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Power Allocation for Energy-Harvesting-based Fading Cognitive Multiple Access Channels: with or without Successive Interference Cancellation

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***Abstract*—This paper considers a fading cognitive multiple access channel (CMAC), where multiple secondary users (SUs), who share the spectrum with a primary user (PU), transmit to a cognitive base station (CBS). A power station is assumed to harvest energy from the nature and then provide power to the SUs. We investigate the power allocation problems for such a CMAC to maximize the SU sum rate under the interference power constraint, the sum transmit power constraint and the peak transmit power constraint of each individual SU. In particular, two scenarios are considered: with successive interference cancellation (SIC) and without SIC. For the first scenario, the optimal power allocation algorithm is derived. For the second scenario, a heuristic algorithm is proposed. We show that the proposed algorithm with SIC outperforms the algorithm without SIC in terms of the SU sum rate, while the algorithm without SIC outperforms the algorithm with SIC in terms of the number of admitted SUs for a high sum transmit power limit and a low peak transmit power limit of each individual SU.**

***Keywords*—Cognitive radio, power allocation, multiple access channels, energy harvesting**

I. Introduction

In recent years, various wireless multimedia services have been increasingly popular, which causes huge demand for radio spectrum resources. However, most radio spectrum resources have been allocated to various wireless services. This poses a big challenge to the spectrum allocation regulation. In this respect, many field spectrum usage measurements show that most of the allocated spectrum is under-utilized. For example, in [1], it is shown that the average spectrum usage is approximately 15.2% in Beijing, China, for spectrum bands from 440 MHz to 2700 MHz. Thus, it is urgent to improve the spectrum utilization rather than finding more spectrum resources. In this regard, cognitive radio (CR) [2], is a promising solution to solve the spectrum scarcity problem by allowing secondary users (SUs) to utilize the spectrum allocated to primary users (PUs). A very popular model in CR is spectrum sharing which limits the interference power from the SUs to the PUs.

Power allocation is very important for controlling the interference to the PUs. Subject to the interference power constraint and the transmit power constraint, the power allocation problems for a fading CR point to point channel to maximize the ergodic capacity and the outage capacity were investigated in [3]. In [4], under the same system model as in [3], we proposed power allocation algorithms that can balance between the goal of maximizing the ergodic capacity and the goal of minimizing the outage probability. Besides the CR point to point channel model investigated in current studies such as [3], [4], a more complicated cognitive multiple access channel (CMAC) model has also attracted a lot of attention. In [5], the power allocation problems for maximizing the ergodic capacity of the fading CMAC under various power constraints were investigated. In [6], the optimal power allocation to achieve the outage capacity region boundary for the fading CMAC was derived. In [7], the power allocation problem for the fading CMAC to maximize the ergodic capacity was reformulated as a geometrical problem and the optimal power allocation was then derived. In [8], the optimal power allocation for an interference-limited CMAC to minimize the outage probability of the SUs was proposed. In [9], the weighted sum rate maximization power allocation problem for a CMAC was studied. All these works in [5]–[9] assumed that successive interference cancellation (SIC) is available and there is no mutual interference between the SUs. For the case that SIC is unavailable, [10] derived the optimal power allocation strategies to maximize the ergodic capacity and the outage capacity for a two-user CMAC under the interference power constraint, [11] proposed a greedy power allocation policy to achieve the effective capacity region boundary for a fading CMAC under the interference power constraint, [12] investigated the power allocation problems to maximize the ergodic sum rate for a fading CMAC under various power constraints, and [13] proposed two heuristic power allocation algorithms to maximize the ergodic sum rate of a fading CMAC under the PU outage constraint.

Different from the previous works in investigating power allocation problem in CMAC, this paper studies the power allocation problem for a fading CMAC based on energy harvesting to maximize the sum rate of the SUs with or without SIC. Energy harvesting is a promising approach to decrease the energy cost in wireless networks. We consider a scenario where a power station harvests energy from the environments and then provides power to the SUs. In this scenario, since the harvested energy is shared among the SUs, a sum transmit power constraint for the SUs is introduced. It is noted that such constraint was not considered in the previous works such as [5]–[13]. The related works in energy-harvesting-based CR networks can be found in [14]–[16]. However, to our best knowledge, the scenario of energy-harvesting-based

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CMAC considered in this paper has not been considered yet in literature.

The main contributions and the key findings of this paper are listed below:

- The power allocation problems for the SUs to maximize the SU sum rate for the fading energy-harvesting-based CMAC under the interference power constraint, the sum transmit power constraint and the peak transmit power constraint of each individual SU are investigated considering two scenarios: (1) with SIC ; (2) without SIC.
- We derive the optimal power allocation algorithm for the first scenario. For the second scenario, a heuristic power allocation algorithm is proposed.
- It is shown that, although the proposed power allocation algorithm with SIC always achieves higher SU sum rate than that of the algorithm without SIC, the number of admitted SUs achieved by the algorithm without SIC is higher than that achieved by the algorithm with SIC for a high sum transmit power limit and a low peak transmit power limit of each individual SU. It is also shown that the number of admitted SUs increases as the peak transmit power limit of each individual SU decreases.

The rest of the paper is organized as follows. The system model is described in Section II. The power allocation problems to maximize the SU sum rate for the fading energy-harvesting-based CMAC with SIC and without SIC are investigated in Section III and Section IV, respectively. Section V gives simulation results to verify the proposed algorithms. Section VI concludes the paper.

II. System Model

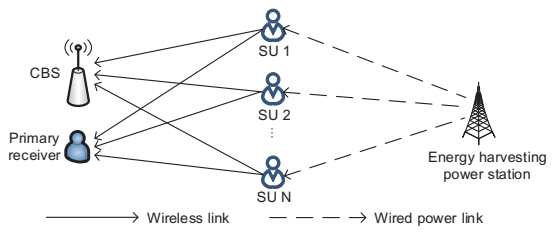

Fig. 1. System model.

We consider a CMAC as shown in Fig. 1, which consists of one receiving cognitive base station (CBS), N SUs, and one power station. The power station continuously harvest energy from nature, and then transfer the energy to the SUs through wired power lines[1]. It is assumed that the CMAC shares the same narrow band with the PU. Block-fading channel model is assumed for all the channels involved, i.e., the channels keep constant in one transmission block, but may change from one transmission block to another. The instantaneous channel power gains from SU i to CBS and SU i to the PU receiver are denoted by h_i and g_i, respectively. We assume that the SUs and the power station are not equipped with rechargeable batteries, and the total power harvested and available for the SUs for one transmission block is $\bar{P}$. Note that the value of $\bar{P}$ changes from one block to another. The noise power is denoted by σ^2.

Let $\mathbf{p} = [p_1, \ldots, p_N]$ denote the power allocation vector of the SUs, where p_i denote the transmit power of SU i. The interference power constraint is adopted to protect the PU transmission as given by

$$\sum_{i=1}^{N} p_i g_i \leq \bar{Q}, \tag{1}$$

where $\bar{Q}$ is the predefined interference power limit. The sum transmit power of the SUs can not be higher than the power available at the power station, as given by[2]

$$\sum_{i=1}^{N} p_i \leq \bar{P}. \tag{2}$$

Besides, the transmit power of each individual SU is restricted due to hardware limit as

$$p_i \leq P_i^{max}, i = 1, \ldots, N, \tag{3}$$

where P_i^{max} denotes the peak transmit power limit of SU i.

With SIC capability at the CBS, the sum rate of the SUs is given by

$$R(\mathbf{p}) = \ln\left(1 + \frac{\sum_{i=1}^{N} p_i h_i}{\sigma^2}\right). \tag{4}$$

On the other hand, the sum rate of the SUs without SIC capability at the CBS is given by

$$R(\mathbf{p}) = \sum_{i=1}^{N} \ln\left(1 + \frac{p_i h_i}{\sigma^2 + \sum_{j=1, j\neq i}^{N} p_j h_j}\right). \tag{5}$$

III. Power allocation with SIC

This section investigates the problem of power allocation for the SUs to maximize their sum rate with SIC capability at the CBS. The problem is formulated as follows

$$\text{P1}: \max_{\boldsymbol{p}} \ln\left(1 + \frac{\sum_{i=1}^{N} p_i h_i}{\sigma^2}\right) \tag{6}$$

$$\text{s.t.} \sum_{i=1}^{N} p_i g_i \leq \bar{Q}, \tag{7}$$

$$\sum_{i=1}^{N} p_i \leq \bar{P}, \tag{8}$$

$$p_i \leq P_i^{max}, i = 1, \ldots, N \tag{9}$$

$$p_i \geq 0, i = 1, \ldots, N \tag{10}$$

[1]It is noted that using an energy harvesting power station to provide energy to the SUs is suitable for the scenarios when the SUs are not mobile and fixed at some locations. If energy harvesting is performed at the SUs, the amount of harvested energy is limited due to limited capability of the SUs. Thus, using a more powerful energy harvesting power station can provide more harvested energy to the SUs. Besides, a central power station provides more flexible and efficient power allocation compared to distributed energy supply.

[2]It is noted that the sum transmit power constraint in (2) is applied due to introducing the central power station. Compared to distributed energy supply which does not require the constraint in (2), the sum transmit power constraint leads to more complicated power allocation as the transmit powers of the SUs are coupled in (2).

It is easy to verify that the objective function in (6) is concave with respect to $\boldsymbol{p}$ and thus P1 can be solved by the convex optimization [17]. The Lagrangian of P1 can be written as

$$L(\lambda,\mu,\boldsymbol{\nu},\boldsymbol{\varepsilon},\boldsymbol{p}) = \ln\left(1+\frac{\sum_{i=1}^{N}p_ih_i}{\sigma^2}\right)-\lambda\left(\sum_{i=1}^{N}p_ig_i\le\bar{Q}\right)$$
$$-\mu\left(\sum_{i=1}^{N}p_i-\bar{P}\right)-\sum_{i=1}^{N}\nu_i(p_i-P_i^{max})+\sum_{i=1}^{N}\varepsilon_ip_i, \quad (11)$$

where λ, μ, ν_i and ε_i are the non-negative dual variables associated with the constraints in (7), (8), (9) and (10), respectively, $i=1,\ldots,N$, with $\boldsymbol{\nu}=[\nu_1,\ldots,\nu_N]$ and $\boldsymbol{\varepsilon}=[\varepsilon_1,\ldots,\varepsilon_N]$. According to the KKT conditions [17], the optimal solutions of P1 shall satisfy the following equations

$$\frac{h_i}{\sigma^2+\sum_{n=1}^{N}p_nh_n}-\lambda g_i-\mu-\nu_i+\varepsilon_i=0,\forall i \quad (12)$$
$$\nu_i(p_i-P_i^{max})=0,\forall i \quad (13)$$
$$\nu_i\ge 0,\forall i \quad (14)$$
$$p_i\le P_i^{max},\forall i \quad (15)$$
$$\varepsilon_ip_i=0,\forall i \quad (16)$$
$$\varepsilon_i\ge 0,\forall i \quad (17)$$
$$p_i\ge 0.\forall i \quad (18)$$

From the above KKT optimality conditions, we can obtain the following lemma:

Lemma 1: The solution of P1 has at most one SU's transmit power, denoted by p_i that satisfies $0<p_i<P_i^{max}$.

Proof: Supposing that there are two SUs i and j with $0<p_i<P_i^{max}$ and $0<p_j<P_j^{max}$, then, from (13), (14), (16), (17), we have $\nu_i=\nu_j=0$ and $\varepsilon_i=\varepsilon_j=0$. Using these results, from (12), we have

$$\frac{h_i}{\lambda g_i+\mu}=\sigma^2+\sum_{n=1}^{N}p_nh_n, \quad (19)$$
$$\frac{h_j}{\lambda g_j+\mu}=\sigma^2+\sum_{n=1}^{N}p_nh_n. \quad (20)$$

Then, from the above two equations, we have

$$\frac{h_i}{\lambda g_i+\mu}=\frac{h_j}{\lambda g_j+\mu}. \quad (21)$$

Since $\{h_i,i=1,\ldots,N\}$ and $\{g_i,i=1,\ldots,N\}$ are sets of independent random variables and μ is constant, the probability of satisfying the above equality is zero. Thus, it can be concluded that there is at most one SU i with $0<p_i<P_i^{max}$.

Lemma 1 is thus proved. □

Lemma 1 indicates that most of $p_i,i=1,\ldots,N$ is either 0 or P_i^{max}. The following lemma shows the relationship between the SU whose transmit power is zero and the SU whose transmit power is nonzero.

Lemma 2: Supposing that any two SUs, i and j satisfy $p_i=0$ and $p_j>0$, then the following inequality must be satisfied:

$$\frac{h_i}{\lambda g_i+\mu}\le\frac{h_j}{\lambda g_j+\mu}. \quad (22)$$

Proof: Supposing that any two arbitrary SUs i and j with $p_i=0$ and $p_j>0$, from (13), (14), (16), (17) we have $\nu_i=0$ and $\varepsilon_j=0$. Then, from (12), we have

$$\frac{h_i}{\lambda g_i+\mu-\varepsilon_i}=\sigma^2+\sum_{n=1}^{N}p_nh_n, \quad (23)$$
$$\frac{h_j}{\lambda g_j+\mu+\nu_j}=\sigma^2+\sum_{n=1}^{N}p_nh_n. \quad (24)$$

From the above two equations and using the fact that $\varepsilon_i\ge 0$ and $\nu_j\ge 0$, we get

$$\frac{h_i}{\lambda g_i+\mu}\le\frac{h_j}{\lambda g_j+\mu}. \quad (25)$$

Lemma 2 is thus proved. □

Lemma 3: Supposing that any two SUs, i and j satisfy $0<p_i<P_i^{max}$ and $p_j=P_j^{max}$, then the following inequality must be satisfied:

$$\frac{h_i}{\lambda g_i+\mu}\le\frac{h_j}{\lambda g_j+\mu}. \quad (26)$$

Proof: Supposing that any two arbitrary SUs i and j with $0<p_i<P_i^{max}$ and $p_j=P_j^{max}$, from (13), (14), (16), (17) we have $\nu_i=0$, $\varepsilon_i=0$ and $\varepsilon_j=0$. Then, from (12), we have

$$\frac{h_i}{\lambda g_i+\mu}=\sigma^2+\sum_{n=1}^{N}p_nh_n, \quad (27)$$
$$\frac{h_j}{\lambda g_j+\mu+\nu_j}=\sigma^2+\sum_{n=1}^{N}p_nh_n. \quad (28)$$

Using the fact that $\nu_j\ge 0$, we get

$$\frac{h_i}{\lambda g_i+\mu}\le\frac{h_j}{\lambda g_j+\mu}. \quad (29)$$

Lemma 3 is thus proved. □

Without loss of generality, we assume that the SUs are sorted such that $\frac{h_1}{\lambda g_1+\mu}\ge\frac{h_2}{\lambda g_2+\mu}\ge\ldots\ge\frac{h_N}{\lambda g_N+\mu}$. Assuming that K SUs can transmit with nonzero power, from Lemma 1, Lemma 2 and Lemma 3 we have $p_i=p_i^{max},i=1,\ldots,K-1$ and $p_j=0,j=K+1,\ldots,N$. The transmit power of SU K satisfies $0<p_K\le p_K^{max}$. The following lemma gives the expression of p_K and the way to obtain the value of K.

Lemma 4: The transmit power of SU K is

$$p_K=\min\left(p_K^{max},\frac{1}{h_K}\left(\frac{h_K}{\lambda g_K+\mu}-\sigma^2-\sum_{n=1}^{K-1}p_n^{max}h_n\right)\right). \quad (30)$$

The value of K is the largest value of k such that

$$\sum_{n=1}^{k-1}p_n^{max}h_n<\frac{h_k}{\lambda g_k+\mu}-\sigma^2. \quad (31)$$

Proof: Supposing that $0<p_K<p_K^{max}$, then, from (13), (14), (16), (17), we have $\nu_K=0$ and $\varepsilon_K=0$. Inserting these results into (12), we have

$$\frac{h_K}{\lambda g_K+\mu}=\sigma^2+\sum_{n=1}^{N}p_nh_n. \quad (32)$$

Using $p_i = p_i^{max}, i = 1, \ldots, K-1$ and $p_j = 0, j = K+1, \ldots, N$, we can rewrite the above expression as

$$\frac{h_K}{\lambda g_K + \mu} = \sigma^2 + \sum_{n=1}^{K-1} p_n^{max} h_n + p_K h_K, \tag{33}$$

from which we have

$$p_K = \frac{1}{h_K}\left(\frac{h_K}{\lambda g_K + \mu} - \sigma^2 - \sum_{n=1}^{K-1} p_n^{max} h_n\right). \tag{34}$$

Since $p_K \leq p_K^{max}$, it then follows that

$$p_K = \min\left(p_K^{max}, \frac{1}{h_K}\left(\frac{h_K}{\lambda g_K + \mu} - \sigma^2 - \sum_{n=1}^{K-1} p_n^{max} h_n\right)\right). \tag{35}$$

Since $p_K > 0$, from (34), we have

$$\sum_{n=1}^{K-1} p_n^{max} h_n < \frac{h_K}{\lambda g_K + \mu} - \sigma^2. \tag{36}$$

For SUs $j = K+1, \ldots, N$, since $p_j = 0$ and $\nu_j = 0$, from (12), we have

$$\frac{h_j}{\lambda g_j + \mu - \varepsilon_j} = \sigma^2 + \sum_{n=1}^{K-1} p_n^{max} h_n + p_K h_k. \tag{37}$$

Since $p_K \leq P_K^{max}$ and $p_j = 0, j = K+1, \ldots, N$, we have

$$\frac{h_j}{\lambda g_j + \mu - \varepsilon_j} - \sigma^2 \leq \sum_{n=1}^{j-1} p_n^{max} h_n. \tag{38}$$

Consider the fact that $\varepsilon_j \geq 0$, we have

$$\frac{h_j}{\lambda g_j + \mu} - \sigma^2 \leq \sum_{n=1}^{j-1} p_n^{max} h_n. \tag{39}$$

The above inequality indicates that any SU $j \in \{K+1, \ldots, N\}$ does not satisfies the inequality $\sum_{n=1}^{j-1} p_n^{max} h_n < \frac{h_j}{\lambda g_j + \mu} - \sigma^2$. Thus, From (36), we conclude that the value of K is the largest value of k that satisfies the inequality in (31).

Lemma 4 is thus proved. □

Finally, the values of λ and μ can be obtained by the sub-gradient-based method. Our proposed optimal solution to P1 is summarized in Algorithm 1.

Remark 1: In the proof of Lemma 1, we assume that $\frac{h_i}{\lambda g_i + \mu} = \frac{h_j}{\lambda g_j + \mu}$ is impossible for arbitrary $i \in \{1, \ldots, N\}$ and $j \in \{1, \ldots, N\}, j \neq i$ due to the fact that the probability of satisfying the equality is zero. However, for extreme case when the equality holds, Lemma 1 will not be tenable. In such extreme case, Lemma 2 and Lemma 3 still hold, and in Lemma 4, it is easy to verify that the value of K is still the largest value of k such that the inequality in (31) is satisfied. Thus, based on above discussions, if $\frac{h_{K-1}}{\lambda g_{K-1} + \mu} = \frac{h_K}{\lambda g_K + \mu}$ does not hold, then Lemma 4 still establishes and Algorithm 1 does not need to be modified. Otherwise, if there exists a M $(1 \leq M < K)$ that satisfies the equality $\frac{h_M}{\lambda g_M + \mu} = \frac{h_{M+1}}{\lambda g_{M+1} + \mu} \cdots = \frac{h_K}{\lambda g_K + \mu}$, then we have $p_i = p_i^{max}, i = 1, \ldots, M-1$, $0 < p_j \leq p_j^{max}, j = M, \ldots, K$ and $p_k = 0$, $k = K+1, \ldots, N$. Thus, in this case, we need to determine $p_j, j = M, \ldots, K$, and we propose a heuristic scheme as follows. First, we let $p_j = p_j^{max}, j = M, \ldots, K-1$ and calculate p_K from (30). If p_K satisfies $0 < p_k \leq p_K^{max}$, then we have obtained $p_j, j = M, \ldots, K$. Otherwise, we iteratively decrease $p_j, j = M, \ldots, K-1$ as $p_j = \pi p_j, j = M, \ldots, K-1$ until $p_k = \frac{1}{h_K}\left(\frac{h_K}{\lambda g_K + \mu} - \sigma^2 - \sum_{n=1}^{M-1} p_n^{max} h_n - \sum_{n=M}^{K-1} p_n h_n\right) > 0$, where π is a bit less than 1.

Algorithm 1 Optimal power allocation with SIC.

1: Initialize $\lambda(0)$, $\mu(0)$, $t = 0$
2: Sort the N SUs such that $\frac{h_1}{\lambda g_1 + \mu} \geq \frac{h_2}{\lambda g_2 + \mu} \geq \ldots \geq \frac{h_N}{\lambda g_N + \mu}$.
3: **Repeat**
4: Find the largest value of k that satisfies $\sum_{n=1}^{k-1} p_n^{max} h_n < \frac{h_k}{\lambda g_k + \mu} - \sigma^2$ and denote it by K.
5: Set $p_i = p_i^{max}$ for $i = 1, \ldots, K-1$, $p_j = 0$ for $j = K+1, \ldots, N$ and obtain p_K from (30).
6: $d_1 = \sum_{i=1}^{N} p_i g_i - \bar{Q}$.
7: $d_2 = \sum_{i=1}^{N} p_i - \bar{P}$.
8: $\lambda(t+1) = (\lambda(t) + \theta_1 d_1)^+$.
9: $\mu(t+1) = (\mu(t) + \theta_2 d_2)^+$.
10: $t = t + 1$.
11: **Until** $|\lambda(t) - \lambda(t-1)| \leq \zeta_1$ and $|\mu(t) - \mu(t-1)| \leq \zeta_2$ are satisfied simultaneously.

where t is the iteration number, θ_1 and θ_2 are scalar step sizes, ζ_1 and ζ_2 denote the error tolerances.

IV. Power allocation without SIC

This section investigates the problem of power allocation for the SUs to maximize their sum rate without SIC capability at the CBS. The problem is formulated as

$$\text{P2}: \max_{\boldsymbol{p}} \sum_{i=1}^{N} \ln\left(1 + \frac{p_i h_i}{\sigma^2 + \sum_{j=1, j\neq i}^{N} p_j h_j}\right) \tag{40}$$

$$\text{s.t.} \sum_{i=1}^{N} p_i g_i \leq \bar{Q}, \tag{41}$$

$$\sum_{i=1}^{N} p_i \leq \bar{P}, \tag{42}$$

$$p_i \leq P_i^{max}, i = 1, \ldots, N \tag{43}$$

$$p_i \geq 0, i = 1, \ldots, N \tag{44}$$

It is easy to observe that the objective function in (40) is non-concave and thus P2 can not be solved by convex optimization. To solve the above problem, we introduce the following lemmas:

Lemma 5: At least one of the constraints in (41)-(44) is satisfied with equality for the optimal solution of P2.

Proof: Denote $f(\boldsymbol{p}) = \sum_{i=1}^{N} \ln\left(1 + \frac{p_i h_i}{\sigma^2 + \sum_{j=1, j\neq i}^{N} p_j h_j}\right)$ as the objective function in (40). We prove the lemma by contradiction. We assume that all of the constraints in (41)-(44) are satisfied with strict inequality for the optimal solution of P2. For an arbitrary $i \in \{1, \ldots, N\}$, the first derivative of

$f(\boldsymbol{p})$ with respect to p_i can be obtained as

$$\frac{\partial f(\boldsymbol{p})}{\partial p_i} = \frac{1}{\left(1+\frac{p_i h_i}{\sigma^2+\sum_{j=1,j\neq i}^{N} p_j h_j}\right)} \times \frac{h_i}{\sigma^2+\sum_{j=1,j\neq i}^{N} p_j h_j} - \sum_{n=1,n\neq i}^{N} \frac{1}{1+\frac{p_i h_i}{\sigma^2+\sum_{j=1,j\neq n}^{N} p_j h_j}} \times \frac{p_n h_n h_i}{\left(\sigma^2+\sum_{j=1,j\neq n}^{N} p_j h_j\right)^2} = \frac{h_i}{\sigma^2+\sum_{j=1}^{N} p_j h_j}\left(1-\sum_{n=1,n\neq i}^{N} \frac{p_n h_n h_i}{\sigma^2+\sum_{j=1,j\neq n}^{N} p_j h_j}\right). \tag{45}$$

It is observed that $\frac{h_i}{\sigma^2+\sum_{j=1}^{N} p_j h_j}$ is strictly positive and $1-\sum_{n=1,n\neq i}^{N} \frac{p_n h_n h_i}{\sigma^2+\sum_{j=1,j\neq n}^{N} p_j h_j}$ is a strictly increasing function of p_i. Therefore, $\frac{\partial f(\boldsymbol{p})}{\partial p_i}=0$ has one unique solution and is denoted as $p_i = x_i$. We discuss the optimal p_i in the following two cases:

Case 1: $x_i < 0$. In this case, for $0 \le p_i \le \min\left(P_i^{max}, \frac{\bar{Q}-\sum_{j=1,j\neq i}^{N} p_j g_j}{g_i}, \bar{P}-\sum_{j=1,j\neq i}^{N} p_j\right)$, we have $\frac{\partial f(\boldsymbol{p})}{\partial p_i} > 0$ and thus $f(\boldsymbol{p})$ is a increasing function of p_i. Thus, we can increase p_i to $\min\left(P_i^{max}, \frac{\bar{Q}-\sum_{j=1,j\neq i}^{N} p_j g_j}{g_i}, \bar{P}-\sum_{j=1,j\neq i}^{N} p_j\right)$ to increase the value of $f(\boldsymbol{p})$. This will make one of the constraints in (41)-(43) become equality constraint. This contradicts with our assumption that the constraints in (41)-(43) are satisfied with strict inequality.

Case 2: $x_i > P_i^{max}$. In this case, for $0 \le p_i \le \min\left(P_i^{max}, \frac{\bar{Q}-\sum_{j=1,j\neq i}^{N} p_j g_j}{g_i}, \bar{P}-\sum_{j=1,j\neq i}^{N} p_j\right)$, we have $\frac{\partial f(\boldsymbol{p})}{\partial p_i} < 0$ and thus $f(\boldsymbol{p})$ is a decreasing function of p_i. Based on this, we can decrease p_i to 0 to increase the value of $f(\boldsymbol{p})$. This will lead to equality constraint in (44) for the arbitrary $i \in \{1,\ldots,N\}$, which contradicts with our assumption that the constraints in (44) are satisfied with strict inequality.

Case 3: $0 \le x_i \le P_i^{max}$. In this case, we have $\frac{\partial f(\boldsymbol{p})}{\partial p_i} < 0$ for $0 \le p_i < x_i$ and $\frac{\partial f(\boldsymbol{p})}{\partial p_i} > 0$ for $p_i > x_i$, and thus $f(\boldsymbol{p})$ first decreases as p_i increases for $0 \le p_i < x_i$ and then increases as p_i increases for $p_i > x_i$. Thus, the value of $f(\boldsymbol{p})$ can be increased by increasing p_i to $\min\left(P_i^{max}, \frac{\bar{Q}-\sum_{j=1,j\neq i}^{N} p_j g_j}{g_i}, \bar{P}-\sum_{j=1,j\neq i}^{N} p_j\right)$ or decreasing p_i to 0. This results in one of the constraints in (41)-(44) becoming equality constraint, which contradicts with our assumption that all of the constraints in (41)-(44) are satisfied with strict inequality.

Lemma 5 is thus proved. □

Lemma 6: At most one SU's transmit power satisfies $0 < p_i < P_i^{max}, i \in \{1,\ldots,N\}$ for the optimal solution of P2.

Proof: If both the two constraints in (41) and (42) are satisfied with strict inequality, then according to Lemma 5, $p_i, \forall i$ is either 0 or P_i^{max}. Thus, there is no SU's transmit power satisfies $0 < p_i < P_i^{max}$.

Next, considering the case that at least one of the two constraints in (41) and (42) is satisfied with equality. For arbitrary $x \in \{1,\ldots,N\}$ and $y \in \{1,\ldots,N\}, y \neq x$, we assume that p_x and p_y satisfies $0 < p_x < P_x^{max}$ and $0 < p_y < P_y^{max}$, respectively. First, we consider the case that $h_x > h_y$ and $\sum_{i=1}^{N} p_i = \bar{P}$. Then p_y can be written as $p_y = \bar{P} - p_x - A$, where $A = \sum_{j=1,j\neq x,j\neq y}^{N} p_i$. Let $B = \sum_{j=1,j\neq x,j\neq y}^{N} p_j h_j$ and $C_i = \sum_{j=1,j\neq i,j\neq x,j\neq y}^{N} p_j h_j$. Then, $f(\boldsymbol{p})$ can be rewritten as

$$\begin{aligned} f(\boldsymbol{p}) &= \sum_{i=1}^{N} \ln\left(1+\frac{p_i h_i}{\sigma^2+\sum_{j=1,j\neq i}^{N} p_j h_j}\right) \\ &= \ln\left(1+\frac{p_x h_x}{\sigma^2+(\bar{P}-p_x-A)h_y+B}\right) \\ &+ \ln\left(1+\frac{(\bar{P}-p_x-A)h_y}{\sigma^2+p_x h_x+B}\right) \\ &+ \sum_{i=1,i\neq x,i\neq y}^{N} \ln\left(1+\frac{p_i h_i}{\sigma^2+p_x h_x+(\bar{P}-p_x-A)h_y+C_i}\right) \end{aligned} \tag{46}$$

The first derivative of $f(\boldsymbol{p})$ with respect to p_x can then be obtained as

$$\begin{aligned} \frac{\partial f(\boldsymbol{p})}{\partial p_x} &= \frac{1}{\sigma^2+B+(\bar{P}-A)h_y+(h_x-h_y)p_x} \\ &\times \left(\frac{(\sigma^2+B)h_x+(\bar{P}-A)h_x h_y}{\sigma^2+B+(\bar{P}-A)h_y-h_y p_x}\right. \\ &- \frac{(\sigma^2+B)h_y+(\bar{P}-A)h_x h_y}{\sigma^2+B+h_x p_x} \\ &\left.+ \sum_{i=1,i\neq x,i\neq y}^{N} \frac{-p_i h_i(h_x-h_y)}{\sigma^2+C_i+(\bar{P}-A)h_y+(h_x-h_y)p_x}\right). \end{aligned} \tag{47}$$

It is observed that, since $h_x > h_y$, $1/\sigma^2+B+(\bar{P}-A)h_y+(h_x-h_y)p_x$ is positive, and the remaining part of $\frac{\partial f(\boldsymbol{p})}{\partial p_x}$ is a increasing function of p_x. Thus, the solution to $\frac{\partial f(\boldsymbol{p})}{\partial p_x}=0$ is unique, and is denoted as z. It is easy to observed that $\left.\frac{\partial f(\boldsymbol{p})}{\partial p_x}\right|_{p_x=0} < 0$. Thus, we have $z > 0$. We discuss the optimal p_x in the following three cases:

Case 1: $z > P_x^{max}$. In this case, $f(\boldsymbol{p})$ is a decreasing function of p_x for $0 \le p_x \le P_x^{max}$. Thus, we can decrease p_x to 0 to increase the value of $f(\boldsymbol{p})$. This contradicts with our assumption that $0 < p_x < P_x^{max}$.

Case 2: $0 < z \le P_x^{max}$. In this case, $f(\boldsymbol{p})$ first decreases as p_x increases for $0 \le p_x < z$ and then increases as p_x increases for $p_x > z$. Thus, the value of $f(\boldsymbol{p})$ can be increased by increasing p_i to P_x^{max} or decreasing p_i to 0. This contradicts with our assumption that $0 < p_x < P_x^{max}$.

For the other cases such as $h_y > h_x$ and $\sum_{i=1}^{N} p_i g_i = \bar{Q}$, we can write $p_x = \frac{\bar{Q}-p_y g_y-\sum_{i\neq x,i\neq y}^{N} p_i g_i}{g_x}$ and use the same method to prove that $0 < p_y < P_y^{max}$ is not optimal.

Lemma 6 is thus proved □

Based on Lemma 5 and Lemma 6, we propose a heuristic algorithm to solve P2 as shown in Algorithm 2.

V. Simulation Results

In this section, simulations are presented to verify the performance of the proposed algorithms. All the channels involved are assumed to follow Rayleigh fading with unit mean. The noise power is set to 1. For simplicity, the peak

Algorithm 2 Heuristic power allocation without SIC.

1: Sort the N SUs such that $g_1 P_1^{max} < g_2 P_2^{max} < \ldots < g_N P_N^{max}$.
2: Set $p_i = P_i^{max}, i = 1, \ldots, N$.
3: If $\sum_{i=1}^{N} p_i g_i \leq \bar{Q}$, then set $n = N$; if $p_1 g_1 > \bar{Q}$, then set $n = 0$; otherwise, find the value of n such that $\sum_{i=1}^{n} p_i g_i < \bar{Q}$, $\sum_{i=1}^{n+1} p_i g_i > \bar{Q}$, and set $p_i = 0, i = n+1, \ldots, N$.
4: **While** $\sum_{i=1}^{N} p_i > \bar{P}$ **do**
5: Set $p_n = 0$.
6: $n = n - 1$.
7: **End while**
8: If $n < N$, then set $p_{n+1} = \min\left(\frac{\bar{Q} - \sum_{i=1}^{n} p_i g_i}{g_{n+1}}, \bar{P} - \sum_{i=1}^{n} p_i\right)$.

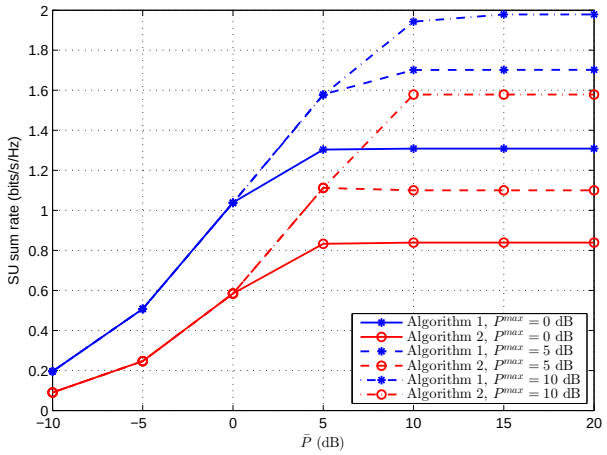

Fig. 2. SU sum rate against $\bar{P}$ ($N = 5$, $\bar{Q} = 0$ dB).

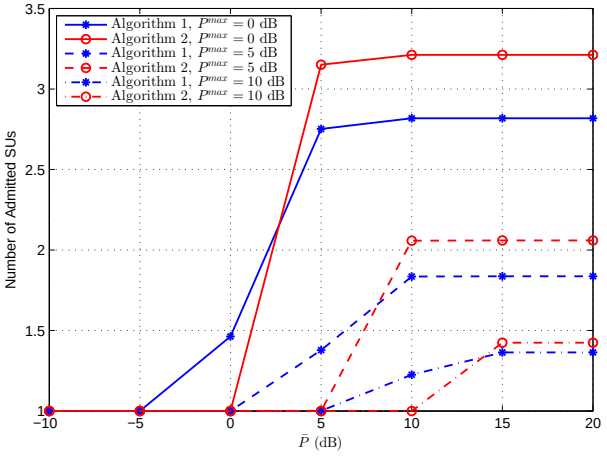

Fig. 3. Number of admitted SUs against $\bar{P}$ ($N = 5$, $\bar{Q} = 0$ dB).

transmit power limits of all the SUs are assumed to be the same and denoted as P^{max}. The following results are obtained by averaging over 1000 simulation runs.

Figs. 2 and 3 plot the SU sum rate and the number of admitted SUs, respectively, against $\bar{P}$ under various values of P^{max}. It is noted that the admitted SUs' transmit powers are nonzero. It is seen that the SU sum rate achieved by Algorithm

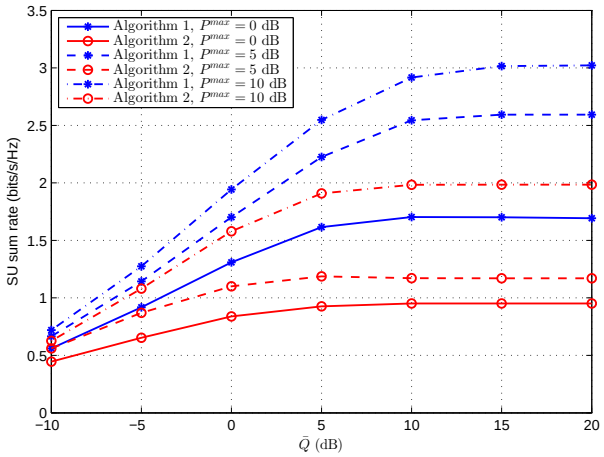

Fig. 4. SU sum rate against $\bar{Q}$ ($N = 5$, $\bar{P} = 10$ dB).

1 with SIC is always higher than that achieved by Algorithm 2 without SIC. This is due to the fact that SIC can cancel mutual interference between the SUs and thus enhance the performance of the SUs. It is also seen that the SU sum rates under various values of P^{max} are the same and increase as $\bar{P}$ increases for a small value of $\bar{P}$, and then saturate to different levels for a high value of $\bar{P}$. This is due to the fact that the SU performance is limited by $\bar{P}$ for a small value of $\bar{P}$ and increasing the value of P^{max} does not improve the SU sum rate. It is seen that higher P^{max} results in higher saturated SU sum rate. In addition, it is observed that the number of admitted SUs increases as $\bar{P}$ increases and then keep constant for a high value of $\bar{P}$. This is because that increasing $\bar{P}$ results in more transmit power that can be allocated to the SUs for a small value of $\bar{P}$. It is also observed that, compared to the number of admitted SUs achieved by Algorithm 2, Algorithm 1 achieves higher number of admitted SUs when $\bar{P}$ is medium and achieves lower number of admitted SUs when $\bar{P}$ is high. It is seen that such transition point of $\bar{P}$ decreases as P^{max} decreases. This indicates that, although Algorithm 2 without SIC performs worse than Algorithm 1 with SIC in terms of the SU sum rate, it can admit more number of SUs for a large $\bar{P}$. Furthermore, it is seen that a lower value of P^{max} leads to larger number of admitted SUs. This is due to the fact that a lower P^{max} will cause the SU with better channel condition to consume less transmit power and thus more SUs can be admitted.

Figs. 4 and 5 plot the SU sum rate and the number of admitted SUs, respectively, against $\bar{Q}$ under various values of P^{max}. Similar to Fig. 2, it is seen that Algorithm 1 achieves higher SU sum rate than that of Algorithm 2. It is also seen that the SU sum rates under various values of P^{max} are different and increase as $\bar{Q}$ increases and then saturate to different values for a high value of $\bar{Q}$. This is because that the performance of the SUs is limited by $\bar{Q}$ for a small value of $\bar{Q}$ and increasing the value of $\bar{Q}$ can improve the SU sum rate, while for a large value of $\bar{Q}$, the performance of the SUs is limited by $\bar{P}$ and increasing the value of $\bar{Q}$ can not improve the SU performance. In addition, it is observed that the number of admitted SUs increases as $\bar{Q}$ increases and then

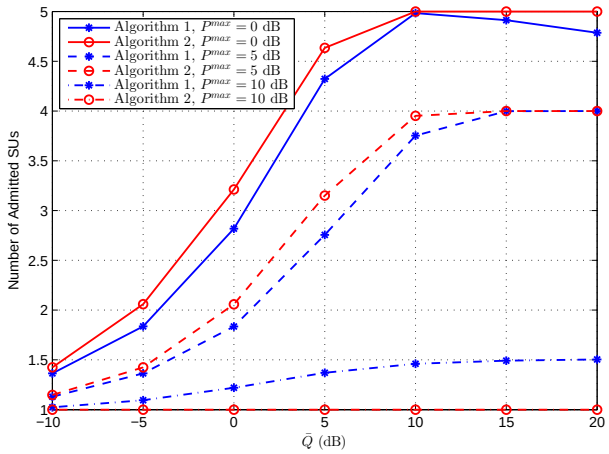

Fig. 5. Number of admitted SUs against $\bar{Q}$ ($N = 5$, $\bar{P} = 10$ dB).

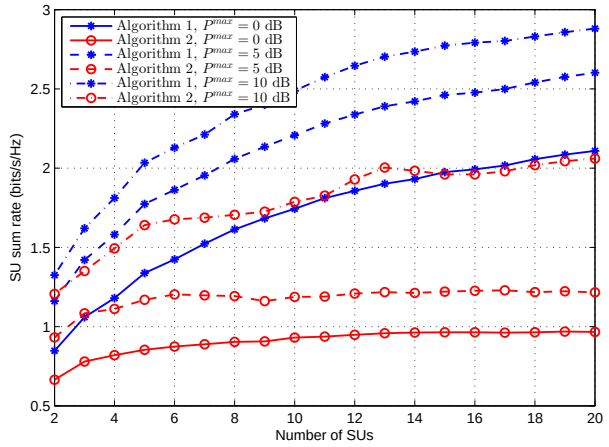

Fig. 6. SU sum rate against N ($\bar{P} = 10$ dB, $\bar{Q} = 0$ dB).

keep constant in most cases for a high value of $\bar{Q}$. This is owning to the fact that increasing $\bar{Q}$ results in more transmit power that can be allocated to the SUs for a small value of $\bar{Q}$, while for a large $\bar{Q}$, $\bar{P}$ restricts the transmit power of the SUs and thus increasing $\bar{Q}$ can not increase the number of admitted SUs. Furthermore, it is observed that Algorithm 2 achieves lower number of admitted SUs than that of Algorithm 1 when P^{max} is high, and achieves higher number of admitted SUs than that of Algorithm 1 when P^{max} is low. This indicates that, Algorithm 2 without SIC performs better than Algorithm 1 with SIC in terms of the number of admitted SUs for a small P^{max}.

Figs. 6 and 7 plot the SU sum rate and the number of admitted SUs, respectively, against the number of SUs N under various values of P^{max}. It is seen that the SU sum rate increases with the increase of N. This is because that a larger value of N results in higher probability that more SUs are in good channel conditions which leads to higher SU sum rate. It is also seen that the impact of increasing N on the SU sum rate for a large N is weaker than that for a small N. In addition, It is seen that the number of admitted SUs increases as N increases except the case that P^{max} is very large for Algorithm 2. This may be due to the fact that a larger value of N leads to higher probability that more SUs are in good

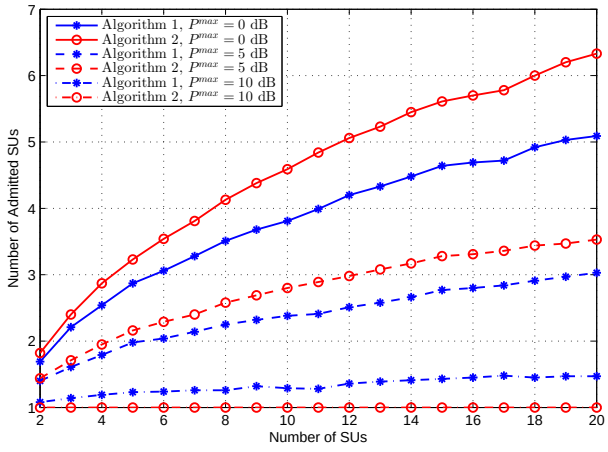

Fig. 7. Number of admitted SUs against N ($\bar{P} = 10$ dB, $\bar{Q} = 0$ dB).

channel conditions which results in higher number of admitted SUs, while for a very large P^{max}, there may exist some SUs with good channel conditions that consume all the transmit power resources which results in constant number of admitted SUs.

VI. Conclusions

We consider a fading CMAC with a power station harvesting energy from the nature and then providing power to the SUs. The problems of power allocation for such a CMAC to maximize the SU sum rate under the interference power constraint, the sum transmit power constraint and the peak transmit power constraint of each individual SU are investigated for two scenarios: with SIC and without SIC. The optimal power allocation algorithm for the first scenario and a heuristic algorithm for the second scenario are proposed respectively. It is shown that the proposed algorithm with SIC achieves higher SU sum rate than that of the algorithm without SIC, while the number of admitted SUs achieved by the algorithm without SIC is higher than that achieved by the algorithm with SIC for a high sum transmit power limit and a low peak transmit power limit of each individual SU. It is also shown that the number of admitted SUs increases with the decrease of the peak transmit power limit of each individual SU.

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