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The Method of Reversible Circuits Design with One-gate Prediction

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***Abstract*—This paper presents an original method of designing reversible circuits. This method is destined to most popular gate set with three types of gates CNT (Control, NOT and Toffoli). The presented algorithm based on graphical representation of the reversible function is called s-maps. This algorithm allows to find optimal or quasi-optimal reversible circuits. The paper is organized as follows. Section 1 recalls basic concepts of reversible logic. Especially the cascade of the gates as realization of reversible function is presented. In Section 2 there is introduced a classification of minterms distribution. The s-maps are the representation of the reversible functions where the minterms distribution is presented. The choice of the first gate in the cascade depends on possibility of improving the distribution. Section 3 describes the algorithm, namely how to find the optimal or quasi-optimal solutions of the given function.**

***Keywords*—reversible logic, reversible circuits, reversible gate, CNT set of the gates**

I. Introduction

SYNTHESIS of the optimal reversible circuits especially for more than four variables is a complex problem and requires time consuming programs and large space of memory. Researches try to find various algorithms whose implementation is easy by programs [1-7]. The method presented in this paper is a complement to previous works [8]. There was presented a graphical representation of reversible function. There were introduced s-maps in order to illustrate the minterm distribution for given reversible function. To achieve a better transparency the method will be presented only for reversible functions with three variables but it is scalable to more variables.

In this paper we will be use the CNT set of the gates containing 12 gates. Four gates with XOR gate on line Y_0 are shown in Fig. 1.

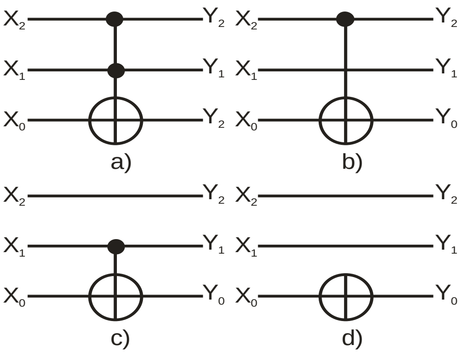

Fig.1. Four reversible gates with XOR on line Y_0: a) T0, b) C0-2, c) C0-1, d) N0

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The gates from Fig. 1 implement the reversible function:
$Y_2 = X_2$
$Y_1 = X_1$
$Y_0 = X_0 \oplus ab$ where:
a = X_2 and b= X_1 Fig. 1a
a = X_2 and b= 1 Fig. 1b
a = 1 and b= X_1 Fig. 1c
a = 1 and b= 1 Fig. 1d

The other eight gates are defined in the same manner: four with XOR on line Y_2 (T2, C2-1, C2-0, N2) and four on line Y_1 (T1, T1-2, T1-0, N1).

The implementation of the given reversible function is the cascade of reversible gates. There are two types of cascades shown in Fig. 2.

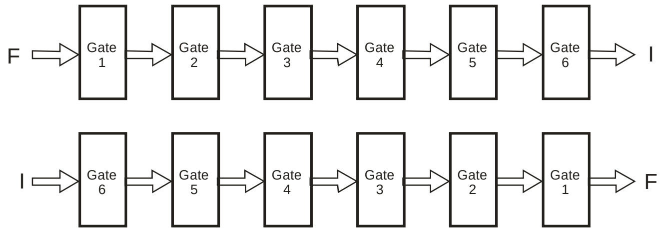

Fig. 2. Cascades with 6 reversible gates

The upper cascade in Fig. 2 transforms the function F into I (I is the identical function - left side of the true table in Table I) and the bottom cascade transforms function I into the given function F.

Let be given the reversible function of three variables presented in Table I.

TABLE I
EXAMPLE *OF THREE VARIABLE REVERSIBLE FUNCTION*

No.	$X_2X_1X_0$	$Y_2Y_1Y_0$
0	000	000
1	001	001
2	010	010
3	011	100
4	100	011
5	101	101
6	110	110
7	111	111

The function from Table I could be presented as minterms permutation i.e. <0,1,2,4,3,5,6,7>. This is a sequence of the vectors $Y_2Y_1Y_0$. Only two vectors (100 and 011) are in improper places relative to the identical function.

The graphical representation of the identical function I is shown in the Fig. 3. There are three s-maps, one for each line Y_i. In the upper row of each map there are minterms containing zeros for the variable X_i and in the bottom row there are minterms containing one.

Y_2

N2	C2-0	C2-1	T2
0	1	2	3
4	5	6	7

Y_1

N1	C1-0	C1-2	T1
0	1	4	5
2	3	6	7

Y_0

N0	C0-1	C0-2	T0
0	2	4	6
1	3	5	7

Fig. 3. Graphical representation of function I <0,1,2,3,4,5,6,7>.

The columns of these maps are marked by the reversible gates names. These gates swapped the minterms in given columns. The gates Ti swapped the minterms in the last column (right columns in every s-maps). The gates C swapped the minterms in suitable column and the minterms in the last column. The gate Ni swaps minterms in all columns in a given s-map.

In Table II there are collected the gates and suitable minterms of function I swapped when the gate is used.

TABLE II
SWAPPED MINTERMS FOR REVERSIBLE GATES

Gate	Swapped rows
T0	6,7
C0-1	2,3 & 6,7
C0-2	4,5 & 6,7
N0	0,1 & 2,3 & 4,5 & 6,7
T1	5,7
C1-0	1,3 & 5,7
C1-2	4,6 & 5,7
N1	0,2 & 1,3 & 4,6 & 5,7
T2	3,7
C2-0	1,5 & 3,7
C2-1	2,6 & 3,7
N2	0,4 & 1,5 & 2,6 & 3,7

In Fig. 4 there is shown the graphical representation of the reversible function from Table I. The fields in s-maps indicated by minterms from Fig. 3 will be called the proper fields for given minterms.

Y_2

N2	C2-0	C2-1	T2
0	1	2	4
3	5	6	7

Y_1

N1	C1-0	C1-2	T1
0	1	3	5
2	4	6	7

Y_0

N0	C0-1	C0-2	T0
0	2	3	6
1	4	5	7

Fig. 4. Graphical representation of function <0,1,2,4,3,5,6,7>.

Two maps from Fig. 3 and Fig. 4 differ from the position of minterms 3 and 4.

Y_2

N2	C2-0	C2-1	T2
0	1	2	**4**
3	5	6	7

Y_1

N1	C1-0	C1-2	T1
0	1	**3**	5
2	**4**	6	7

Y_0

N0	C0-1	C0-2	T0
0	2	**3**	6
1	**4**	5	7

Fig. 5. Graphical representation of function <0,1,2,4,3,5,6,7> with highlighted fields.

Both minterms 3 and 4 on all s-maps in Fig. 4 are on the improper fields and are called improper minterms. These improper fields on the s-maps in Fig. 5 are highlighted. We will highlight only these minterms if in a given s-map for the reversible function they are in a different row than in the s-map for an identical function.

For example, there is given the function <4, 0, 3, 2, 7, 6, 5, 1>. The true table of this function is shown in Table III.

TABLE III
EXAMPL***E OF FUNCTION*** <4, 0, 3, 2, 7, 6, 5, 1>

No.	$X_2X_1X_0$	$Y_2Y_1Y_0$
0	000	100
1	001	000
2	010	011
3	011	010
4	100	111
5	101	110
6	110	101
7	111	001

The three s-maps for the function from Table III are presented in Fig. 6. To design the reversible circuits implementation this function should eliminate all highlighted fields. The first gate in the cascade must give a better minterms distribution.

Y_2

N2	C2-0	C2-1	T2
4	0	3	2
7	6	5	**1**

Y_1

N1	C1-0	C1-2	T1
4	0	**7**	**6**
3	2	**5**	**1**

Y_0

N0	C0-1	C0-2	T0
4	**3**	**7**	**5**
0	**2**	**6**	1

Fig. 6. The s-maps for the function <4, 0, 3, 2, 7, 6, 5, 1> with highlighted fields.

Minterm 0 is on place 1. This place is not highlighted for Y_2 and Y_1 line because minterm 0 is in the upper row and X_2 and X_1 have the same value as minterm 1. For line Y_0 this minterm is highlighted because it is placed in the bottom row and has different value than the minterm 1.

Assuming one gate cascade with the gate C1-2 and on the input of this gate there will be our given function. This gate swaps two pairs rows: 4 with 6 and 5 with 7. For the given function on the place 4 there is minterm 7, on the place 6 is minterm 5, on the place 5 there is minterm 6 and on the place 7 there is minterm 1. On the output of the gate C1-2 there will be the function G. The s-maps of the function G are shown in Fig. 7.

Y_2

N2	C2-0	C2-1	T2
4	0	3	2
5	**1**	7	6

Y_1

N1	C1-0	C1-2	T1
4	0	5	1
3	2	7	6

Y_0

N0	C0-1	C0-2	T0
4	**3**	**5**	**7**
0	**2**	1	**6**

Fig. 7. The s-maps for the function <4, 0, 3, 2, 5, 1, 7, 6>

The function shown in Fig. 7 is called the rest function after gate C1-2. The s-map corresponding to the line Y_1 does not have highlighted fields. All minterms in the upper row have the same value (zero) as the minterms of the function I and all minterms in the bottom row have the same value (one) as the minterms of the function I. The s-maps of the function G have a better minterms distribution.

Definition 1. The line Y_i is ordered if s-map corresponding with this line does not have highlighted fields.

If we compare the minterms distributions between s-maps from Fig. 6 and Fig. 7 we will see that the gate C1-2 swap two minterms pair in two last columns of the s-map Y_1. In the remaining s-maps these pairs are located in the same rows. The first pair (5 and 7) is in the bottom row of the s-map Y_2 and in the upper row of the s-map and in the upper row of the s-map Y_0. The second pair (1 and 6) is in the bottom row of both s-maps. In general case each gate with XOR on line Y_i swaps the minterms in columns of the s-map Y_i (columns swapping) and in the rows of the remaining s-maps (rows swapping).

The problem of synthesising the reversible function is to eliminate the highlighted fields. It is particularly important to find a cascade with minimal number of gates. These cascades will be called the optimal cascades. To find the optimal cascade a designer must find the gate which may be an initial gate in the cascade (the gate 1 in the upper cascade in Fig. 2). The same must be repeated for the rest function and so on. Usually exist more than one optimal cascade for given function. The designer must find the set of the gates as candidates for first gates of optimal cascades. This set will be called the set of the best gates. In order to find this set will be introduced the criterions for selection the gates as the members of this set.

II. Distribution of improper minterms

The each gate in optimal cascade must bring the rest functions closer to identical function. The rest function after each gate have to leads to sequentially ordering each s-maps. To ordering any s-map the designer should find such gate that the rest function after this gate will have better minterms distribution. Better distribution means distribution requiring fewer gates to order the s-map.

For example, for the given function from Fig. 5 one should order the s-map Y_2. It is necessary to shift minterm 3 to the last position in the bottom row and then the gate T2 will be ordered this s-map. The minterm 3 is located in the s-maps Y_1 and Y_0 in one before last column. The gate C1-2 and C0-2 swap minterm 3 respectively with minterm 6 and minterm 5. Four pairs of gates C1-2 and T0, C1-2 and C0-2, C0-2 and T1, C0-2 and C1-2 shift the minterm 3 to the last column in thes-map Y_2. Finally the gate T2 ordering s-map Y_2. In this case we say that the gates C0-2 and C1-2 improve the minterms distribution.

In order to eliminate the highlighted fields in the s-map Y_i we should shift in rows these fields to create highlighted columns. It is possible to use the gates on the lines Y_j when $j \neq i$. The ordering is ended by the gate with XOR on line Y_i.

Definition 1. The output function Yi will be called ordered if all minterms contained zero on the position X_i of the identical function I are in upper row of s-map for Y_i. All minterms contained one on position X_i of identical function I are in the bottom row of the s-map for Y_i.

Two fields in all s-maps for the given function have special attributes. The field <000> (left field in the upper row) and the field <111> (right field in bottom row) contain the same minterms. From Table II we can observe that each gate with XOR on the line Y_i swapped in the column minterm placed in the field <111>. Moreover, minterm placed in the field <000> is swapped in the column only by gate N_i.

Lemma 1. If in field <000> there is minterm 0 then there exist an optimal cascade without gate N_i.

Proof. Minterm 0 is in the same fields in all s-maps. Using the gates C and T it is possible to swap minterms 4, 2 and 1 under the field <000> respectively in the lines Y_2, Y_1 and Y_0. Then the first columns in all s-maps will contain the proper minterm.

Conclusion 1. To improve minterms distribution of the reversible function with minterm 0 on the field <000>it is necessary to swap improper minterm from fields under the field <000>.

Conclusion 2. The gate which minimizes the number of highlighted fields improves minterms distribution of the reversible function with the minterm 0 on the field <000>.

Conclusion 3. The gate which changes columns with one highlighted field into a column with both highlighted fields improve minterms distribution of the reversible function with the minterm 0 on the field <000>.

Conclusion 4. The gate which maximizes the number of highlighted fields improves minterms distribution of the reversible function whose minterm on the field <000> is different than 0.

From these conclusions there will be introduced criterions in order to find a set of the best gates for the first place in the cascade.

III. Gates punctuation

The punctuation of the gates will be proposed as the measurement of the improvement degree of the minterms distribution. Based on this punctuation, a set of the gates will be selected and one of them is the first gate in the cascade. 9 criterions will be introduced

1. If in the s-map of the given function all highlighted fields are corresponding to any gate G this gate receives +2 points. All remaining gates on this line receive -2 points because they destroy this minterms distribution.
2. If the s-map of the rest function after the gate G has all highlighted fields corresponding to any gate the gate G receive +2 points. In this case a iven line could be ordered by the next gate.
3. If the s-map of the rest function after the gate G has destroyed the highlighted fields corresponding to any gate the gate G receives -2 points.

Four additional criterions only for s-maps of given functions when minterm 0 is in proper field:

4. If the s-map of the given function and the rest function after the gate G have the same number of highlighted fields but more columns with highlighted fields than the gate G receive +1 point.

5. If the s-map of the given function and the rest function after the gate G have the same number of highlighted fields but fewer columns with highlighted fields then the gate receives -1 point.
6. If the s-map of the rest function after the gate G has fewer highlighted columns, the gate G receive +k points, where k is the difference between the number of highlighted columns in function s-maps and the rest function s-maps.
7. If the s-map of the rest function after the gate G has more highlighted columns, the gate G receives -k points, where k is the difference between the number of highlighted columns in function s-maps and the rest function s-maps.

There are two additional criterions only for s-maps of given functions when minterm 0 is in an improper field:

8. If the s-map of the rest function after the gate G has all highlighted fields corresponding to any gate then the gate G receives +2 point.
9. If the s-map of the rest function after the gate G contains minterm 0 on the proper place the gate receives +2 points.

If any of these criterions are not met, the algorithm requires consideration of all gates as best gates.

IV. Example

To illustrate the gates punctuation we will take the reversible function from Fig 6. The results of the punctuation are collected in Table IV.

TABLE IV
RANKING OF ALL 12 GATES

Gate	Punctuation for swapping row $Y_2Y_1Y_0$	Criterion of punctuation
T0	0,0,0	
C0-2	0,0,+1	6
C0-1	0,0,+1	6
N0	0,0,+2	9
T1	0,-2,0	
C1-2	0,+2,0	1
C1-0	0,-2,0	
N1	0,-2,0	
T2	0,-2,0	3
C2-1	-1,-2,0	3
C2-0	-1,-2,0	3
N2	-2,-2,0	3

A set with four best gates: C0-2, C0-1, N0 and C1-2 is the result of the punctuation from Table IV In this place the algorithm receives four branches and all of them must be considered by the designer. The consideration gives only three ways to obtain the optimal cascades. The branch with gate C0-1 will be eliminated. There is also another way to receive an optimal cascade after the gate C2-0 is chisen. This case will be discussed in a future paper. Now we will consider one of these branches.

Let us assume the gate C0-2 as the first gate in the cascade. To find the second gate in the cascade we should repeat this procedure. The new minterm distribution of the rest function after gate C0-2 is shown in Fig. 8.

Y_2				Y_1				Y_0			
N2	C2-0	C2-1	T2	N1	C1-0	C1-2	T1	N0	C0-1	C0-2	T0
4	0	3	2	4	0	6	7	4	3	6	1
6	7	1	5	3	2	1	5	0	2	7	5

Fig. 8. Graphical representation of rest function after the gate C0-2

The punctuation of the gates is collected in Table V.

TABLE V
RANKING OF ALL 12 GATES

Gate	Punctuation for swapping row $Y_2Y_1Y_0$	Criterion of punctuation
T0	0,0,0	
C0-2	0,0,-1	
C0-1	0,0,+1	6
N0	0,+2,0	1
T1	0,-2,0	1
C1-2	+1,+2,0	4,2
C1-0	0,-2,0	1
N1	0,-2,0	1
T2	-1,-2,0	1
C2-1	-1,-2,0	1
C2-0	-2,-2,0	1
N2	-2,-2,0	1

The result of the punctuation from Table V is a set with three best gates: C0-1, N0 and C1-2. Let us assume the gate N0 as the next gate in the cascade. The minterm distribution of the rest function after two gates in the cascade (C0-2 and N0) is shown in Fig. 9 and the punctuation of the gates is collected in Table VI.

TABLE VI
RANKING OF ALL 12 GATES

Gate	Punctuation for swapping row $Y_2Y_1Y_0$	Criterion of punctuation
T0	0,-2,0	3
C0-2	-1,-2,0	3
C0-1	0,-2,0	3
N0	-2,-2,0	3
T1	+1,-2,0	6,3
C1-2	+1,+2,0	6,1
C1-0	0,-2,0	3
N1	0,-2,0	3
T2	0,-2,0	3
C2-1	-1,-2,0	3
C2-0	0,-2,0	3
N2	-2,-2,0	3

Y_2				Y_1				Y_0			
N2	C2-0	C2-1	T2	N1	C1-0	C1-2	T1	N0	C0-1	C0-2	T0
0	4	2	3	0	4	7	6	0	2	7	5
7	6	5	1	2	3	5	1	4	3	6	1

Fig. 9. The s-maps of rest function after the gates C0-2, N0

The result of punctuation from Table VI is only one best gate C1-2. The minterm distribution of the rest function after gates C0-2, N0 and C1-2 is in Fig. 10.

Y_2				Y_1				Y_0			
N2	C2-0	C2-1	T2	N1	C1-0	C1-2	T1	N0	C0-1	C0-2	T0
0	4	2	3	0	4	5	1	0	2	5	7
5	1	7	6	2	3	7	6	4	3	1	6

Fig. 10. Graphical representation of rest function after the gates C0-2, N0 and C1-2

After three gates in the cascade (C0-2, N0 and C1-2) the line Y_1 is ordered. Only the gates with XOR on the lines Y_2 and Y_0 will be used. In Table VII the punctuation of the gates is collected.

TABLE VII
RANKING OF ALL 12 GATES

Gate	Punctuation for swapping row $Y_2Y_1Y_0$	Criterion of punctuation
T0	0,-,+1	3
C0-2	-1,-,+1	3
C0-1	0,-,0	3
N0	0,-,0	3
T1	-	
C1-2	-	
C1-0	-	
N1	-	
T2	-1,-,-1	3
C2-1	-2,-,0	3
C2-0	+2,-,+2	3
N2	-2,-,0	3

The set of the best gates contains two gates: T0 and C2-0. If we try to select T0 gate we obtain more gates than after the choice gate C2-0. It is a result of the big advantage of the gate C2-0 over the gate T0. The rest function after the gates C0-2, N0, C1-2 and C2-0 is shown in Fig. 11.

Y_2				Y_1				Y_0			
N2	C2-0	C2-1	T2	N1	C1-0	C1-2	T1	N0	C0-1	C0-2	T0
0	1	2	6	0	1	5	4	0	2	5	7
5	4	7	3	2	6	7	3	1	6	4	3

Fig. 11. Graphical representation of rest function after the gates C0-2, N0, C1-2 and C2-0

The result of the gates punctuation of the function from Fig. 11 is collected in Table VIII. The best gate set contains only one gate T2. In this place of algorithm we are to find the part of cascade with five gates:

C0-2, N0, C1-2, C2-0, T2

The s-maps of the rest function after the gates C0-2, N0, C1-2, C2-0 and T2 is shown in Fig. 12.

TABLE VIII
RANKING OF ALL 12 GATES

Gate	Punctuation for swapping row $Y_2Y_1Y_0$	Criterion of punctuation
T0	-2,-,0	3
C0-2	-2,-,+1	3
C0-1	-2,-,0	3
N0	-2,-,0	3
T1	-	
C1-2	-	
C1-0	-	
N1	-	
T2	+2,-,+2	3
C2-1	-2,-,0	3
C2-0	-2,-,0	3
N2	-2,-,0	3

Y_2				Y_1				Y_0			
N2	C2-0	C2-1	T2	N1	C1-0	C1-2	T1	N0	C0-1	C0-2	T0
0	1	2	3	0	1	5	4	0	2	5	7
5	4	7	6	2	3	7	6	1	3	4	6

Fig. 12. Graphical representation of rest function after the gates C0-2, N0, C1-2, C2-0 and T2

Although the algorithm requires a ranking table we can see that there exists only one best gate C0-2. The s-maps of the rest function after the gates C0-2, N0, C1-2, C2-0, T2 and C0-2 are shown in Fig. 13. These s-maps are the s-maps of the identical function I. The found cascade with six gates is the optimal cascade for this function.

Y_2				Y_1				Y_0			
N2	C2-0	C2-1	T2	N1	C1-0	C1-2	T1	N0	C0-1	C0-2	T0
0	1	2	3	0	1	4	5	0	2	4	6
4	5	6	7	2	3	6	7	1	3	5	7

Fig. 13. Graphical representation of rest function after the gates C0-2, N0, C1-2, C2-0, T2 and C0-2

In practice we found four optimal cascades because two neighbouring gates with XOR on the same line could be exchanged. The found sequence contains two such pairs. They are underlined below:

C0-2, N0, C1-2, C2-0, T2, C0-2

Then our four cascades are:

C0-2, N0, C1-2, C2-0, T2, C0-2

C0-2, N0, C1-2, C2-0, C0-2, T2

N0, C0-2, C1-2, C2-0, C0-2, T2

N0, C0-2, C1-2, C2-0, T2, C0-2

In fact this function has 23 optimal solutions. The proposed here algorithm leads to 21 cascades. During the first step of the algorithm we obtain the set with four best gates: C0-2, C0-1, N0 and C1-2. Then we select only the C0-2 gate. Other branches give seven cascades with the C1-2 gate as the first gate and ten cascades with N0 gate (we can see this from table IV). When the gate C0-1 as the first gate is selected in the cascade then we obtain cascades with seven gates i.e. non-optimal.

The one-gate prediction algorithm does not find two optimal cascades. These cascades begin with the C2-0 gate. In the first step of the above algorithm this gate receives -3 points (-1,-2,0) and does not find place in the best gates set. The s-maps of the rest function after gate C2-0 are shown in Fig. 14.

Y_2

N2	C2-0	C2-1	T2
4	6	3	1
7	0	5	2

Y_1

N1	C1-0	C1-2	T1
4	6	7	0
3	1	5	2

Y_0

N0	C0-1	C0-2	T0
4	3	7	5
6	1	0	2

Fig. 14. Graphical representation of rest function after the gates C2-0

The ranking table of this function is shown in Table IX. The best gates set contains four gates is : C0-2, N0, C1-2 and C1-0.

TABLE IX
RANKING OF ALL 12 GATES

Gate	Punctuation for swapping row $Y_2Y_1Y_0$		Criterion of punctuation
T0	+1,-1,0	8	
C0-2	+2,0,0	6	6
C0-1	+1,-2,0	7	6
N0	0,0,+2	9	9
T1	0,-1,0	5	
C1-2	0,+2,0	3	1
C1-0	0,+2,+1	4	
N1	0,0,0	A	
T2	0,-1,0	2	3
C2-1	-1,-2,0	0	3
C2-0	+1,0,0	1	3
N2	0,0,0	b	3

After further algorithm application only the C0-2 gate gives us the minimal solution. The rest function after the gates C2-0 and C0-2 is shown in Fig. 15.

Y_2

N2	C2-0	C2-1	T2
4	6	3	1
0	7	2	5

Y_1

N1	C1-0	C1-2	T1
4	6	0	7
3	1	2	5

Y_0

N0	C0-1	C0-2	T0
4	3	0	2
6	1	7	5

Fig. 15. Graphical representation of rest function after the gates C2-0, C0-2

In Table X there is collected the punctuation of the rest function from Fig. 15 ea. the rest function after gates C2-0 and C0-2. The set of the best gates contains five gates: C1-0, T2, C2-1, C2-0 and N2.

The further procedure application will give us only two minimal cascades with six gates:

C2-0,C0-2,C1-0,N2,T0,C2-0
C2-0,C0-2,N2,C1-0,T0,C2-0

This last consideration leads us to the statement that the presented algorithm allows to find only some of optimal solutions. There are optimal cascades which require the prediction more than one gate and then a different punctuation.

TABLE X
RANKING OF ALL 12 GATES

Gate	Punctuation for swapping row $Y_2Y_1Y_0$		Criterion of punctuation
T0	0,-2,-1	8	
C0-2	0,-2,-2	6	6
C0-1	0,-2,-1	7	6
N0	0,-2,-2	9	9
T1	0,-2,0	5	
C1-2	-1,-2,0	3	1
C1-0	0,+2,+1	4	
N1	0,-2,0	A	
T2	0,+2,0	2	3
C2-1	-1,+2,0	0	3
C2-0	0,+2,0	1	3
N2	0,+2,0	b	3

V. CONCLUSIONS

The main aim of this paper is to create a design of optimal reversible cascades which enables implementation of the given function. The presented algorithm for the synthesis of three variable reversible functions allows to design optimal reversible circuits for the majority of the functions. Presented algorithm may be performed also by a “manual” process. This algorithm is scalable for more variables. Especially it is possible to transform this algorithm into a software algorithm.

Further works will be provided to find a new punctuation which will allow to find all optimal cascades.

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