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OFDM for Cognitive Radio Systems: Novel Power Allocation and Bit Loading Algorithms

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***Abstract*—A novel method to improve the performance of the frequency band is cognitive radio that was introduced in 1999. Due to a lot of advantages of the OFDM, adaptive OFDM method, this technique is used in cognitive radio (CR) systems, widely. In adaptive OFDM, transmission rate and power of subcarriers are allocated based on the channel variations to improve the system performance. This paper investigates adaptive resource allocation in the CR systems that are used OFDM technique to transmit data. The aim of this paper is to maximize the achievable transmission rate for the CR system by considering the interference constraint. Although secondary users can be aware form channel information between each other, but in some wireless standards, it is impossible for secondary user to be aware from channel information between itself and a primary user. Therefore, due to practical limitation, statistical interference channel is considered in this paper. This paper introduces a novel suboptimal power allocation algorithm. Also, this paper introduces a novel bit loading algorithm. In the numerical results sections, the performance of our algorithm is compared by optimal and conventional algorithms. Numerical results indicate our algorithm has better performance than conventional algorithms while its complexity is less than optimal algorithm.**

***Keywords*—Cognitive Radio; OFDM, Interference Constraints, Power Allocation, Adaptive Modulation, Bit Loading**

I. INTRODUCTION

DUE to increase wireless systems and users, spectrum channel is becoming more and more rare resource. Hence, improving spectrum performance is an important research topic in academia [1]. Several technologies have introduced by researchers to improve the system performance. An attractive technique to improve the channel utilization performance is a cognitive radio (CR) that it uses an unused portion of the spectrum channels [2]. In the CR, secondary users (SU) can utilize spectrum bands of primary users (PU). SU can consider interference that is introduced on the PU and keep is less than specified threshold [3]. In some CR systems, SUs and PUs are presented in adjacent frequency bands. In these systems, SUs use unused portions of frequency band between PUs. In this case, both primary and secondary users introduce harmful interference to spectrum band of each other. This interference can reduce the system performance [4]. Therefore, some constraints should be considered by SU to prevent any harmful interference on the PUs band [5].

Adaptive resource allocation such as power and modulation level can improve the system performance. In the adaptive power and modulation level allocation technique, power and modulation level are allocated based on channel fading gains [6]. This technique guarantees SU can utilize the channel in an optimal manner [7-8]. Modern wireless communication systems have been used M-QAM modulation technique to broadcast, widely [9-10]. By using adaptive modulation, the system can transmit its data with maximum transmission rate in all situations [11].

A powerful technique to use unused portions of the band is OFDM technique [12]. Adaptive resource allocation can improve the performance of the OFDM systems. In this technique more power is allocated to subcarriers that have higher channel fading gain and vice versa [13].

There are several challenges for OFDM-based CR systems; first, calculating an optimal power allocation is too difficult, second, it is difficult or may be impossible for SU to obtain of instantaneous channel fading gain between SU and PU and, third, allocating discrete number of bits to each subcarrier. These challenges have investigated in some papers. For instance, in [14] authors introduced a suboptimal algorithm to calculate a transmit power, but this algorithm can be used only when both PU and SU use the same OFDM systems. The method was proposed in [15] calculated transmission power of SU based on distance between each subcarrier and the PU in the frequency domain, but this method is to complex. A low-complexity method was introduced in [16]. In this paper, researchers assumed all subcarriers introduce equal interference on PU bands. In some papers such as [17, 18], authors prefer to use an iterative algorithm to calculate transmit power of SU.

This paper considers two problems; first calculate a suboptimal power allocation algorithm to allocate transmit power to subcarriers while the SU uses M-QAM modulation. In addition, the instantaneous channel fading information between SU transmitter and receiver (SUT and SUR) and statistical property information of channel between secondary user transmitter and primary user receiver are available at SUT side. The second aim of this paper is bit loading where discrete bit numbers is allocated to subcarriers based on the allocated power.

This paper is organized as follows. System model, proposed power allocation algorithm and conventional power loading algorithms are described in sections II, III and IV, respectively. In addition, bit loading algorithm is introduced in section V. In the section VI, numerical results are explained and conclusion is described in the section VII.

II. MODEL SYSTEM

Figure 1 indicated the system model in the frequency domain where a SU user and several PUs are located near to each other. SU uses OFDM modulation to use frequency holes. Frequency holes are divided to N flat subcarriers with

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bandwidth *Δf.* The PUs spectrum bands are equal to B. maximum interference threshold for PUs is Q_{th}. At the SUT side, SUT be aware from instantaneous SUT-SUR channel and statistical SUT-PUR channel state information.

Figure 2, shows a system model. SUR-SUR channel gain is shown by h_i^{ss}, where i is subcarrier index. SUT- ℓ-th PUR channel fading gain is shown by h_ℓ^{sp}. In addition, we assume SUT uses an ideal Nyquist pulse to transmit its signals.

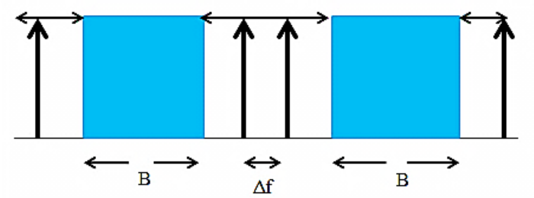

Fig. 1. Spectrum band of the primary and secondary users

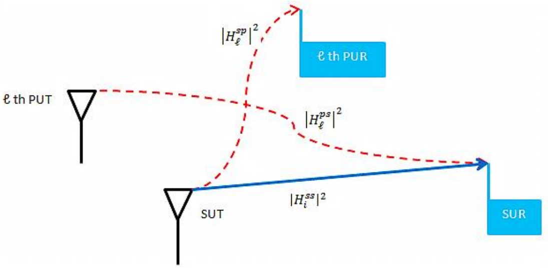

Fig. 2. Location of PUs and SU

When the SUT uses M-QAM policy, the BER can be approximated by following equation [19]:

$$BER_i \approx 0.2\exp\left(\frac{-1.5P_i\left|h_i^{ss}\right|^2}{\left(M_i-1\right)\left(\sigma^2+\sum_{\ell=1}^{L}J_i^\ell\right)}\right) \tag{1}$$

where, P_i indicates transmission power of SU on ith subcarriers. σ^2 indicates the variance of the AWGN noise, J_i^ℓ is an interface that introduce by the ℓ-th PU on the i-th subcarrier and BER_0 is BER target. Therefore, the modulation level at i-th OFDM subcarrier can be calculated as follows:

$$M_i = 1+\frac{-1.5}{\ln\left(5BER_0\right)}\frac{P_i\left|h_i^{ss}\right|^2}{\sigma^2+\sum_{\ell=1}^{L}J_i^\ell} \tag{2}$$

Each transmission symbol includes b bits. The value of b is calculated by Equation (3) as follows:

$$b_i = \log_2\left(M_i\right) = \log_2\left(1+\frac{-1.5}{\ln\left(5BER_0\right)}\frac{P_i\left|h_i^{ss}\right|^2}{\sigma^2+\sum_{\ell=1}^{L}J_i^\ell}\right) \tag{3}$$

If symbol duration be equal to T_s, the transmission rate is calculated by following equation:

$$C_i = b_i \times \frac{1}{T_s} \tag{4}$$

The purpose is to maximize transmission rate of the SU by allocating adaptive power to OFDM subcarriers. SU should consider interference threshold (Q_{th}) that is introduced by it on the PUR. In addition, maximum allocated power is limited by practical constraint. Therefore, the optimization problem can write mathematically as follows:

$$C = \max_{P_i} \frac{1}{T_s}\sum_{i=1}^{N}\log_2\left(1+\frac{-1.5}{\ln\left(5BER_0\right)}\frac{P_i\left|h_i^{ss}\right|^2}{\sigma^2+\sum_{\ell=1}^{L}J_i^\ell}\right) \tag{5}$$

Subject to:

$$Pr.\left\{\sum_{i=1}^{N}I_i^\ell \le Q_{th}\right\} \ge a \ ; \ \forall \ell \tag{6}$$

$$\sum_{i=1}^{N}P_i \le P_{\max} \tag{7}$$

$$P_i \ge 0 \ ; \ \forall i \tag{8}$$

The interference that is introduced by the i-th subcarrier of SU on the PUR is related to SU's transmission power, symbol duration, and i-th OFDM subcarrier and PU spectral distance ($d_{i\ell}$) i. e.:

$$I_i^\ell = P_i K_i^\ell \tag{9}$$

where,

$$K_i^\ell = \left|h_\ell^{sp}\right|^2 T_s \int_{d_{i\ell}-B_\ell/2}^{d_{i\ell}+B_\ell/2}\left(\frac{\sin\left(\pi f T_s\right)}{\pi f T_s}\right)^2 df \tag{10}$$

The Equation (6) indicates the probability for being interference less than specified threshold is more than value a. This equation is used because the system is utilizing the statistical channel properties. It is assumed the SUT-PUR channel has Rayleigh distribution. Therefore the Equation (6) can be rewritten as follows:

$$1-\exp\left(-\frac{Q_{th}}{2\sigma_\ell^2\sum_{i=1}^{N}P_iK_i^\ell}\right) \ge a \tag{11}$$

where σ is Rayleigh parameter. The Equation (11) can be rewritten as follows:

$$\sum_{i=1}^{N}P_iK_i^\ell \le \frac{Q_{th}}{2\sigma_\ell^2\ln\left(\frac{1}{1-a}\right)} = I_{th} \qquad \forall \ell \tag{12}$$

This problem is convex optimization; therefore we apply KKT conditions to calculate optimal power at each subcarrier. The optimal transmit power at each subcarrier is obtained by following equation:

$$P_i^* = \max\left\{0, \frac{1}{\beta+\sum_{\ell=1}^{L}\gamma_\ell K_i^\ell} - \frac{\sigma^2+\sum_{\ell=1}^{L}J_i^\ell}{\frac{-1.5}{\ln\left(5BER_0\right)}\left|h_i^{ss}\right|^2}\right\} \tag{13}$$

Proof: the proof is given in Appendix.

III. PROPOSED SUBOPTIMAL ALGORITHM

Optimal solution is to complex because it should consider L+1 constraints due to coexist L PU in the adjacent of the SU and power budget limitation, simultaneously. Therefore, this solution cannot use in practical applications. In the suboptimal algorithm, constraints are considered, independently.

In a suboptimal algorithm two important issues should be considered; first, a constraint that introduces by SU on PU. This issue is so important because the SU should not produce harmful interference at PURs. The second issue is a sum of the noise and constraint that introduces by PUs on SU's subcarriers. Indeed, our purpose is to maximize overall transmit power of SU, to achieve maximum transmission rate. Hence, we should consider noise and interference at SUR and SUT-SUR channel fading information. Based on Equation (9), interference power is related to K_i. Based on Equation (9), allocated power to subcarriers has an inverse relationship to value of the K_i, i. e. less power should be assigned to subcarrier with high value of the K_i and vice versa. This policy helps us to guarantee required QOS for PUs. Also, we know noise and interference and channel power gain can affect the performance of the systems. Therefore, less power should be allocated to noisy channels and vice versa. We consider these criteria for both PU and SU and introduce novel suboptimal algorithm due to ℓ-th PU constraint as follows:

$$P_i^{\ell} = X\,\frac{\left|h_i^{ss}\right|^2}{\left(\sigma^2+\sum_{\ell=1}^{L} J_i^{\ell}\right)} \qquad (14)$$

Where, X is a constant value. By assuming strict equality in Equation (6), we calculate this constant value as follows:

$$X = \frac{I_{th}}{\sum_{i=1}^{N}\left(\dfrac{\left|h_i^{ss}\right|^2 \times K_i^{\ell}}{\sigma^2+\sum_{\ell=1}^{L} J_i^{\ell}}\right)} \qquad (15)$$

Now, by substation Equation (15) into Equation (14), the transmission power of SU due to considering constraints of ℓ-th PU activity is obtained as follows:

$$P_i^{\ell} = \frac{\left|h_i^{ss}\right|^2 \times I_{th}}{\left(\sigma^2+\sum_{\ell=1}^{L} J_i^{\ell}\right)\sum_{i=1}^{N}\left(\dfrac{K_i^{\ell}\times\left|h_i^{ss}\right|^2}{\sigma^2+\sum_{\ell=1}^{L} J_i^{\ell}}\right)} \qquad (16)$$

By considering power budget constraint and use water filling algorithm, transmission power of SU due to maximum power budget can be obtained as follows:

$$P_i^{\max} = \max\left\{0, \frac{1}{\mu} - \frac{\sigma^2+\sum_{\ell=1}^{L} J_i^{\ell}}{\dfrac{-1.5}{\ln\left(5BER_0\right)}\left|h_i^{ss}\right|^2}\right\} \qquad (17)$$

In Equation (17), μ represents Lagrange parameter. The Lagrange parameter can be obtained by filling the Equation (17) into Equation (7).

For L PUs, Equation 14 is calculated L times. The minimum power that is obtained from Equation (14) and Equation (17) can satisfy all problem constraints. Hence, the allocated power to i^{th} OFDM subcarrier is calculated as follows:

$$P_i = \min\left\{P_i^1, P_i^2, \ldots, P_i^L, P_i^{\max}\right\} \qquad (18)$$

IV. CONVENTIONAL ALGORITHMS

Several policies are used to allocate transmission power in OFDM systems. These algorithms need to encounter some changes for utilizing in CR systems. However, because of simplicity of these algorithms, in some applications they may be used. In this section, we describe two important algorithms.

A. *Uniform Loading Algorithm*

In this algorithm, equal power is allocated to all subcarriers without considering any constraints. It is obvious this method is so simple. For determining transmit power of subcarriers in OFDM-based systems, both interference power and maximum power constraints should be considered. By assuming Equation (12) as equality and solving it, the transmit power at each OFDM subcarrier is obtained as follows:

$$P_U^{\ell} = \frac{I_{th}}{\sum_{i=1}^{N} K_i^{\ell}} \qquad (19)$$

Similarity, by considering Equation (7) as equality, the transmit power due to this constraint is calculated as follows:

$$P_U^{\max} = \frac{P_{\max}}{N} \qquad (20)$$

The final allocated power to subcarriers based on uniform loading algorithm is calculated by Equation (21). Equation (21) satisfies all constraints in Equation (12) and Equation (7):

$$P = \min\left\{P_U^1, P_U^2, \ldots, P_U^L, P_U^{\max}\right\} \qquad (21)$$

B. *Water filling algorithm*

In the conventional OFDM systems, water filling algorithm is an optimal power allocation policy. However, in the OFDM-based CR systems, due to coexistence the PUs and a SU, this algorithm needs some changes. Also, water filling is not an optimal solution in the CR systems. Similar to standard water filling, the transmission power at each subcarrier is obtained as follows:

$$P_i^{WF} = \max\left\{0, \frac{1}{\mu} - \frac{\sigma^2+\sum_{\ell=1}^{L} J_i^{\ell}}{\dfrac{-1.5}{\ln\left(5BER_0\right)}\left|h_i^{ss}\right|^2}\right\} \qquad (22)$$

Where, μ is Lagrange parameter. It is necessary to consider all constraints in Equation (12) and Equation (7).

For calculating transmission power at i-th subcarrier due to ℓ-th PU constraint, we should use following equations:

$$P_{\max}^{u\ell} = N \times P_U^{\ell} \qquad (23)$$

$$\sum_{i=1}^{N}\max\left\{0, \frac{1}{\mu} - \frac{\sigma^2+\sum_{\ell=1}^{L} J_i^{\ell}}{\dfrac{-1.5}{\ln\left(5BER_0\right)}\left|h_i^{ss}\right|^2}\right\} = P_{\max}^{u\ell} \qquad (24)$$

For calculating Lagrange parameter due to maximum power budget we use following equation:

$$\sum_{i=1}^{N} \max\left\{0, \frac{1}{\mu} - \frac{\sigma^2 + \sum_{\ell=1}^{L} J_i^{\ell}}{\frac{-1.5}{\ln(5BER_0)}\left|h_i^{ss}\right|^2}\right\} = P_{\max} \tag{25}$$

Similar to previous algorithms, the final allocated power to subcarriers is a minimum value of the calculated power from Equation (24) and Equation (25).

V. BIT LOADING ALGORITHM

In the previous sections, optimal and suboptimal allocated power for OFDM-based CR systems is calculated. When the SU uses M-QAM modulation scheme, modulation level of all subcarriers can be obtained based on the allocated power to subcarriers. However, the value of the modulation level that is obtained by Equation (2) and Equation (3) is continues number, while, a number of bits per symbol should be integer. Several methods were introduced by researchers to allocate integer bits in symbol. In all papers, authors introduced an algorithm to allot maximum bits to symbol based on allocated power with considering all constraints in Equation (12) and Equation (7). In [14], [20] and [21] authors introduced three algorithms for allocating discrete bits to each subcarrier. Now, a novel suboptimal bit loading algorithm is introduced in this section.

Figure 3 indicates flowchart of this proposed algorithm. In this algorithm we calculate the integer number of bits per symbol at first and then obtain the required power for providing these bits for subcarriers. The function $\lceil x \rceil$ rounds x to the nearest integer that is greater than x. Now, we consider maximum power and interference power threshold constraints, separately. In each step, the algorithm calculates Δp for all subcarriers to determine worth of each extra bit at each subcarrier. Δp helps us to decrease maximum extra power at subcarriers by eliminating minimum number of bits. This step surveys maximum power budget constraint and then if total allocated power satisfies this constraint, the algorithm goes to next step otherwise the loop is repeated. In each repetition the algorithm decreases one bit at subcarrier which has maximum Δp. Transmit power corresponding to one bit, Δp, is obtained by Equation (26):

$$\Delta P_i = P_i(b_i) - P_i(b_i - 1) = \gamma_i\left(2^{b_i - 1}\right) \tag{26}$$

Where, γ is obtained by following equation:

$$\gamma_i = \frac{\sigma^2 + \sum_{\ell=1}^{L} J_i^{\ell}}{\frac{-1.5}{\ln(5BER_0)}\left|h_i^{ss}\right|^2} \tag{27}$$

In the next step, the algorithm investigates interference power constraints for all PUs and the loop is repeated until all constraints are fulfilled. Such as pervious section, in this step, the algorithm uses Equation (26) to decreases one bit at subcarrier which has a maximum Δp. At the last step of the algorithm, we select the minimum obtained power. The value of the bits is corresponding to minimum power is the solution of the problem.

VI. NUMERICAL RESULTS

This section investigates the proposed algorithms for power allocation and bit loading by employing numerical examples. It is assumed that number of primary users are equal to two (L=2) and the bandwidth of PU (B) is equal to 2MHz. Also, SU uses OFDM technique for using unused parts of spectrum bands and it divides spectrum into 6 subcarriers (N=6) with bandwidth (Δf) equal to 0.3125 MHz. Symbol duration for SU (T_s) is 4µs. It is assumed the distribution of the channels is Rayleigh. Average power gain for $|h_i^{ss}|^2$, $|h_1^{sp}|^2$ and $|h_2^{sp}|^2$ are -5, -10 and -7 dB, respectively. The values of J_i^{ℓ} and σ^2 are random values with averages 10^{-6} and 10^{-8} watt, respectively. The value of BER_0 is equal to 10^{-3}. The algorithm is run

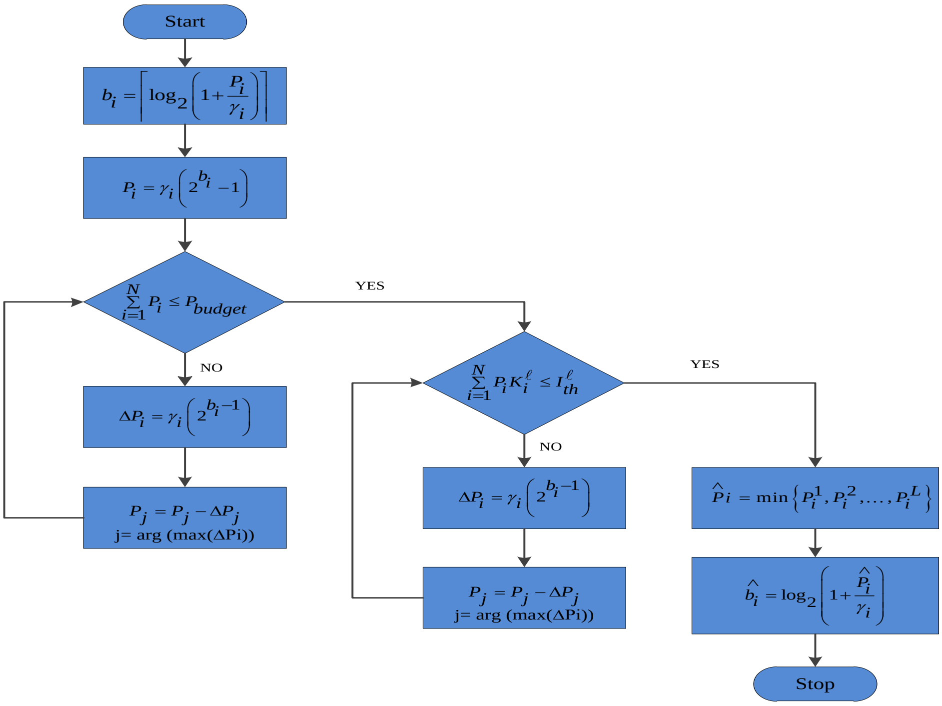

Fig. 3. Flowchart of discrete bit loading algorithm

100,000 times, independently, and results are an average results of these runs.

Figure 4, shows transmission rate vs. maximum power budget for all algorithms. The value of the interference threshold is 2×10^{-6} watt. Based on this figure, we can conclude the efficiency of the optimal algorithm is better than other algorithms. In addition, the performance of the proposed suboptimal algorithm is better than both conventional power loading algorithms. Also, we can conclude, the efficiency of the OFDM-based CR system is improved by increasing the amount of maximum power budget. It is obvious, because when the amount of maximum power budget increases, e more power can be allocated to subcarriers and therefore transmission rate increases.

Transmission rate for different values of the interference threshold for all algorithms is shown in figure 5. We assume the value of maximum power budget for SU is 5×10^{-4}. We observe the transmission rate increases by increasing maximum interference power threshold. When the value of the maximum interference power threshold increases, SU has chance to allocate more power to subcarriers. Therefore, the efficiency of the system increases. Moreover, in this figure we see the suboptimal algorithm that is introduced in this paper has a better performance than uniform loading and water filling algorithm and worse performance than optimal algorithm.

As mentioned in the previous sections, the performance of adaptive modulation system is related to the BER target. Figure 6, shows the performance of system vs. BER target for all algorithms. Similar to previous results, an optimal algorithm has the best efficiency. In addition, transmission rate that is obtained by proposed algorithm is more than the transmission rate when system uses uniform loading and water filling algorithms.

Complexity of system is an important factor to select an algorithm for OFDM-based systems.

The complexity of all algorithms is shown in Table 1. This table shows the proposed suboptimal power allocation algorithm complexity is more than uniform loading algorithm and equal to water filling algorithm. Also, optimal algorithm has the highest complexity.

Figure 7 indicates the performance of bit loading algorithm. The BER target, maximum power budget and interference power threshold are assumed to be 10^{-3}, 5×10^{-4} and 2×10^{-6}, respectively. The proposed suboptimal algorithm is used to determine allocated power of subcarriers. This figure indicates number of bits that are obtained by suboptimal power allocation algorithm may not integer, therefore we establish bit loading algorithm to assign integer number of bits to subcarriers. It is observed, although by applying the suboptimal bit loading algorithm, in the majority of subcarriers the number of bits per symbol decreases, in some subcarriers the number of bits increases. This enhancement is happened because we use [x] function in our algorithm. In fact, we try to satisfy constraints and allocate integer bits to subcarriers by decreasing minimum power.

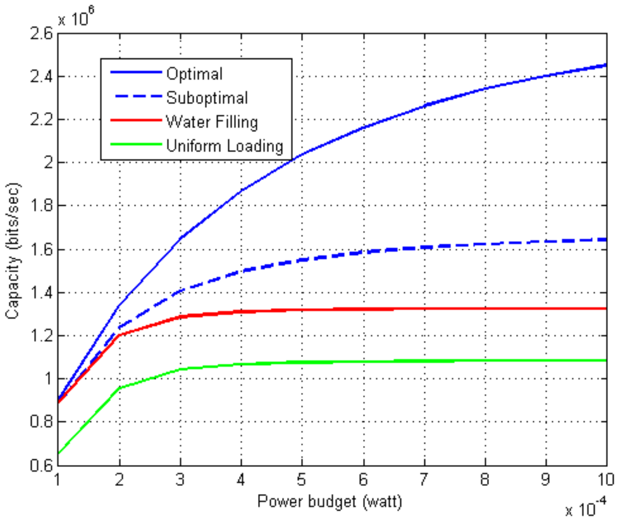

Fig. 4. Transmission rate for different values of maximum power budget

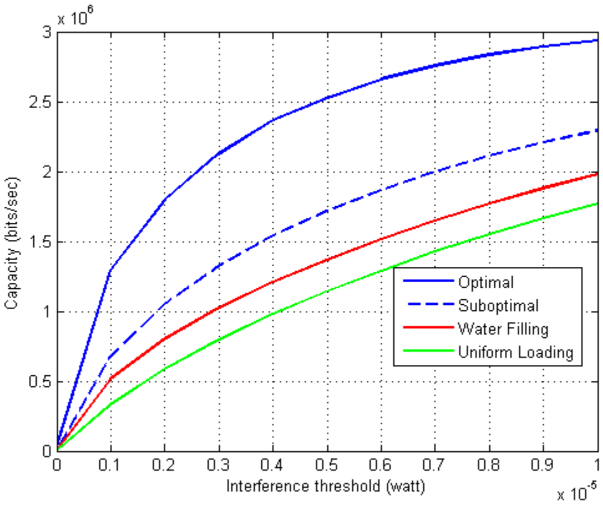

Fig. 5. Transmission rate vs. interference power threshold

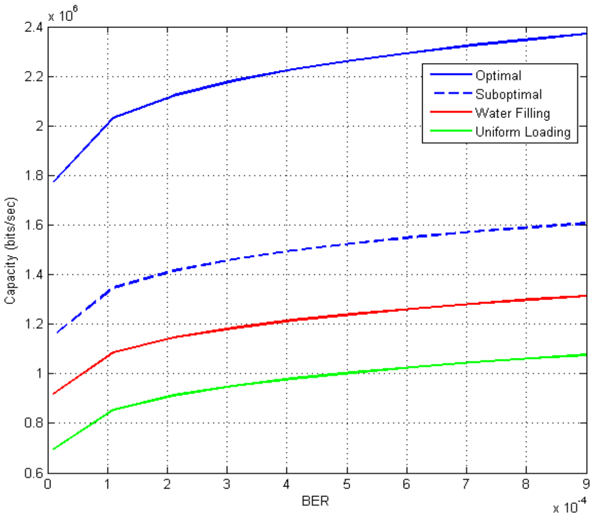

Fig. 6. Transmission rate for different value of BER target

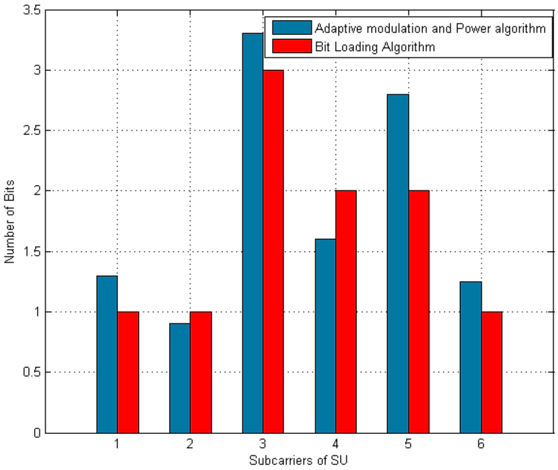

Fig. 7. Performance bit loading algorithm

TABLE I
Algorithms complexity

Algorithm	**Complexity**
Optimal Algorithm	O(N^3)
Proposed Suboptimal Algorithm	O(LN)+ O(N log(N))
Water Filling	O(LN)+ O(N log(N))
Uniform Loading	O(LN)

VII. CONCLUSION

A low-complexity suboptimal algorithm was proposed in this paper for allotting transmits power to subcarriers in OFDM-based CR systems. This suboptimal algorithm is based on interference power threshold that introduces by SUT on PURs, SUT-PUR channel state information and variance of noise and interferences product on SUR. Due to practical limitations, statistical interference constraint is considered in this paper. The proposed suboptimal power allocation algorithm has the better performance than water filling and uniform loading algorithms. In addition, the complexity of this algorithm is less than optimal power allocation algorithms. Problem is formulated such that we can calculate of modulation level of subcarriers based on allocated power. Indeed, in this method modulation level and transmission power allocated based on channel variations therefore it helps system to have the best performance in all situations. However, in practical applications the amount of modulation level must be power of 2 for example, 2, 4, 8, and etc., but modulation level that is obtained based allocated power may not satisfy this conditions. Therefore, we introduced an algorithm to determine the modulation level of subcarriers based on allocated power and practical conditions.

APPENDIX

The optimal solution is obtained by using Lagrange method for solving convex problems. Hence, we should use Lagrange method at first and then use KKT conditions as follows to obtain optimal solution for the problem.

$$L=-\sum_{i=1}^{N}\log_2\left(1+\frac{-1.5}{\ln(5BER_0)}\frac{P_i\left|h_i^{ss}\right|^2}{\sigma^2+\sum_{\ell=1}^{L}J_i^{\ell}}\right) \tag{A1}$$

$$-\alpha_i P_i+\beta\left(\sum_{i=1}^{N}P_i-P_{\max}\right)+\sum_{\ell=1}^{L}\sum_{i=1}^{N}\gamma_\ell\left(I_i-I_{th}\right)$$

$$\frac{\partial L}{\partial P_i}=0\Rightarrow\frac{1}{P_i+\dfrac{\sigma^2+\sum_{\ell=1}^{L}J_i^{\ell}}{\dfrac{-1.5}{\ln(5BER_0)}\left|h_i^{ss}\right|^2}}+\alpha_i-\beta-\sum_{\ell=1}^{L}\gamma_\ell K_i^{\ell}=0 \tag{A2}$$

$$\beta\geq 0 \tag{A3}$$

$$\gamma_\ell\geq 0\quad;\quad\forall\ell \tag{A4}$$

$$\alpha_i\geq 0\quad;\quad\forall N \tag{A5}$$

$$\beta\left(\sum_{i=1}^{N}P_i-P_{budget}\right)=0 \tag{A6}$$

$$\gamma_\ell\left(\sum_{i=1}^{N}I_i-I_{th}\right)=0\quad;\quad\forall\ell \tag{A7}$$

$$\alpha_i P_i=0 \tag{A8}$$

Removing α_i from A2 and then:

$$\frac{1}{P_i+\dfrac{\sigma^2+\sum_{\ell=1}^{L}J_i^{\ell}}{\dfrac{-1.5}{\ln(5BER_0)}\left|h_i^{ss}\right|^2}}\leq\beta+\sum_{\ell=1}^{L}\gamma_\ell K_i^{\ell} \tag{A9}$$

$$\frac{P_i}{P_i+\dfrac{\sigma^2+\sum_{\ell=1}^{L}J_i^{\ell}}{\dfrac{-1.5}{\ln(5BER_0)}\left|h_i^{ss}\right|^2}}-P_i\beta-P_i\sum_{\ell=1}^{L}\gamma_\ell K_i^{\ell}=0 \tag{A10}$$

If $\beta+\sum_{\ell=1}^{L}\gamma_\ell K_i^{\ell}\leq\dfrac{\dfrac{-1.5}{\ln(5BER_0)}\left|h_i^{ss}\right|^2}{\sigma^2+\sum_{\ell=1}^{L}J_i^{\ell}}$, then Equation (A9) can only hold if $P_i^*\geq 0$ and by solving Equation (A10), we have:

$$P_i^*=\frac{1}{\beta+\sum_{\ell=1}^{L}\gamma_\ell K_i^{\ell}}-\frac{\sigma^2+\sum_{\ell=1}^{L}J_i^{\ell}}{\dfrac{-1.5}{\ln(5BER_0)}\left|h_i^{ss}\right|^2} \tag{A11}$$

Optimal allocated power i^{th} OFDM subcarrier can be obtained as:

$$P_i^*=\max\left\{0,\frac{1}{\beta+\sum_{\ell=1}^{L}\gamma_\ell K_i^{\ell}}-\frac{\sigma^2+\sum_{\ell=1}^{L}J_i^{\ell}}{\dfrac{-1.5}{\ln(5BER_0)}\left|h_i^{ss}\right|^2}\right\} \tag{A12}$$

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