, (e-mail: beata.krupanek@polsl.pl).

Second Author is from Silesian University of Technology, Faculty of Electrical Engineering, Institute of Measurement Science, Electronics and Control, Akademicka 10, 44-100 Gliwice, Poland, (e-mail: ryszard.bogacz@polsl.pl).

II. Mathematical Basis of the Proposed Description of Delays

A wireless network consists of a collection of nodes, each capable of transmitting to or receiving from other nodes [7]. The moments in which the specified nodes sends the information are largely random. This is the result of random time decision-making by the user and the algorithm of medium access – in most wireless networks it is the CSMA/CA algorithm. As the complexity of wireless networks increases, there is a need to develop better way of describing some of transmission parameters such a delays. In many networking applications delay is a key performance metric along with throughput [7]. In this context the good mathematical model of delay is key to achieving the quality of service required by the application.

A. *The Model of Transmission Delays Between Two Nodes*

Let us consider the simple situation shown in Fig. 1, where **A** and **B** denote nodes of a wireless network which communicate directly.

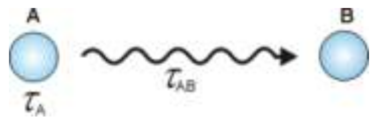

Fig. 1. Sources of delays during transmission from node A to B, τ_A – time necessary to obtain the access to a communication medium, τ_{AB} – time of the message transmission

Let us take then that the transmission of a message from node **A** to **B** is activated at moment t_0 and this message is received by the node **B** at moment t_B. The difference between these two moments:

$$\tau_{tot} = t_B - t_0 \tag{1}$$

is the total time of the transmission and can be interpreted as the communication delay of the message transmitted from node **A** to **B**.

One can determine two main sources of partial delays being components of the total delay τ_{tot}. The first one τ_A is associated with activity inside node **A** and in the considered situation can be qualified as the time necessary to obtain the access to the communication medium. The second component τ_{AB} describes the time necessary to transmit a message from **A** to **B** trough the medium. Denoting the moment when **A** gets the access by t_A, the total delay (1) can be written as the sum of partial delays:

$$\tau_{\mathrm{tot}} = t_{\mathrm{B}} - t_{\mathrm{A}} + t_{\mathrm{A}} - t_0 = \tau_{\mathrm{AB}} + \tau_{\mathrm{A}}. \quad (2)$$

A model of the total delay has to contain all possible values of the delays occurring in the described conditions. If one takes into account that access procedures commonly used in wireless networks are of random character [1], the total delay τ_{tot} should be described in probabilistic categories, too. To obtain a compact model of delays, there is assumed that all the delays in Eq. (2) are treated as uncorrelated random variables, which enables determining the probability density function of the total delay as:

$$g_{\mathrm{tot}}(\tau_{\mathrm{tot}}) = g_{\mathrm{A}}(\tau_{\mathrm{A}}) \otimes g_{\mathrm{AB}}(\tau_{\mathrm{AB}}), \quad (3)$$

where $\otimes$ is the symbol of convolution and $g_{\mathrm{A}}(\tau_{\mathrm{A}})$ and $g_{\mathrm{AB}}(\tau_{\mathrm{AB}})$ are the probability density functions of the partial random delays τ_{A} and τ_{AB}.

To perform convolution (3), it is necessary to describe the constant delay τ_{AB} in probabilistic categories. Such a description can be obtained by using the Dirac's delta [6] as a probability density function. Assuming b to be a constant value, the delta function for the delay τ can be written as:

$$\delta(\tau - b) = \begin{cases} \infty & \text{for} \quad \tau = b \\ 0 & \text{for} \quad \tau \neq b. \end{cases} \quad (4)$$

The convolution of the probability density function $g(\tau)$ with the delta function (4) gives:

$$g(\tau) \otimes \delta(\tau - b) = g(\tau - b), \quad (5)$$

which means that this convolution shifts the function $g(\tau)$ in the horizontal axis by the value b. It allows simplifying transformations by making such displacements instead of performing the convolutions. Basing on properties on delta function [6] the probability density function of the access delay can be write in the form:

$$g_{\mathrm{A}}(\tau_{\mathrm{A}}) = g_{\mathrm{Apat}}(\tau_{\mathrm{A}} - \tau_0), \quad (6)$$

where τ_0 is the expected value of $g_{\mathrm{A}}(\tau_{\mathrm{A}})$.

The probability density function $g_{\mathrm{Apat}}(.)$ has the same shape as $g_{\mathrm{A}}(\tau_{\mathrm{A}})$ but its mean value is equal to zero. This function plays an important role in when it is used as a kind of a pattern in the delay model built on the basis of measurements. The results enable obtaining a pattern for ZigBee module as the normal probability density function $g_{\mathrm{Apat}}(\tau_{\mathrm{A}}) = N(0, 1)$ ms properly truncated because $g_{\mathrm{A}}(\tau_{\mathrm{A}}) > 0$.

B. The Delay Model of Communication with Retransmissions

The radio signal being the communication medium in wireless networks is exposed to many different disturbances both of passive and active nature [4]. The passive disturbances are caused by terrain obstacles, walls, etc. while the active ones result from influences of electromagnetic and electrostatic fields. Generally, the appearance of disturbances cause that the received message is not correct. Information about this fact is sent back to the transmitter which usually tries to send the message once more. The number of retransmissions depends on construction of the wireless module and can be determined by its user.

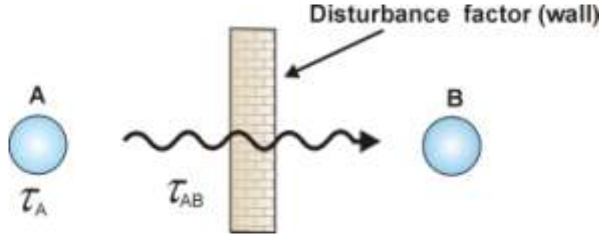

Fig. 2. Transmission between two nodes disturbed by a wall

In Fig. 2 there is shown the situation when two nodes A and B communicate directly but a disturbance, represented in this figure by a wall, causes that some transmissions have to be repeated, i.e. the message must be retransmitted. It means that the partial delay τ_{AB} connected with the correct transmission of a message is not constant as it has been considered assumed previously but depends on the number of retransmissions. In the work [4], it has been experimentally proved that in this case τ_{AB} can be represented by the following delta function sequence:

$$g_{\mathrm{AB}}(\tau_{\mathrm{AB}}) = a_0\delta(\tau_{\mathrm{AB}} - b_0) + a_1\delta(\tau_{\mathrm{AB}} - b_1) + \ldots + \\ + a_k\delta(\tau_{\mathrm{AB}} - b_k), \quad (7)$$

where $\delta(.)$ denotes the delta function defined by (3), k is the number of retransmissions. Both $a_0, a_1, \ldots, a_k$ and $b_0, b_1, \ldots, b_k$ are constant coefficients with non-negative values.

To obtain the total communication delay τ_{tot}, realizations of partial delays τ_{A} and τ_{AB} are added up accordingly with Eq. (2). It means that if they are treated as uncorrelated random variables, the probability density function of the total delay can be obtained as convolution (3). After introducing Eq. (7) and (5) to (3) and taking into account that convolution is a linear transformation, one can write (3) as the sequence of partial convolutions:

$$g_{\mathrm{tot}}(\tau_{\mathrm{tot}}) = a_0 g_{\mathrm{A}}(\tau_{\mathrm{tot}} - b_0) + a_1 g_{\mathrm{A}}(\tau_{\mathrm{tot}} - b_1) + \ldots + \\ + a_k g_{\mathrm{A}}(\tau_{\mathrm{tot}} - b_k) \quad (8)$$

which is the sum of duplicates of the probability density function $g_{\mathrm{A}}(\cdot)$ properly moved in time by the constant values $b_0, b_1, \ldots, b_k$ and multiplied by the constant coefficients $a_0, a_1, \ldots, a_k$, which describe the probability of occurring the succeeding retransmissions.

Equation (8) is the probabilistic model of the total communication delay in a situation when disturbances affect the transmission, which causes the necessity of retransmissions. Instead of using the function $g_{\mathrm{A}}(\cdot)$ in this model, it is better to introduce the pattern $g_{\mathrm{Apat}}(.)$ described by (6), which has the expected value equal to 0. In this case the time displacements $b_0, b_1, \ldots, b_k$ can be determined in relation to the vertical axis as constant delays $\tau_0, \tau_1, \ldots, \tau_k$, which allows calculating them on the basis of measurement results. Taking this into account, on can write the parameters of the model describing the delays of the disturbed communication on the way between nodes A and B as:

$$\mathbf{Delay}_{\mathrm{AB}}\{g_{\mathrm{Apat}}(.), (a_0, \tau_0), (a_1, \tau_1), \ldots, (a_k, \tau_k)\}. \quad (9)$$

To verify the thesis that this model is a good description of delays in wireless networks exposed to disturbances, the total delays in real networks were measured with use of the developed system and parameters of the model were identified for several kinds of external disturbances.

III. Measurement of Delays

THE main practical application of the proposed mathematical apparatus consists in description of communication delays by the model, parameters of which depend on properties of factors disturbing the transmission medium. To identify the model, it is necessary to measure delays in selected conditions.

A. Measurement Setup

The general scheme of the system used for measurements of communication delays in wireless networks is shown in Fig.3.

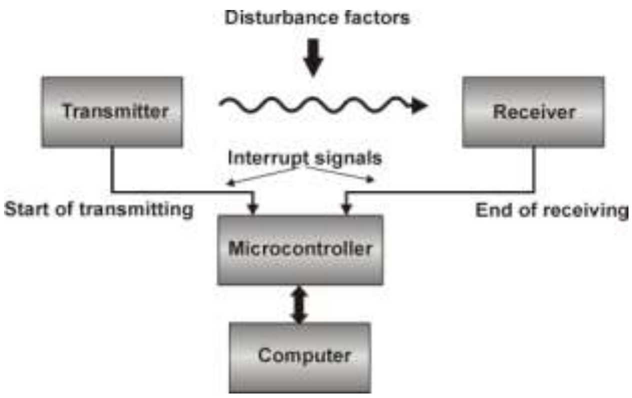

Fig 3. Scheme of the system for measurements of transmission delays

The investigated communication channel consists of two elements: a transmitter and a receiver working in ZigBee standard [4, 8]. Then used radio modules were Xbee's from Digi International. Transmission of data having a constant length is disturbed by different kind of factors such as walls, electromagnetic or electrostatic fields and other wireless transmissions. The delay, defined as the time between the start of sending data in the transmitter and the moment when data are completed in the receiver, is measured by a microcontroller. The measured results are sent to the computer where they are processed to the form of a histogram.

B. Results and Discussion

The exemplary histogram shown in Fig. 4 was obtained from the described system in the situation characterized in Fig. 2. The signal transmitted between the same ZigBee wireless modules was absorbed by the wall built from reinforced concrete 35 cm thick which caused that a part of communicates had to be retransmitted. The graphical representation of 10 000 delay measurement results in the form of the histogram shown in Fig. 4 consists of 2 modes describing the probability of appearance of the first and the second transmission, respectively.

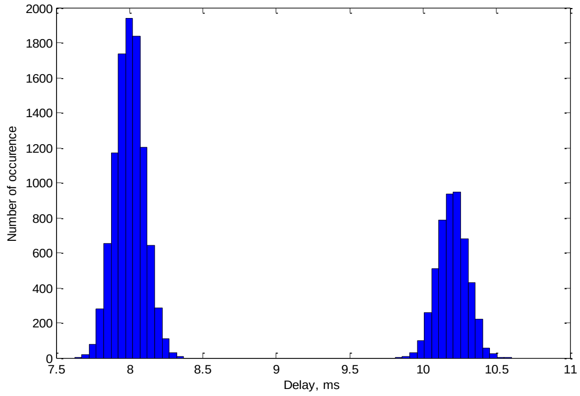

Fig. 4. Exemplary histogram of the total delay with one retransmission obtained from the measurement system

In real measurement conditions when transmitter is separated from receiver by a obstacle like wall or a several walls (especially in buildings) then the strength of radio signal is attenuated.

The data packets are deformed or lost what reflects in a additional modes in the delay histogram. Figures 5 and 6 shows the obtained histograms from measurement conditions such as:

a) one wall between transmitter and receiver,

b) three walls between transmitter and receiver.

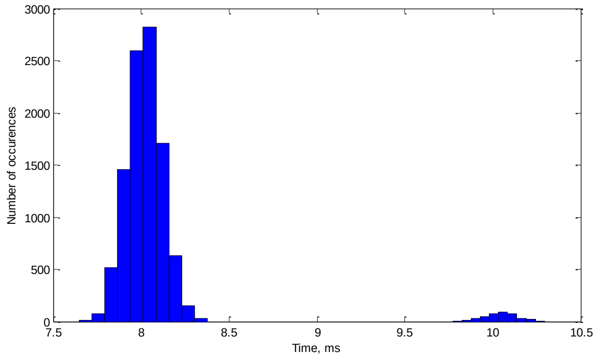

Fig. 5. Transmission through one brick wall

It can be seen that the retransmissions occurs when the packet must be sent another time because of transmission disturbances. The more obstacle the more retransmissions. In the extreme case all of the available retransmissions are used but sill we have packets witch didn't reach the receiver. Then such a packets are deleted by radio module. User can increase the number of retransmissions when the module is working in hard conditions.

Having measured a set of the delay values obtained for the selected couple of wireless modules in the investigated situation, i.e. when the transmission is disturbed by the chosen factor, one can perform identification of the model parameters. To start with, one has to dispose the pattern $g_{Apat}(.)$ determined for this couple of modules. Having it, one can realize the identification procedure in the following steps:

- At first, it is necessary to present both the pattern and the set of delays as two different histograms with the same width of classes.
- Next, the comparison of the first mode of the total delay with the pattern is performed in order to determine

coefficient a_0 and time displacement τ_0. Both a_0 and τ_0 are obtained in the same process which consists in multiplying the pattern by coefficient a_0 and shifting it along axis τ_0 as long as the difference between the classes of the first mode and the calibrated pattern comes up to a minimum.

- The procedure described above should be repeated for all the rest modes of the histogram.

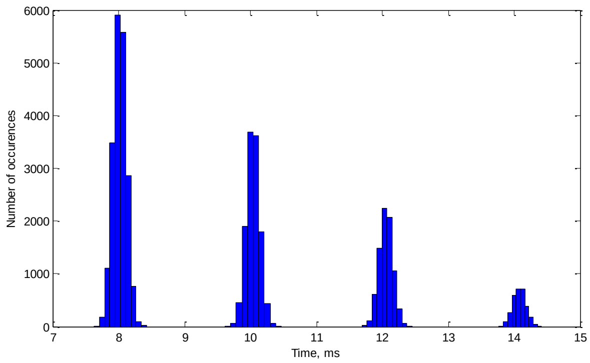

Fig. 6. Transmission through three brick walls. Rest of packets that has been not sent in 3 retransmissions was deleted

After realizing the whole procedure, one obtains coefficients $a_0, a_1, \ldots, a_k$ and $\tau_0, \tau_1, \ldots, \tau_k$ determined for the probability density function $g_{\mathrm{Apat}}(.)$, which means that the identification of the total delay model in the sense of expression (10) has been performed. To calculate the inaccuracy of the identified model, it is necessary to determine the total delay histogram on the basis of the obtained parameters using the Monte Carlo method and compare this histogram (let us call it as the template) with the one determined in the measurement way. The mean-squared error of the identification can be calculated as:

$$e_{\mathrm{id}} = \frac{1}{N}\sqrt{\sum_{n=1}^{N}\left[y_{\mathrm{meas}}(n) - y_{\mathrm{sim}}(n)\right]^2} \qquad (10)$$

where $y_{\mathrm{meas}}(n)$ is the height of n-th bar of the total delay histogram obtained in the measurement way, $y_{\mathrm{sim}}(n)$ is the height of n-th bar of the histogram calculated by using the Monte Carlo method, N is the number of all bars.

Let us assume that the delay model has been identified in the described way on the basis of measurements presented in the form of the histogram from Fig. 4. The obtained parameters of the model can be written in the simplified form as:

$$\mathbf{Delay}_{\mathrm{AB}}\begin{Bmatrix} N(0,1)\ \mathrm{ms},\ (0.75,\ 8.2\,\mathrm{ms}), \\ (0.25,\ 10.4\,\mathrm{ms}) \end{Bmatrix}. \qquad (11)$$

Comparing the histogram with the template composed of the shifted patterns, calculated by using the Monte Carlo method for these parameters, one obtains the error values shown in Fig.5.

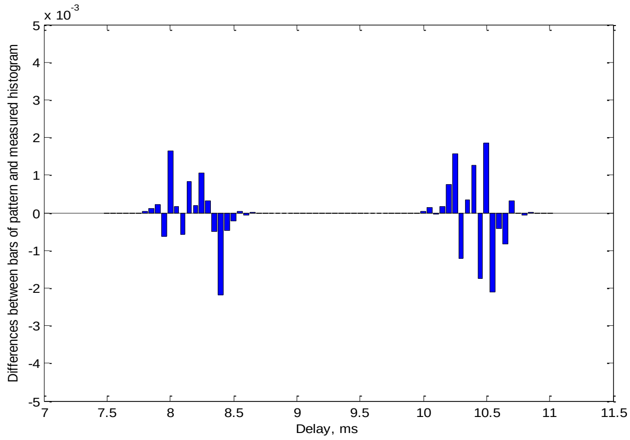

Fig. 5. Differences between the bars of the template and the measured histogram versus delay

The mean-squared error calculated on the basis of Eq. (16) $e_{\mathrm{id}} = 0.15\mathrm{E}{-3}$. Therefore, one can state that the delay model (17) of the communication channel has been identified in the proposed way with the inaccuracy less than 0.02E-3.

IV. CONCLUSIONS

The essence of the method of modeling delays in wireless networks presented in the paper consists in using the delta function sequence treated as the probability density function of the delay in the communications chain with retransmissions. The chain must be divided into elementary channels consisting of two nodes: a transmitter and a receiver communicating directly. The transmission between them can be identified in the measurement way presented in the paper and should be described in probabilistic categories as the delta function sequence. The total delay in a composed chain can be obtained by using convolution of the partial descriptions. The partial model describing the delay of the transmission between two nodes is relatively simple and its important feature is the possibility of including influence of disturbances on the delay.

REFERENCES

[1] Yuan L., Zhu Y. (2006) Modeling and Simulating Wireless Sensor Transportation Monitoring Network. *The Sixth World Congress on Intelligent Control and Automation*, Vol. 2, pp. 8640-8644.

[2] Chaudhary D. D., Waghmare L. M. (2012) Quality of service analysis in wireless sensor network by controlling end-to-end delay. *7th IEEE Conference on Industrial Electronics and Applications*, pp. 703-708.

[3] Xiong X., Tan J., Lin X. (2013) Study on Communication Architecture Design of Wide-Area Measurement System. *IEEE Transactions on Power Delivery*, Vol. 28, Issue 3, pp. 1542-1547.

[4] Topór-Kamiński T., Krupanek B., Homa J. (2013) Delays Models of Measurement and Control Data Transmission Network. *Advanced Technologies for Intelligent Systems of National Border Security*, Studies in Computational Intelligence, 440, pp. 257-279.

[5] Krupanek B. (2009) Simulations of experimental set up based on ZigBee standard using Opnet. *International PhD Workshop OWD 2009*, Wisła, pp. 262-267.

[6] Gupta S.C. (1964) Delta Function. *IEEE Transactions on Education*, vol. 7, iss. 1., pp. 16-22.

[7] Gamal A., Mammen J., Prabhakar B., Shah D. (2006) Optimal Throughput–Delay Scaling in Wireless Networks—Part I: The Fluid Model. *IEEE Transactions on Information Theory*, vol. 52, no. 6, pp. 2568-2592.

[8] Krupanek B., Bogacz R. (2014) Investigations of transmission delays in ZigBee networks. *Przegląd Elektrotechniczny*, iss.1., pp. 70-75.