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ZA-APA with Adaptive Zero Attractor Controller for Variable Sparsity Environment

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***Abstract*—The zero attraction affine projection algorithm (ZA-APA) achieves better performance in terms of convergence rate and steady state error than standard APA when the system is sparse. It uses l_1 norm penalty to exploit sparsity of the channel. The performance of ZA-APA depends on the value of zero attractor controller. Moreover a fixed attractor controller is not suitable for varying sparsity environment. This paper proposes an optimal adaptive zero attractor controller based on Mean Square Deviation (MSD) error to work in variable sparsity environment. Experiments were conducted to prove the suitability of the proposed algorithm for identification of unknown variable sparse system**.

Keywords— Zero Attraction APA, sparse channel, convergence, steady state mean square error,variable zero attraction controller

I. Introduction

A SYSTEM is said to be sparse when the number of zero coefficients is more than the number of non-zero coefficients [1]. Some of the areas where sparsity can occur includes underwater acoustics [2], wireless multipath channel [3], hands free communication channel [4] etc. Not only sparse, in many of the real time systems, the level of sparsity also varies with time. Adaptive filters are type of filter with adjustable filter coefficients. They find application in various fields such as system identification, echo cancellation etc., The Least Mean Square (LMS), Normalized LMS and Affine Projection Algorithm (APA) are the famous type of adaptive algorithms. The LMS and NLMS are widely used because of their simplicity. APA is used because of the property of faster convergence and lower steady state error even though they have more complexity. Conventional adaptive filter do not have sparsity awareness term in them and hence their performance does not improve with increase in sparsity. Nowadays due to development in compression sensing, several sparsity aware norm penalized adaptive filters has gained more importance due to their improved performance for sparse channel [5]. The prominent ones are Zero attraction (ZA) type and reweighted ZA algorithms. ZA algorithms work by including l_1 norm penalty term into the original update equation of adaptive algorithm. The work of the zero attractor term is to attract the near to zero coefficients so that the convergence speed is accelerated. RZA algorithms selectively induces filter taps with small magnitude to zero rather than uniformly attract all filter taps to zero. This results in RZA to have better performance than ZA algorithm.

But selection of the parameters to obtain uniform shrinkage is a difficult task in RZA algorithm especially if the system is time varying. The common major drawback of these ZA sparsity aware adaptive filters is that they work well only when the level of sparsity is more and their performance detoriates with lesser sparsity, much lower than conventional algorithm. The ZA-APA which belong to this family has improved performance than conventional APA and LMS and ZA-LMS when the system is sparse [6,7]. One common problem seen in the ZA-APA is that they work well only for sparse environment and their performance decreases when the sparsity level is decreased. From the theoretical analysis [8] it is found that the zero attractor controller plays a key role in the final steady state error which need to be changed based on the sparsity level. The problem of time varying sparsity is solved in [9] using combinational approach .The major drawback of combinational approach is that the complexity is more. Variable zero attractor controller is proposed in [10, 16] for time varying sparse system. An adaptive zero Attractor for l_0 based LMS Algorithm is proposed [12] . The updation of zero attractor controller is obtained by maximizing the decrease in transient MSD. Simulations indicate the suitability of the algorithm for time varying environments. This is the motivation behind the proposed approach.

Hence in this paper, we resolve this variable sparsity environment problem by proposing an optimal adaptive zero attractor controller which is based on increasing the decrease in transient MSD. Firstly an optimal zero attractor controller is derived based on MSD error. Then an update rule is proposed which tend to vary the zero attractor controller based on the level of sparsity. A practical update rule is also proposed. It can be found that the proposed algorithm, during the sparse environment works similar to optimal ZA-APA and it works better than ZA-APA during semi sparse and non-sparse environments and thus makes the algorithm as a suitable candidate for variable sparsity environments. Finally the simulation in the context of sparse system identification verifies the performance of the proposed algorithm**.**

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This paper is organized as follows .Review of ZA-APA is provided in section 2.In section 3, optimal adaptive zero attractor controller is proposed. Simulations results are provided in section 4.Finally conclusions are discussed in section 5.

II.REVIEW OF ZA-APA

Consider an unknown system with impulse response w^o. The input signal is given by

$\boldsymbol{x}(n) = [x(n)\; x(n-1)\; x(n-2) x(n-N+1)]^T$.

The input is passed through the system with impulse response w^o to obtain the desired signal $d(n)$, which is modeled as a linear regression model given by

$\boldsymbol{d}(n) = \boldsymbol{x}(n)\boldsymbol{w}^o + v(n)$

Here $\mathbf{w^o}$ is the unknown weight vector which is needed to be estimated. v(n) is the measurement noise taken as a white Gaussian noise with zero mean and variance σ_v^2. n is the time index and N is the length of the input.The conventional APA with regularization (δ) computes the weight vector at each iteration given as

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu A^T(n)(\delta I + A(n)A^T(n))^{-1}\boldsymbol{e}(n) \quad (1)$$

Here I is the identity matrix of order N X N , μ is the step size and $\boldsymbol{e}(n)$ is the error vector given by

$\boldsymbol{e}(n) = \boldsymbol{d}(n) - \boldsymbol{y}(n)$

where $\boldsymbol{y}(n) = A(n)\boldsymbol{w}(n)$ is the estimated output. The desired response is given by

$\boldsymbol{d}(n) = [d(n)\; d(n-1) \dots\dots\dots\dots\dots d(n-P+1)]^T$ and$A(n) = [\boldsymbol{x}(n)\boldsymbol{x}(n-1)\boldsymbol{x}(n-2) \dots.\boldsymbol{x}(n-P+1)]^T$ is the projection vector obtained by taking the delayed version of input vector .Here P is projection order. Usually P is less than or equal to N. If l_1 norm penalty is included and if Lagrange's multiplier is used, the update equation of ZA-APA [7] is given by (2) as

$$w(n+1) = w(n) + \mu A^T(n)\big(\delta I + A(n)A^T(n)\big)^{-1}e(n) + \alpha\left(A^T(n)\big(\delta I + A(n)A^T(n)\big)^{-1}A(n)\right)sgn(w(n)) - \alpha sgn(w(n)) \quad (2)$$

Equation (2) consists of four terms .The first two terms are same as traditional APA whereas the third and the fourth terms are called as the zero attraction terms .They are responsible for the attraction of filter coefficients to zero when their magnitude is close to zero. It should be noted that the magnitude of attraction is controlled by the parameter α called as the zero attractor controller and $sgn(\boldsymbol{w}(n))$ is the component wise sign function defined as

$$sgn(w(n)) = \left\{\begin{matrix} \dfrac{w(n)}{|w(n)|} & if \;\; w(n) \neq 0 \\ 0 & if \;\; w(n) = 0 \end{matrix}\right\}$$

The literatures related to ZA-APA shows that the steady state performance of ZA-APA depends on the zero attractor controller [11]. Also it is found that ZA-APA cannot outperform APA when the system is non-sparse[8] .As the work of zero attractor controller is to attract the zero coefficients to zero and the application of attraction strength evenly to all filter taps, it is required to change the value of zero attractor controller based on the level of sparsity. Thus in order to make ZA-APA capable of working in all environmental conditions, we propose an optimal zero attractor controller which is also adaptive .The optimal value is based on the largest decrease in transient MSD error.

III.ADAPTIVE OPTIMAL ZERO ATTRACTOR CONTROLLER

In this section we first obtain an optimal zero attractor controller by maximizing the decrease in transient MSD from one iteration to the next. Thus the new update recursion of ZA-APA with adaptive zero attractor controller is given by

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu A^T(n)\big(\delta I + A(n)A^T(n)\big)^{-1}\boldsymbol{e}(n) + \alpha(n+1)\left(A^T(n)\big(\delta I + A(n)A^T(n)\big)^{-1}A(n)\right)sgn(\boldsymbol{w}(n)) - \alpha(n+1)sgn(\boldsymbol{w}(n)) \quad (3)$$

If the weight error vector is given by (1)

$\tilde{\boldsymbol{w}}(n) = \boldsymbol{w}^o - \boldsymbol{w}(n)$,then (2) in terms of weight error vector in recursive form can be written as

$$\tilde{w}(n+1)\; \tilde{w}(n) - \mu A^T(n)\big(\delta I + A(n)A^T(n)\big)^{-1}e(n) - \alpha A^T(n)\big(\delta I + A(n)A^T(n)\big)^{-1}A(n)sgn\big(w(n)\big) + \alpha sgn(w(n)) \quad (4)$$

If $e(n)$ is written in terms of weight error vector as $(n) = e_a(n) + v(n) = A(n)\,\tilde{w}(n) + v(n)$, we get

$$\tilde{\boldsymbol{w}}(n+1) = \big[\mathrm{I} - \mu A^T(n)\big(\delta I + A(n)A^T(n)\big)^{-1}A(n)\big]\,\tilde{\boldsymbol{w}}(n) - \mu A^T(n)\big(\delta I + A(n)A^T(n)\big)^{-1}\boldsymbol{v}(n) - \alpha A^T(n)\big(\delta I + A(n)A^T(n)\big)^{-1}A(n)sgn(\boldsymbol{w}(n)) + \alpha E[sgn(\boldsymbol{w}(n))] \quad (5)$$

In order to simplify the analysis ,we make use of the following assumptions

A.1.The input is independent and identically distributed (i.i.d) with zero mean and covariance R_x.

A.2. The noise is i.i.d with zero mean and variance σ_v^2 and is assumed to be independent of regressor (n) .

The system impulse response contains unknown coefficients and since the nature of these coefficients have different effect on the performance of the algorithm, it is necessary to classify the filter coefficients into two categories[17] so that these can be analyzed individually. Therefore the entire filter coefficients are classified as

Non zero coefficients (w_{NZ}):$w = w_{NZ}\; if\; |w(n)| > 0$

Zero coefficients (w_Z):$w = w_Z\; if\; w(n) = 0$

where 0 < n < N ,$w_{NZ} \cup w_Z = N$ and $w_{NZ} \cap w_Z = \emptyset$.If Q is the number of non zero filter coefficients ,then the number of zero filter coefficients is N-Q .The sparsity level is given by Q/N. Lesser value of Q / N gives higher level of sparsity

In order to obtain the transient MSD, we make use of IWV method. In [12], individual weight error variance vector (IWV) method is used to analyse the transient performance of ZA-NLMS algorithm. The advantage of IWV analysis is that it replaces the weight error covariance matrix by a column vector which separates the input terms without any approximations. Thus it relieves the dependency of the performance model on the metric matrix $\sum$.

If equation (3) is multiplied by its transpose, and taking expectation on both sides and using Kronecter product on both sides and by using Vec (abc) = ($c^T \otimes$ a) Vec (b), we get the following

$$Vec(E(\tilde{\boldsymbol{w}}(n+1)\tilde{\boldsymbol{w}}^T(n+1))) = Vec(E(\tilde{\boldsymbol{w}}(n)\tilde{\boldsymbol{w}}^T(n))) + \mu^2(E(Y \otimes Y))Vec(E(\tilde{\boldsymbol{w}}(n)\tilde{\boldsymbol{w}}^T(n))) + \mu^2\sigma_v^2 \text{Vec}(\text{E}(Z)) + \alpha^2(E(Y \otimes Y))Vec(E(sgn(\boldsymbol{w}(n))sgn(\boldsymbol{w}(n))^T)) + \alpha^2 Vec(E(sgn(\boldsymbol{w}(n))sgn(\boldsymbol{w}(n))^T)) - \mu(I \otimes E(Y))Vec(E(\tilde{\boldsymbol{w}}(n)\tilde{\boldsymbol{w}}^T(n))) - \alpha(I \otimes E(Y))Vec(E(sgn(\boldsymbol{w}(n))\tilde{\boldsymbol{w}}^T(n))) + \alpha Vec(E(sgn(\boldsymbol{w}(n))\tilde{\boldsymbol{w}}^T(n))) - \mu(E(Y) \otimes I)Vec(E(\tilde{\boldsymbol{w}}(n)\tilde{\boldsymbol{w}}^T(n))) + \mu\alpha E(Y \otimes Y)Vec(E(sgn(\boldsymbol{w}(n))\tilde{\boldsymbol{w}}^T(n))) - \mu\alpha(E(Y) \otimes I)Vec(E(sgn(\boldsymbol{w}(n))\tilde{\boldsymbol{w}}^T(n))) - \alpha(E(Y) \otimes I)Vec(E(\tilde{\boldsymbol{w}}(n)sgn(\boldsymbol{w}(n))^T)) + \mu\alpha(E(Y \otimes Y))Vec(E(\tilde{\boldsymbol{w}}(n)sgn(\boldsymbol{w}(n))^T)) - \alpha^2(E(Y) \otimes I)Vec(E(sgn(\boldsymbol{w}(n))sgn(\boldsymbol{w}(n))^T)) + \alpha Vec(E(\tilde{\boldsymbol{w}}(n)sgn(\boldsymbol{w}(n))^T)) - \mu\alpha(I \otimes E(Y))Vec(E(\tilde{\boldsymbol{w}}(n)sgn(\boldsymbol{w}(n))^T)) - \alpha^2(I \otimes \text{E}(Y))Vec(E(sgn(\boldsymbol{w}(n))sgn(\boldsymbol{w}(n))^T) \tag{6}$$

Where $= A^T(n)\left(A(n)A^T(n)\right)^{-1}A(n)$, $Z = A^T(n)\left(A(n)A^T(n)\right)^{-2}A(n)$ and I is the identity matrix of appropriate dimension. If $\text{P} = (I \otimes \text{E}(Y)) + (\text{E}(Y) \otimes I)$, $\text{Q} = \text{E}(Y \otimes Y)$,$\text{R} = \text{E(Z)}$ then

$$Vec(E(\tilde{\boldsymbol{w}}(n+1)\tilde{\boldsymbol{w}}^T(n+1))) = [I - \mu P + \mu^2 Q]\ Vec(E(\tilde{\boldsymbol{w}}(n)\tilde{\boldsymbol{w}}^T(n))) + \mu^2\sigma_v^2 Vec(R) + \alpha[(I - \mu(\text{E}(Y) \otimes I) - (I \otimes \text{E}(Y)) + \mu\,\text{E}(Y \otimes Y)Vec(E(sgn(\boldsymbol{w}(n))\tilde{\boldsymbol{w}}^T(n))) + (I - \mu(I \otimes \text{E}(Y)) - (\text{E}(Y) \otimes I) + \mu\,\text{E}(Y \otimes Y)Vec(E(\tilde{\boldsymbol{w}}(n)sgn(\boldsymbol{w}(n))^T))] + \alpha^2[I - P + Q]Vec(E(sgn(\boldsymbol{w}(n))sgn(\boldsymbol{w}(n))^T)) \tag{7}$$

If trace is taken on both sides of (7), we get the MSD of ZA-APA after some taking Vec^{-1} on both sides. As done in [12] if $Tr(XY) = \left(Vec(X)\right)^T Vec(Y)$ and $X = I$, we obtain

$$Tr\left(E\left(\tilde{\boldsymbol{w}}(n+1)\tilde{\boldsymbol{w}}^T(n+1)\right)\right) = Tr\left(E\left(\tilde{\boldsymbol{w}}(n)\tilde{\boldsymbol{w}}^T(n)\right)\right) + Vec(I)^T[-\mu P - \mu^2 Q]Vec\left(E\left(\tilde{\boldsymbol{w}}(n)\tilde{\boldsymbol{w}}^T(n)\right)\right) + Vec(I)^T\,\mu^2\sigma_v^2 Vec(R) + \alpha Vec(I)^T[X_1 Vec(E(sgn(\boldsymbol{w}(n))\tilde{\boldsymbol{w}}^T(n))) + X_2 Vec(E(\tilde{\boldsymbol{w}}(n)sgn(\boldsymbol{w}(n))^T))] + \alpha^2 Vec(I)^T[I - P + Q]Vec(E(sgn(\boldsymbol{w}(n))sgn(\boldsymbol{w}(n))^T)) \tag{8}$$

where $X_1 = [(I - \mu(\text{E}(Y) \otimes I) - (I \otimes \text{E}(Y)) + \mu\,\text{E}(Y \otimes Y))$ and $X_2 = (I - \mu(I \otimes \text{E}(Y)) - (\text{E}(Y) \otimes I) + \mu\,\text{E}(Y \otimes Y)$.If α is zero, then (8) reduces to conventional APA. If the step size is chosen to be as per [8], which is assumed to be a constant, then (7) is a second order quadratic equation with α as the polynomial. It order to get minimum MSD , the optimal value of α is obtained by differentiating (7) with respect α and equating it to zero. Thus we get

$$\alpha_o = -\frac{(Vec(I))^T(X_1 Vec(E(sgn(\boldsymbol{w}(n))\tilde{\boldsymbol{w}}^T(n))) + X_2 Vec(E(\tilde{\boldsymbol{w}}(n)sgn\left(\boldsymbol{w}(n)\right)^T)}{Vec(I)^T(2[I-P+Q]Vec(E\left(sgn\left(\boldsymbol{w}(n)\right)sgn(\boldsymbol{w}(n))^T\right)))} \tag{9}$$

In order to find optimal value α_o we need to eliminate the nonlinear terms. The optimal value is applicable for sparse system only because any value of α is not applicable for ZA-APA when the system is non-sparse [8] and ZA-APA cannot outperform APA when the system is non-sparse. Thus we make use of the assumption that the i[th] component of the weight deviation is assumed to follow Gaussian distribution [12]. Let $\gamma_1(\text{n}) = \text{E}(\text{sgn}(\mathbf{w}(\text{n})\text{sgn}(\mathbf{w}(\text{n}))^\text{T})$. If the approximation 1 of [12] is used, then

$$\gamma_1(\text{n}) = (\text{E}(\text{sgn}(\mathbf{w}(\text{n})\text{sgn}(\mathbf{w}(\text{n})^\text{T})))_{\text{i,k}} = \begin{Bmatrix} 1\ if\ i = k \\ E(sgn(w_i(n)))E(sgn(w_k(n)^T))\ if\ i \neq k \end{Bmatrix}$$

Let $\gamma_2(\text{n})$ be defined as $\gamma_2(\text{n}) = E(\tilde{\boldsymbol{w}}(n)sgn(\boldsymbol{w}(n))^T = \boldsymbol{w}^0 E(sgn(\boldsymbol{w}(n)^T)) - E(\boldsymbol{w}(n)sgn(\boldsymbol{w}(n)^T)))$

Using approximation 1 of [12], $E(\boldsymbol{w}(n)sgn(\boldsymbol{w}(n)^T))$ can be written as

$$E(\boldsymbol{w}(n)sgn(\boldsymbol{w}(n)^T))_{i,k} = \begin{Bmatrix} E(\boldsymbol{w}(n)_i E(sgn(\boldsymbol{w}(n)^T))_k\ if\ i \neq k \\ E(\|\boldsymbol{w}(n)_i\|\quad if\ i = k \end{Bmatrix} \tag{10}$$

If the weights are assumed to follow Gaussian distribution [9,12] with zero mean and variance $\sigma_{w,j}^2$ and if it is assumed that $E(|\boldsymbol{w}_j(n)_i| = \boldsymbol{w}_j^0$,then using folded normal distribution we can write as

$$E(\|\boldsymbol{w}(n)_i\| = w^0[1 - 2\,erf\left(-\frac{w^0}{\sigma_w}\right) + \sqrt{\frac{2\sigma_{w,j}^2}{\pi}}\,exp\left(-\frac{{w^0}^2}{2\sigma_w^2}\right) \quad (11)$$

where the error function and sign function are defined as $\mathrm{erf}(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{x^2}{2}\right) dx$ and $E(\mathrm{sgn}(w(n))) = -\mathrm{erf}\left(-\frac{w^0}{\sigma_w}\right)$. As the system is made of zero and non zero filter coefficients we can write as $\gamma_2(n)_{ii} = \sum_{j\in Z} E(\|w_j(n)_i\|) - |w_j^0| + \sum_{j\in NZ} E(\|w_j(n)_i\|) - |w_j^0|$.If the weight $j \in NZ$ and if $\sigma_{w,j}$ is small then we can write as follows , If $w_{o,j} > 0$ then $\mathrm{erf}\left(-\frac{w_0}{\sigma_{w,i}}\right) \cong 0$.If $w_{o,j} < 0$ then $\mathrm{erf}\left(-\frac{w_0}{\sigma_{w,i}}\right) \cong 1$ and for both positive and negative $w_{o,j}$ then $\exp\left(-\frac{{w_{o,j}}^2}{2\sigma_{w,j}^2}\right) \cong 0$ [15].Thus for $j \in NZ$ $E(\|w_j(n)_i\|) = |w_j^0|$. For $\in Z$, $w_j^0 = 0$. Thus $\gamma_2(n)_{ii} = \sum_{j\in Z} \sqrt{\frac{2\sigma_{w,j}^2}{\pi}}$.In orders to get a feasible solution, only diagonal elements are taken. Moreover if the input is Gaussian, then we the optimal zero attractor controller as

$$\alpha_o = -\frac{Vec(I)^T X_1 Vec(E(sgn(w(n))\tilde{w}^T(n))) + X_2 Vec(E(\tilde{w}(n)sgn(w(n))^T}{Vec(I)^T 2[I-P+Q]Vec(E(sgn(w(n))sgn(w(n))^T))}$$

$$= \frac{(X_1+X_2)_{ii}\sqrt{\frac{2\sigma_{w,j}^2}{\pi}}}{2[I-P+Q]_{ii}} \quad (12)$$

Since the variance of the weights is not known, moving average method is adopted to obtain the variance of the weights. Thus

$$\alpha_o(n+1) = \beta\alpha_o + (1-\beta)\left|\frac{(X_1+X_2)_{ii}\sqrt{\frac{2\sigma_{w,j}^2}{\pi}}}{2[I-P+Q]_{ii}}\right| \quad (13)$$

Where $0 \le \beta < 1$ is the smoothing factor. Thus if the variance $\sigma_{w,j}^2$ is changed then accordingly α is changed thereby the algorithm is suitable for variable sparsity environment.

IV. SIMULATIONS

Simulations are performed in the system identification scenario. For this purpose an unknown channel of 16 taps is randomly generated. All the experimentation is tested for colored input. The colored signal is obtained by passing white noise through a first order system with pole at 0.9 (H (z) =1/ (1-0.9z^{-1})).The noise is assumed to be Gaussian with zero mean and unity variance. The SNR is maintained as 30 dB throughout the experiment which is calculated as 10 log $E[y(n)^2]/E[v(n)^2]$. It is assumed that both the filter and system has the same number of taps. The initial values of all the filter coefficients are zero. The regularization is chosen to be 0.001 and the projection order is chosen to be 4. The length of the sample is chosen to be 1000 and the results are averaged over 200 independent runs.

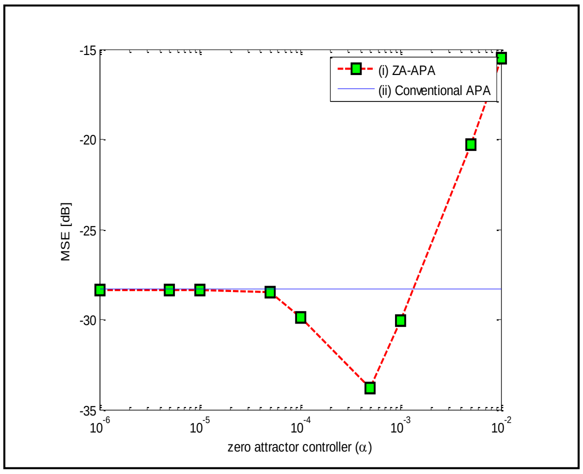

Fig. 1. Steady state MSE of ZA-APA with different zero attractor controller

To illustrate the selection of α on the performance of ZA-APA, the steady state MSE of ZA-APA is plotted for different values of α. The value of α is changed between 10^{-6} to 10^{-2}. For this purpose, the weights of sparse system are assumed to follow Gaussian distribution with the zero mean and the variance 0.5 with sparseness of 0.9375. From (12) the value of α obtained is around 10^{-3}.The graph also shows steady state MSE of conventional APA with the same step size and projection order. It is seen that around 1×10^{-3} the MSE of ZA-APA is lowest which proves the effectiveness of (12) for the selection of α. Also from Fig 1 it should be noted that very small value of α reduces the zero attraction strength which makes it to perform like conventional APA.

The second experiment is conducted to demonstrate the effect of α on non-sparse and sparse system. For this purpose, the weights of non-sparse system are assumed to follow Gaussian distribution with the zero mean and the variance 0.5 and the sparse system is same as experiment one. The non-sparse system has the sparseness of 0.0625. As expected, the value of α in sparse environment makes ZA-APA with lesser steady state error and in non sparse environment the value of α makes the ZA-APA to perform worse than APA. Thus we can conclude that ZA-APA provides better performance than conventional APA only when the system is sparse. When the sparsity level is decreased, any value of α tend to only increase the steady state error.

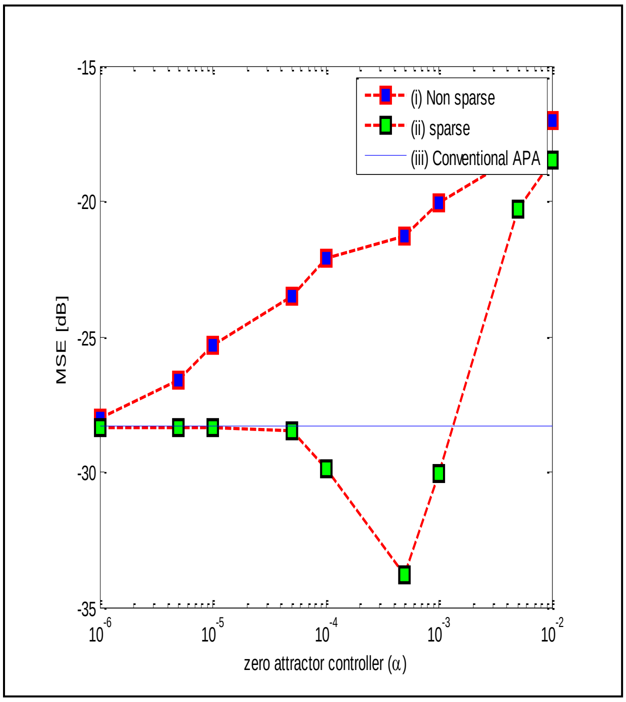

Fig. 2. Steady state MSE analysis of ZA-APA for sparse and non sparse system with different zero attractor controller

Fig 3 is used to illustrate the stability of ZA-APA based on the step size. For this, five different step sizes were chosen as $\mu = 0.01, \mu = 0.1, \mu = 1, \mu = 1.6, \mu = 2.05$and the performance is analysed .Again the system under consideration is a sparse system with number of non zero coefficients equal to 1 with the input is same as first experiment. The value of ρ set as 1×10^{-4} .From the results it is proved that the algorithm is stable only if the step size is chosen to be $0 < \mu < 2$ which is same as the conventional ZA-APA [7].

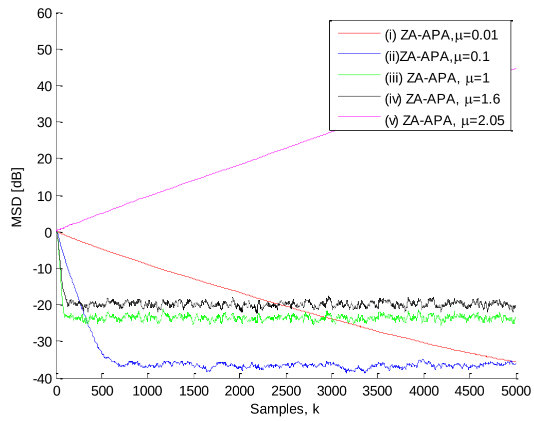

Fig. 3.Stability analysis of ZA-APA with different step size values with SNR=30 db.

Finally in Figure 5 the feasibility of (13) for the variable sparsity environment is proved by simulation under sparse, semi sparse and non-sparse conditions as shown in Fig. 4. The system is taken as the one with 16 coefficients. Initially the system was sparse with 15 zero coefficients and one non zero coefficient. After 5000 time steps, system was made semi sparse with equal number of zero and non-zero filter coefficients. Finally after 10,000 time steps the system was non sparse with no zero filter coefficient. For comparison, the ZA-APA with fixed step size is also plotted.

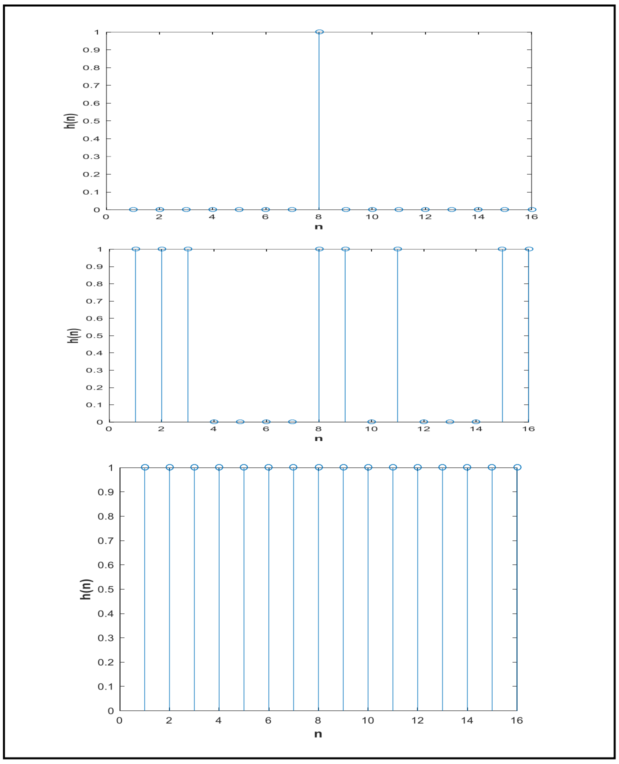

Fig. 4.Impulse response of sparse, semi sparse and non-sparse system

Several interesting findings can be obtained for Fig.5. Firstly the plot confirms that the variable zero attractor controller outperform other fixed ZA-APA in all environments. Secondly the plot verifies whether the variable zero attractor controller ZA-APA can outperform ZA-APA. As per the analysis in previous section and simulations in Fig 2, it is found that variable zero attractor controller ZA-APA cannot outperform ZA-APA when the system is non-sparse. This is further verified in Fig. 3.

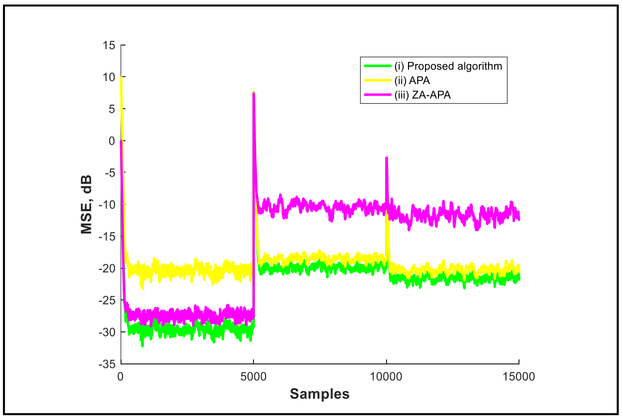

Fig.5. Steady state MSE analysis of the proposed algorithm under variable sparsity environment.

The third experiment is performed for echo cancellation application. Here two systems namely sparse and non-sparse conditions are taken with the length of the impulse response as 512 coefficients. As expected the proposed algorithms perform well at all situation with faster convergence and lower steady state MSE which can be seen from Figure 6.

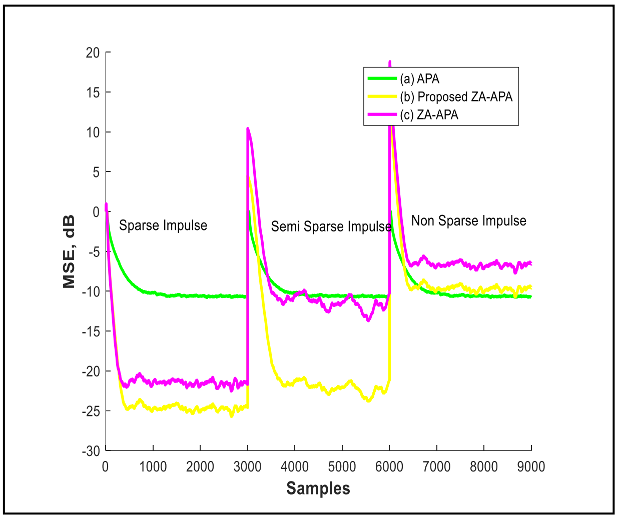

Fig. 6. Steady state MSE analysis of the proposed algorithm for echo cancellation application

CONCLUSION

This paper presents an adaptive zero attractor controller for varying sparsity environment .It is found that any value of zero attractor controllers does not improve the performance of ZA-APA as long as the system is non sparse. Thus an adaptive optimal zero attractor controller based on MSD is proposed. It is found that the performance of the proposed approach is better than ZA-APA under sparse and semi sparse condition and under non-sparse condition also, it is found to be same as ZA-APA as expected. Finally the feasibility of optimal variable zero attractor controller for variable sparsity environment is also proved through simulations. In future , the further extension is to vary the step size along with zero attractor controller and optimize the system.

REFERENCES

[1] R. K. Martin, W. A. Sethares, R. C. Williamson, and C. R. J. Jr., “Exploiting sparsity in adaptive filters,” IEEE Trans. Signal Process., vol. 50, no. 8, pp. 1883-1894, Aug. 2002.

[2] M. Kocic, D. Brady, and M. Stojanovic, “Sparse equalization for real-time digital underwater acoustic communications,” in *Proc. IEEE OCEANS*, pp. 1417–1422,1995.

[3] E. Hansler, “The hands-free telephone problem—An annotated bibliography,”*Signal Process.*, vol. 27, no. 3, pp. 259–271, Jun. 1992.

[4] W. Bajwa, J. Haupt, G. Raz, and R. Nowak, “Compressed channel sensing,” in *Proc. IEEE CISS*,pp. 5–10, 2008.

[5] Y. Chen, Y. Gu, and A. O. Hero, “Sparse LMS for system identification,” ICASSP, pp. 3125-3128, Taiwan, Apr. 2009.

[6] Meng, R. “ Sparsity-aware Adaptive Filtering Algorithms and Application to System Identification “,Doctoral dissertation, University of York,2011.

[7] Meng, Ran, Rodrigo C.de Lamare & Vitor H. Nascimento,"Sparsity-aware affine projection adaptive algorithms for system identification.",Sensor Signal Processing for Defence (SSPD 2011).IET,2011.

[8] Radhika, S., and Arumugam Sivabalan. "ZA-APA with zero attractor controller selection criterion for sparse system identification." *Signal, Image and Video Processing* 12.2 (2018): 371-377.

[9] Das, B.K., Chakraborty, M.: Sparse adaptive filtering by an adaptive convex combination of the LMS and the ZA-LMS algorithm. IEEE Trans. Circuits Syst. I Regul. Pap. **61**(5), 1499–1507 (2014).

[10] Radhika, S., and Sivabalan Arumugam. "Robust Variable Zero Attractor Controller Based ZA-LMS Algorithm for Variable Sparsity Environment." *National Academy science letters* 41.2 (2018): 85-89.

[11] Lima, Markus VS, et al. "Stability and MSE analyses of affine projection algorithms for sparse system identification." Acoustics, Speech and Signal Processing (ICASSP), 2014 IEEE International Conference on. IEEE, 2014.

[12] Y. Gu, J. Jin, and S. Mei, “l_0 Norm constraint LMS algorithm for sparse system identification,” IEEE Signal Process. Lett., vol. 16, no. 9, pp. 774-777, Sep. 2009.

[13] T. Y. Al-Naffouri and A. H. Sayed, “Transient analysis of adaptive filters with error nonlinearities,” IEEE Trans. Signal Process., vol. 51, no. 3, pp. 653-663, Mar. 2003.

[14] Zhang, Sheng, and Jiashu Zhang. "Transient analysis of zero attracting NLMS algorithm without Gaussian inputs assumption." Signal Processing 97 : 100-109,2014.

[15] Price, R. “A useful theorem for nonlinear devices having Gaussian inputs”, IRE Transactions on Information Theory, 4(2), 69-72, 1958.

[16] Sivashanmugam, Radhika, and Sivabalan Arumugam. "Robust Adaptive algorithm by an adaptive zero attractor controller of ZA-LMS algorithm." *Mathematical Problems in Engineering* 2016 (2016).

[17] Shi, K. and Shi, P., 2010. Convergence analysis of sparse LMS algorithms with l1-norm penalty based on white input signal. *Signal Processing*, *90*(12), pp.3289-3293.