INTL JOURNAL OF ELECTRONICS AND TELECOMMUNICATIONS, 2023, VOL. 69, NO. 2, PP. 383-389
Manuscript received March 14, 2023; revised April, 2023. DOI: 10.24425/ijet.2023.144374

Solving the Problem of Discrete Process Control Synthesis Using Optimization on a Sliding Interval

Zhazira Julayeva, Waldemar Wójcik, Gulzhan Kashaganova, Kulzhan Togzhanova, and Saken Mambetov

***Abstract*—The paper presents a solution to the problem of synthesis of control with respect to the sliding interval length for the optimization of a class of discrete linear multidimensional objects with a quadratic performance criterion. The equation of motion of a closed multidimensional discrete system in the general non-stationary case is derived based on the length of the optimization interval and their main properties. The closed-loop is fitted with a signal representing the predicted values averaged over the whole sliding interval of optimization with a certain weight. A problem with a sliding optimization interval may not require a real-time solution by means of a sequence of solutions on compressed intervals. Therefore, the study of control systems with optimization on a sliding interval is of undoubted interest for a number of practically important control problems.**

***Keywords*—discrete process control; optimization; sliding interval; closed loop control system; equation of a closed loop system**

I. Introduction

THE problem of adaptive control synthesis assumes solving the identification problem, which leads to minimizing some pre-selected quality functional. Thus, the minimization of the quadratic functional directs to the least squares method (LSM), whose estimates for interference having a normal distribution are effective, correct and unbiased. The problems of technical cybernetics are important for ensuring the effectiveness of automated technological complexes and individual enterprises.

At controlling complex objects, there are difficulties in cases where the nature of the processes occurring in them is unknown or poorly investigated, that is, one has to deal with incomplete information about the control object. The optimal type of strategy from the point of view of a given criterion in the above stated conditions significantly depends on the restrictions on the set of acceptable strategies that arise due to incomplete information. The elaboration of varied approaches to the development of control systems that optimize control processes under incomplete information conditions is of particular interest for automating and improving the efficiency of diverse production processes, controlling automated technological aggregates as components of integrated automated plants. Therefore, the development of effective methods of operational control and mathematical models of control of technological objects under conditions of specific incompleteness of information about an object is a task of current concern of the research.

II. Setting and Solving the Problem

One of the unsolved problems is the versatility of the types of incomplete information about controlled objects. Therefore, the goal of control is usually presented as a requirement for optimization (minimization or maximization) of some functional **Q** that depends on the nature of the processes of the state of the object – x and control parameters – u, as well as on some externally specified process x_s (setting effect). Thus, in order to obtain the equation of motion of a closed loop system for optimizing the forecast on a sliding interval, it is necessary to solve the problem of optimal control synthesis, which is in minimizing the value of the quality criterion Q in all future states of the control object available for forecasting. It is proposed to solve this problem by searching for the absolute minimum of the Lagrange function because this allows obtaining a solution to the optimal control synthesis problem in a form that explicitly reveals the dependence of optimal control on the length of the sliding interval N.

Let's consider a generalized setting of the control problem with incomplete information about the behavior of the object and external influencing factors.

For the infinite set of discrete time values $N \overset{\Delta}{=} \{0,1,\ldots\}$ we will define the following its subsets:

1. A set of discrete time values $N_y \overset{\Delta}{=} \{0,1,\ldots,N_y\}$ is the control interval. At this interval, we will consider the motion of a controlled object. N_y is the length of the control interval (a finite or infinite number of discretization intervals of the constituents of this interval).

2. A set of discrete time values $N_n \overset{\Delta}{=} \{n, n+1,\ldots,n+N\}$ forms a sliding prediction interval. Here n is the arbitrary (current) moment of the discrete time of the control interval $0 \subseteq n \subseteq N_y$, and N is the length of the sliding prediction, i.e. the number of discretization intervals for which the object movement and the external influences acting on it are predicted (Figure 1).

Zhazira Julayeva is with Academy of logistics and transport, Almaty Technological University, Almaty, Kazakhstan (e-mail: zhazj@mail.ru).
Waldemar Wójcik is with Lublin University of Technology, Lublin, Poland (e-mail: waldemar.wojcik@pollub.pl).
Gulzhan Kashaganova is with Turan University and Satbayev University, Almaty, Kazakhstan (e-mail: guljan_k70@mail.ru).
Kulzhan Togzhanova and Saken Mambetov are with Almaty Technological University, Almaty, Kazakhstan (e-mail: togzhanova_kuljan@mail.ru, e-mail: mambetov.saken@gmail.com).

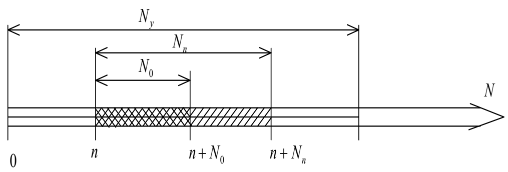

Fig. 1. A set of discrete values of forecast optimization time

3. A set of discrete time values $N_0 \overset{\Delta}{=} \{n, n+1, \ldots, n+N_0\}$ forms the optimization interval. N_0 – the length of the interval, that is, the number of discretization intervals at which forecasts are optimized.

In a general case the lengths of these intervals satisfy the following ordering relationship:

$$N_0 \subseteq N \subseteq N_y . \quad (1)$$

Let us consider the problem of controlling an object that has an r-dimensional control vector $\bar{u}$ and at each current moment of discrete time n is characterized by an m-dimensional vector of the state $\bar{x}$ that can be measured.

Assuming that with the help of some forecasting method, we are able to predict the change of the state vector $\bar{x}$ depending on the law of change of the control vector $\bar{u}$ and external disturbances $\bar{\varpi}$. At the same time, we want to take into account the fact that a reliable forecast is practically possible only for a limited number of cycles ahead N, which form a sliding prediction interval N_n. Without limiting the generality of consideration, we can consider this equivalent to the fact that on the sliding prediction interval we know a functional equation that is satisfied by the predicted values of the state vector $\bar{y}_K$, the control vector $\bar{v}_K$ the perturbation vector $\bar{\varpi}_K$:

$$\bar{y}_{K+1} = F\left(\bar{y}_K, \bar{v}_K, \bar{\varpi}_K\right). \quad (2)$$

Here K is the arbitrary moment of the sliding prediction interval:

$$n \subseteq K \subseteq n+N. \quad (3)$$

The functional equation (2) is set on the interval N_n and is considered under the initial conditions set at the left end of the sliding prediction interval (i.e. at $K = n$):

$$\bar{y}_{K=n} = \bar{x}_n , \quad (4)$$

where $\bar{x}_n$ is the actual value of the state vector at the current moment n.

The generalized indicator of the quality of the functioning of the object **Q** is generally determined by the nature of the change in the actual vector of the object state $\bar{x}_n$ and control $\bar{u}_n$ on the entire control interval N_y and represents the functional:

$$Q = Q[\bar{x}_n, \bar{u}_n]\Big|_{n=0}^{n=N_y} . \quad (5)$$

The task of controlling the object is to find such a sequence of control actions $\bar{u}_n$ that minimizes (maximizes) the optimality criterion provided the above information about the object at each current n (5).

The above formulation of the problem differs from the usually accepted (classical) formulation of the optimal control problem in that usually, instead of the general relationship (1), its special case (6) is considered, when all the intervals N_0, N_n and N_y agree with each other:

$$N_0 = N_n = N_y. \quad (6)$$

In the above setting of the control task the solution is carried out under conditions of specific incompleteness of information about the control object, which consists in the unavailability of information about the object circulation in the interval N_y/N_n for the formation of a management strategy at the current moment n.

Thus, acceptable control strategies are a set of reflections:

$$Z_n(N) \overset{\Delta}{=} \left\{ \begin{matrix} \bar{y}_n, \bar{y}_{n+1}, \ldots, \bar{y}_{n+N} \\ \bar{y}_{3n}, \bar{y}_{3n+1}, \ldots, \bar{y}_{3n+N} \\ \bar{v}_n, \bar{v}_{n+1}, \ldots, \bar{v}_{n+N} \\ \bar{v}_{3n}, \bar{v}_{3n+1}, \ldots, \bar{v}_{3n+N} \end{matrix} \right\}, \quad (7)$$

where the set $U_n : \{Z_n(N) \to \bar{U}_n\}$ is formed by a predictive device (N – step determinant).

The way of solving the problem is mainly indicated in [1, 2], and in the symbols adopted above it boils down to the fact that the optimization of a given functional should be carried out over the entire sliding prediction interval, i.e. $N_0 = N$.

In fact, in all physically feasible processes, the influence applied to an object at the current moment of time can only change its future states. For the optimal control of an object, it is necessary at each current moment to take into account the reaction of an object and the impact of external disturbances throughout the expected control interval. In the theory of optimal control, this circumstance has its mathematical expression in the fact that the equation of motion of an object is defined over the entire control interval ($N = N_y$), and the minimum of the functional Q for objects without consequence (having the Markov property) is achieved using the control, which minimizes the "remaining part" at each current moment: $Q = Q[\bar{x}_i, \bar{u}_i]\big|_n^{N_y}$ (Bellman's optimality principle). The consistent application of this idea to the above-mentioned case of an object with incomplete information leads to a control algorithm with optimization of the forecast at a sliding interval, proposed in [3, 4].

As it was already noted above, this control algorithm consists in building at each current n sequences of future values of command variables $\bar{v}_K^0$ that optimize the predicted value of the quality criterion Q in the prediction interval. The first members of these sequences in the general case form another sequence of command variables implemented on the object (Figure 2).

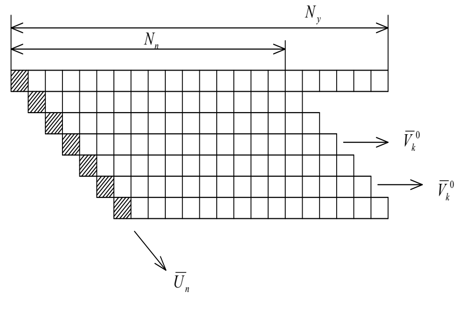

Fig. 2. The sequence of command variables implemented on the object

The properties of the control sequence, the minimum achievable value of the quality criterion Q, the controlled object, etc. depend on the length of the interval N.

The availability of conducting an analytical study of these dependencies strongly depends on the forecasting equations class under consideration (2) and the type of an optimized functional (5). Therefore, we limit ourselves to considering a class of objects whose movement in the prediction interval allows representation as a solution of a system of linear equations, and the quality criterion is a quadratic functional.

Let us assume that some object is characterized at each current moment of time n by state vector $\bar{x}_n$ and control vector $\bar{u}_n$.

At each current n, the given prediction equation (2), defined at N future discrete time points, is a linear difference equation:

$$\bar{y}_{K+1} = A_K \bar{y}_K + B_K \bar{v}_K + W_K \bar{\omega}_K, \qquad n \le K \le n+N. \tag{8}$$

and is considered under initial conditions:

$$\bar{y}_{K=n} = \bar{x}_n, \qquad 0 \le n \le N_y. \tag{9}$$

Here $\bar{y}_K$, $\bar{v}_K$, $\bar{\omega}_K$ are the predicted values of the state, control and disturbance vectors, respectively.

The object is controlled in such a way so that to minimize the value of the quality criterion in all future states of the object available for prediction:

$$Q(\bar{v}_K) = \frac{1}{2}\sum_{K=n}^{n+N}[(\bar{y}_{3K} - \bar{y}_K)^T Q_K (\bar{y}_{3K} - \bar{y}_K) + (\bar{v}_{3K} - \bar{v}_K)^T R_K (\bar{v}_{3K} - \bar{v}_K)].. \tag{10}$$

In this case, the condition for closing the system through optimization of the prediction on a sliding interval takes the form $\bar{u}_n = \bar{v}^0_{K=n}$, where $\bar{v}^0_{K=n}$ are the first members of optimizing sequences $\bar{v}^0_K: Q(\bar{v}^0_K) = \min_{\bar{v}} Q(\bar{v}_K)$.

The task is to research the dynamic properties and, above all, the stability of systems that are closed due to optimization of the forecast on a sliding interval for a given class of objects and functionals.

In order to carry out this study, it is necessary to derive the equation of motion of a closed loop system, for which, first, it is necessary to solve the problem of synthesis of optimal control $\bar{v}^0_K$ (i.e., to find control as a function of phase coordinates) at a sliding optimization interval. The problem of synthesis of optimal control for a linear object and a quadratic functional has been studied by many authors [5-8] under the assumption that optimization is carried out over the entire control interval (fixed or infinite), that is, provided that N_0=N=N_y.

In our study, there is a need to consider the sliding optimization interval, and the solution should be obtained in a form that explicitly reveals the dependence of optimal control on the length of the sliding interval N.

These circumstances do not allow the use of known solutions to the problem of analytical design of regulators [6-10] and require the development of such approaches to solving the problem that take into account its features.

The need to study optimization on a sliding interval of constant length was stipulated by the fact that we considered only reflection (7) as acceptable strategies, which was a mathematical expression of the limitations of real predictable devices. However, the above statement of the problem is quite general and includes a number of tasks with different content, allowing for a different interpretation.

In this regard the works [8, 9] are of particular interest, as they are devoted to calculations related to one class of optimal control systems, thus leading their authors to the need to consider the optimization of an integral quadratic functional on a sliding interval with a one-dimensional linear control object, in which some issues related to optimization on a sliding interval were investigated. As is known, the need to solve the Riccati equation with an initial condition set at infinity under these initial conditions arises in the problems of analytical design of regulators on an infinite control interval.

Since the solution of the Cauchy problem by numerical methods is impossible for a differential equation if the initial conditions are set to infinity, such difficulties are usually solved by the fact that the restriction can be a large but finite integration interval on which the solution is numerically developed in reverse time. In order for the integration interval not to shorten over time, its end, at which the initial conditions are set, is moved during repeated calculations, which leads to the appearance of a sliding optimization interval.

Thus, the solution to the original functional optimization problem on an infinite interval:

$$e = \int_t^{\infty}\left\{\Phi_1\left[Q_1(\sigma) - q_1(\sigma)\right]^2 + \Psi_1\left[M_1(\sigma) - m_1(\sigma)\right]^2\right\}d\sigma, \tag{11}$$

for the object

$$\begin{aligned}\dot{Z}_1(t) &= b_{II} z_1(t) + c_{II} m_1(t) + u_1(t)\\ q_1(t) &= a_{II} z_1(t)\end{aligned}, \tag{12}$$

is replaced by the solution of the functional optimization problem on a sliding interval of constant length τ >0:

$$e = \int_t^{t+\tau}\left\{\Phi_1\left[Q_1(\sigma) - q_1(\sigma)\right]^2 + \Psi_1\left[M_1(\sigma) - m_1(\sigma)\right]^2\right\}d\sigma, \tag{13}$$

where e is error criterion (optimization functional); z_1 – object state coordinate; q_1 – the original coordinate; m_1 – command variable; u_1 – disturbance; Q_1 and M_1 - desired values of the initial coordinate and the impact coordinates; Q_1 and Ψ_1 – of the command variable; Q_1 and M_1 – weight values of the functional; a_{II}, b_{II}, c_{II} – object parameters.

Using the proposed normalization method, the author estimates the shortest interval length τ, at which repeated calculations give estimates of the optimal parameters of the regulator, which differ little from those calculated at an infinite interval.

Thus, the main definitions drawn in [8, 9] relate to the issue of the relationship between the quality of the system and the length of the interval τ over which the output error is averaged, since the interval τ is a value proportional to the time required to calculate the parameters $\hat{R}$.

Therefore, if repetitive calculations performed at high speed are used to obtain a continuous optimum, then the interval τ determines the speed at which changes in the system can be received. This allows determining the limits within which the discrete nature of parameter changes $\hat{R}$ can be neglected.

Further, the authors of [8, 9] come to the conclusion that the relationship between quality and time of calculation results in the relationship between quality and information about the future, necessary for the system operation.

With the decrease in the interval τ, the amount of information about the required output of the system decreases, as a result of which a decrease in the quality of the system should be expected. The increase in the error (with the decrease of τ_n) takes place because an increasing amount of information about the required output is discarded. At $\tau_n = 1$, the increase in error compared to $\tau_n = \infty$ is insignificant – less than 4%, which made it possible to find the invariant $ac\sqrt{\Phi/\Psi} = 1$ that determines the relationship between the quality of the system and the calculation time [8, 9]. Thus, the research on first-order objects with an integral quadratic quality functional was focused around optimization issues on sliding intervals of sufficiently long length, when the difference in optimization results on sliding and endless intervals becomes small.

The main conclusion reached by the authors of [8, 9] as a result of the discussion of the above difficulties is that even in a problem with a sliding interval, it is not always necessary to recalculate the parameters of the regulator. In other words, a problem with a sliding optimization interval may not require a real-time solution by means of a sequence of solutions at compressed intervals. Therefore, the study of control systems with optimization on a sliding interval is of undoubted interest for a number of practically important control tasks.

III. THE CONTROL OF DISCRETE PROCESSES WITH OPTIMIZATION ON A SLIDING INTERVAL

The control object O at each current moment of discrete time n is characterized by an m-dimensional state vector $\bar{x}_n$ and an r-dimensional control vector $\bar{u}_n$.

Let's assume that the future values of the vector of state $\bar{y}_K$, control $\bar{v}_K$, and measured perturbation $\bar{\omega}_K$ on a sliding prediction interval with a length of N cycles satisfy a system of linear difference equations (8): $\bar{y}_{K+1} = A_K\bar{y}_K + B_K\bar{v}_K + W_K\bar{\omega}_K$ under initial conditions $\bar{y}_{K=n} = \bar{x}_n$. Here $n \in N_y$, $N_y = \{0, \infty\}$, $K \in N_n$, $N_n = \{n, n+N\}$, $N \supseteq 0$, A_K - (mxm), B_K - (mxr), W_K - (mxp).

The construction of the matrices A_K, B_K, W_K, $\forall K \in N_n$ makes the content of the forecasting problem, which is not considered in this paper, and the matrices A_K, B_K and W_K are assumed to be already known.

The entire set of reflections is accepted as valid at each current moment of time n of the set of control strategies U_n:

$$U_n : \{Z_n(N) \to \bar{U}_n\},$$

where $Z_n(N)$ is the set of sequences of vectors defined on the prediction interval N_n:

$$Z_n(N) \overset{\Delta}{=} \begin{Bmatrix} \bar{y}_n, & \bar{y}_{n+1}, & \dots, & \bar{y}_{n+N} \\ \bar{y}_{3n}, & \bar{y}_{3n+1}, & \dots, & \bar{y}_{3n+N} \\ \bar{v}_n, & \bar{v}_{n+1}, & \dots, & \bar{v}_{n+N} \\ \bar{v}_{3n}, & \bar{v}_{3n+1}, & \dots, & \bar{v}_{3n+N} \end{Bmatrix}.$$

Here, the sequences of vectors $\bar{y}_{3K}$ *and* $\bar{v}_{3K}$ are the preferred (preset) values of the sequences of state and control vectors on the prediction interval.

The quality criterion is a quadratic functional Q (10) defined on the set $Z_n(N)$:

$$Q[\bar{v}_K] \overset{\Delta}{=} \frac{1}{2}\sum_{K=n}^{n+N}[(\bar{y}_{3K} - \bar{y}_K)^T Q_K(\bar{y}_{3K} - \bar{y}_K) + +(\bar{v}_{3K} - \bar{v}_K)^T R_K(\bar{v}_{3K} - \bar{v}_K)],$$

where $Q_K \supseteq 0$, $R_K > 0$, $\forall K \in N_n$.

Thus, we take the optimization interval to be equal to the prediction interval.

The object is controlled in such a way as to minimize the value of the quality criterion (10) on the set Z_n, that is, on all future states of the object available for prediction. To do this, the system gets interlocked in accordance with the condition:

$$\bar{u}_n = \bar{v}^0_{K=n},$$

where $\bar{v}^0_{K=n}$ are the first members of optimizing sequences:

$$\bar{v}^0_K : \ Q[\bar{v}^0_K] = \min_{\bar{v}_K} Q[\bar{v}_K],$$
$$\forall K \in N_n \ \ and \ \ \forall n \in N_y.$$

The task of control synthesis with forecast optimization on a sliding interval is to find a sequence of control actions $\bar{u}_n$, $\forall n \in N_y$ in the function of the current coordinates of the object state $\bar{x}_n$ and the length of the optimization interval N.

Consider the minimum conditions of functional (10) in the form of a discrete analogue of the Pontryagin maximum principle:

$$\begin{cases} \Delta\bar{y}^0_K = \dfrac{\partial}{\partial\bar{\Psi}^0_K} H^0_K \\ \Delta\bar{\Psi}^0_K = -\dfrac{\partial}{\partial\bar{y}^0_K} H^0_K \\ \dfrac{\partial}{\partial\bar{v}^0_K} H^0_K = \bar{0} \end{cases} \tag{14}$$

under boundary conditions:

$$\begin{cases} \bar{y}^0_{K=n} = \bar{x}_n \\ \bar{\Psi}^0_{K=n+N} = \bar{0}, \end{cases} \tag{15}$$

where the Hamilton function has the form:

$$\begin{aligned} H_K \overset{\Delta}{=} & \bar{\Psi}^T_K (A_K\bar{y}_K + B_K\bar{v}_K + W_K\bar{\omega}_K) - \\ & -\frac{1}{2}\Big[(\bar{y}_{3K} - \bar{y}_K)^T Q_K(\bar{y}_{3K} - \bar{y}_K) + (\bar{v}_{3K} - \bar{v}_K)^T R_K(\bar{v}_{3K} - \bar{v}_K)\Big], \end{aligned} \tag{16}$$

and

$$\begin{aligned} \Delta\bar{y}_K &\overset{\Delta}{=} \bar{y}_{K+1} - \bar{y}_K \\ \Delta\bar{\Psi}_K &\overset{\Delta}{=} \bar{\Psi}_{K+1} - \bar{\Psi}_K. \end{aligned} \tag{17}$$

Here, the first condition (15) expresses the fact that in the prediction equations (8), the current state vector $\bar{x}_n$ is used as the initial conditions $\forall n \in N_y$, and the second boundary condition (15) expresses the fact of free variation of the value of the control optimization interval.

Taking into account the designations (16) and (17), the control system (14) will take the form:

$$\begin{cases} \bar{y}^0_{K+1} = A_K\bar{y}^0_K + B_K\bar{v}^0_K + W_K\bar{\omega}_K \\ \bar{\Psi}^0_{K-1} = -Q_K\bar{y}^0_K + A^T_K\bar{\Psi}^0_K + Q_K\bar{y}_{3K} \\ \bar{v}^0_K = \bar{v}_{3K} + R^{-1}_K B^T_K\bar{\Psi}^0_K \end{cases}. \tag{18}$$

Since for discrete systems, the analogue of the Pontryagin maximum principle is in the general case neither a necessary nor sufficient condition for optimal control, and the main result relating to linear discrete systems belonging to Rozonoer [8-14] is applicable only with a linear quality functional, then we prove the necessity and sufficiency of condition (18) for determining the sequence $\bar{v}_K^0$, the quadratic functional (10).

Proof

Let us consider on the interval N_n the functional $\Phi[\bar{v}_K]$ as a function of 2(N+1) variables

$$Q[\bar{v}_K] = \Phi(\bar{v}_n, \bar{v}_{n+1}, \dots, \bar{v}_{n+N}, \bar{y}_n, \dots, \bar{y}_{n+N}), \quad (19)$$

subject to additional conditions in the form of equalities:

$\bar{y}_{K+1} = A_K\bar{y}_K + B_K\bar{v}_K + W_K\bar{\omega}_K, \bar{y}_{K=n} = \bar{x}_n, \quad \forall K \in N_n$. $\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$ (20)

The problem of minimizing the functional (10) is equivalent to finding the conditional minimum of the function (19).

By deducing the vector of indefinite multipliers $\bar{\lambda}_K$, this problem is reduced to finding the unconditional minimum of the Lagrange function:

$$L \overset{\Delta}{=} \sum_{K=n}^{n+N}[\frac{1}{2}(\bar{y}_{3K} - \bar{y}_K)^T Q_K(\bar{y}_{3K} - \bar{y}_K) + \frac{1}{2}(\bar{v}_{3K} - \bar{v}_K)^T R_K(\bar{v}_{3K} - \bar{v}_K) +$$
$$+\bar{\lambda}_K^T(y_{K+1} - A_K\bar{y}_K - B_K\bar{v}_K - W_K\bar{\omega}_K)]$$

for 2(N+1) variables $\bar{y}_K$ and $\bar{y}_K$ и $\bar{v}_K$.

Since the resulting Lagrange function is quadratic, a necessary and sufficient condition for its minimum is that its gradient is equal to zero for all variables:

$$\begin{cases} \dfrac{\partial}{\partial \bar{y}_K} L = \bar{\lambda}_{K-1} - Q_K(\bar{y}_{3K} - \bar{y}_K) - A_K^T\bar{\lambda}_K = \bar{0} \\ \dfrac{\partial}{\partial \bar{v}_K} L = -R_K(\bar{v}_{3K} - \bar{v}_K) - B_K^T\bar{\lambda}_K = \bar{0}. \end{cases} \quad (21)$$

With the account of the limiting condition for the multiplier $\bar{\lambda}_{K=n+N} = \bar{0}$, the system of equations (20) and (21) coincides exactly to the notation with the system (18).

Thus, the discrete analogue of the Pontryagin maximum principle is a necessary and sufficient condition for the minimum of the quadratic functional (10) under constraints of the equality type in the form of linear difference equations (8), as was to be proved.

The synthesis of discrete process control with optimization on a sliding interval.

Consider the system of equations (18). After shutting off the vector $\bar{v}_K^0$ from the first equation and reducing the first two equations to a matrix form, an equivalent system is obtained:

$$\begin{Vmatrix} \bar{y}_{K+1}^0 \\ \bar{\Psi}_{K-1}^0 \end{Vmatrix} = G_K \begin{Vmatrix} \bar{y}_K^0 \\ \bar{\Psi}^0 \end{Vmatrix} + \begin{Vmatrix} B_K\bar{v}_{3K} + W_K\bar{\omega}_K \\ Q_K\bar{y}_{3K} \end{Vmatrix}, \quad (22)$$

where $G_K \overset{\Delta}{=} \begin{Vmatrix} A_K & P_K \\ -Q_K & A_K^T \end{Vmatrix} - (2m \times 2m)$, and $P_K \overset{\Delta}{=} B_K R_K^{-1} B_K^T$.

The system of equations (18) with boundary conditions (15) forms a two-point boundary value problem, the solution of which will allow finding the desired control as a function of the current coordinates of the state and the length of the optimization interval.

The peculiarity of the discrete analogue of the maximum principle is that if the equation of motion of an object is given in direct differences, then the equation for the conjugate vector $\bar{\Psi}_K^0$ will be obtained in inverse differences – and vice versa. This makes it difficult to present a general solution to the boundary value problem. These difficulties can be avoided by converting system (18) (under certain conditions) into an equivalent system of equations in direct differences.

For this purpose, we will consider the second equation of the system (18) with the independent variable K+1.

In this case, the system (18) will be equivalent to the system

$$\begin{Vmatrix} E & 0 \\ -Q_{K+1} & A_{K+1}^T \end{Vmatrix} \cdot \begin{Vmatrix} \bar{y}_{K+1}^0 \\ \bar{\Psi}_{K+1}^0 \end{Vmatrix} = \begin{Vmatrix} A_K & P_K \\ 0 & E \end{Vmatrix} \cdot \begin{Vmatrix} \bar{y}_{K+1}^0 \\ \bar{\Psi}_{K+1}^0 \end{Vmatrix} + \begin{Vmatrix} B_K\bar{v}_{3K} + W_K\bar{\omega}_K \\ -Q_{K+1}\bar{y}_{3K+1} \end{Vmatrix}, \quad (23)$$

because the determinant $\begin{vmatrix} E & 0 \\ -Q_{K+1} & A_{K+1}^T \end{vmatrix} \equiv |A_{K+1}|$, and the condition $|A_K| \neq 0, \quad \forall K \in N_n$ are necessary and sufficient for the inverse matrix to exist

$$\begin{Vmatrix} E & 0 \\ -Q_{K+1} & A_{K+1} \end{Vmatrix} \equiv \begin{Vmatrix} E & 0 \\ (A_K^T)^{-1} Q_{K+1} & (A_K^T)^{-1} \end{Vmatrix}. \quad (24)$$

Multiplying equation (23) on the left by matrix (24), we obtain a system of equations (25) in direct differences equivalent to the system of equations (18), provided that all matrices A_K are non-singular:

$$\begin{Vmatrix} \bar{y}_{K+1}^0 \\ \bar{\Psi}_{K+1}^0 \end{Vmatrix} = L_K \begin{Vmatrix} \bar{y}_K^0 \\ \bar{\Psi}_K^0 \end{Vmatrix} + L_K^* \begin{Vmatrix} B_K\bar{v}_{3K} + W_K\bar{\omega}_K \\ y_{3K+1} \end{Vmatrix}, \quad (25)$$

where

$$L_K \overset{\Delta}{=} \begin{Vmatrix} A_K & P_K \\ (A_{K+1}^T)^{-1} Q_{K+1} & (A_{K+1}^T)^{-1}(Q_{K+1}P_K + E) \end{Vmatrix}$$

$$L_K^* \overset{\Delta}{=} \begin{Vmatrix} E & 0 \\ (A_{K+1}^T)^{-1} Q_{K+1} & -(A_{K+1}^T)^{-1} Q_{K+1} \end{Vmatrix}.$$

The solution of the system of equations (25) over the entire sliding interval from n to n+N can be represented as:

$$\begin{Vmatrix} \bar{y}_K^0 \\ \bar{\Psi}_K^0 \end{Vmatrix} = \phi(K,n) \begin{Vmatrix} \bar{y}_K^0 \\ \bar{\Psi}_K^0 \end{Vmatrix} + \sum_{i=n}^{K-1} \phi(K, i+1) L_i^* \begin{Vmatrix} B_i\bar{v}_i + W_i\bar{\omega}_i \\ \bar{y}_{3i+1} \end{Vmatrix}, \quad (26)$$

Where $\phi(K,n) \overset{\Delta}{=} \begin{Vmatrix} \phi_{11}(K,n) & \phi_{12}(K,n) \\ \phi_{21}(K,n) & \phi_{22}(K,n) \end{Vmatrix} - (2m \times 2m)$.

The fundamental matrix of solutions is defined by the expression:

$$\phi(K,n) = \begin{cases} \prod_{i=n}^{K-1} L_i & \text{for} \quad K > n \\ E & \quad K = n. \end{cases} \quad (27)$$

From (26) it follows that for K=n+N and taking into account the boundary conditions (15) we get:

$$\begin{Vmatrix} \bar{y}_{n+N}^0 \\ \bar{0} \end{Vmatrix} = \phi(n+N, n) \begin{Vmatrix} \bar{x}_n \\ \bar{\Psi}_n^0 \end{Vmatrix} + \sum_{i=n}^{n+N-1} \phi(n+N, i+1) L_i^* \begin{Vmatrix} B_i\bar{v}_i + W_i\bar{\omega}_i \\ \bar{y}_{3i+1} \end{Vmatrix}. \quad (28)$$

Given the identical equation: $\phi_{22}^{-1}(n+N,n)\phi_{22}(n+N,i+1) \equiv \phi_{22}(n,i+1)$, we solve the second equation of system (28) with respect to the combined vector $\bar{\Psi}_n^0$, or, introducing the new notation, we represent the current vector of the conjugate system as:

$$\bar{\Psi}_n^0 = -K_n(N)\bar{x}_n + \bar{P}_n\left(N,\bar{\omega},\bar{y}_3,\bar{v}_3\right), \qquad (29)$$

where $K_n(N)\overset{\Delta}{=}\phi_{22}^{-1}(n+N,n)\phi_{21}(n+N,n)$ is the gain coefficients matrix, and

$$\bar{P}_n(N)\overset{\Delta}{=}-\phi_{22}^{-1}(n+N,n)\bar{\Psi}(n+N,n),$$

$$\bar{\Psi}_{(K,n)}\overset{\Delta}{=}\sum_{i=n}^{K-1}[\phi_{21}(K,i+1)(B_i\bar{v}_{3i}+W_i\bar{\omega}_i)+\phi_{12}(K,i+1)(A_{i+1}^T)^{-1}Q_{i+1}(B_i\bar{v}_{3i}+W_i\bar{\omega}_i-y_{3i+1})]$$

is the vector that takes into account the forecasts of all external influences affecting the movement of the system throughout the entire optimization interval.

Knowing the dependence (29) of the current vector of the combined system $\bar{\Psi}_n^0$ on the vector of the current state $\bar{x}_n$, it is possible to determine all the variables of interest to us.

The prediction of the optimal sequence of the state vector $\bar{y}_K^0$ on the sliding optimization interval has the form:

$$\bar{y}_K^0=(\phi_{11}(K,n)-\phi_{12}(K,n)K_n(N))\bar{x}_n+(\phi_{12}(K,n)\bar{P}_n(N)+\bar{Y}(K,n)), \qquad (30)$$

where

$$\bar{Y}(K,n)\overset{\Delta}{=}\sum_{i=n}^{K-1}[\phi_{11}(K,i+1)(B_i\bar{v}_{3i}+W_i\bar{\omega}_i)+\phi_{12}(K,i+1)(A_{i+1}^T)^{-1}Q_{i+1}(B_i\bar{v}_{3i}+W_i\bar{\omega}_i-\bar{y}_{3i+1})].$$

The forecast of the optimal sequence of the control vector on the sliding optimization interval has the form:

$$\bar{v}_K^0=\bar{v}_{3K}-R_K^{-1}B_K^T(\phi_{22}(K,n)K_n(N)+\phi_{21}(K,n))\bar{x}_n+$$
$$+R_K^{-1}B_K^T(\phi_{22}(K,n)\bar{P}_n(N)+\bar{\Psi}(K,n))$$

The desired current control action will be described by the equation:

$$\bar{u}_n=\bar{v}_{3n}-R_n^{-1}B_n^TK_n(N)\bar{x}_n+R_n^{-1}B_n^T\bar{P}_n(N,\bar{\omega},\bar{y}_3,\bar{v}_3) \qquad (31)$$

Thus, the control carried out according to the principle of optimizing the prediction of future values of the functional is equivalent to the above linear control law with negative feedback (31) and time-variable coefficients that also depend on the number of cycles N of the optimization interval.

The dependence of the feedback gain coefficient matrix $K_n(N)$ on the number of clock cycles N is given by expressions (27) and (29) and is uniquely determined by the values of the matrix L_K (25) over the entire sliding optimization interval.

Knowing the control (31) implemented on the object at each current moment of time n, it is possible to obtain the equation of motion of a closed loop system, which has the form:

$\bar{x}_{n+1}=A_n\bar{x}_n+B_n\bar{u}_n(\bar{x}_n,N)+W_n\bar{\omega}_n$, and given (31), we get:

$$\bar{x}_{n+1}=A_{3n}(N)\bar{x}_n+B_n\bar{v}_{3n}+W_n\bar{\omega}_n+P_n\bar{P}_n(N,\bar{\omega},\bar{y}_3,\bar{v}_3), \qquad (32)$$

where $A_{3n}(N)\overset{\Delta}{=}\left[A_n-P_nK_n(N)\right]$ is a matrix of closed loop parameters.

The solution of equation (32), that is, the trajectory of a closed loop system from the initial state $\bar{x}_0$ to the current $\bar{x}_n$, can be represented as:

$$\bar{x}_n=F(n,0)\bar{x}_0+\sum_{j=0}^{n-1}F(n,j+1)P_j\bar{P}_j(N,\bar{\omega},\bar{y}_3,\bar{v}_3),$$

where the fundamental matrix of the equation of motion of a closed loop system is defined by the expression:

$$F(n,p)\triangleq\begin{cases}\prod_{l=p}^{n-1}(A_l-p_lK_l(N) & \text{for } n>p\\ E & n=p\end{cases}.$$

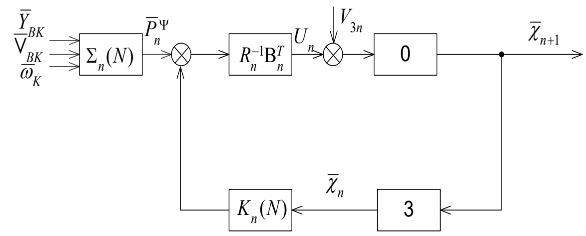

Fig. 3. The block diagram of a closed system equivalent to control with forecast optimization on a sliding interval

The block diagram of a closed system equivalent to control with forecast optimization on a sliding interval is shown in Figure 3.

The block diagram of a closed system equivalent to control with forecast optimization on a sliding interval (Figure 3) contains a feedback loop, as well as an open circle of averaging and transformation of external influences.

IV. Results and Discussion

From the analysis of the expressions obtained above, the following conclusions can be drawn:

1. The equation of motion of a closed loop system has the same order as the equation of motion of an object, so that control with optimization of the forecast, under accepted assumptions, is equivalent to the introduction of linear rigid feedback.
2. The properties of a closed loop are determined by the matrix of parameters of a closed loop system:

   $$A_{3n}(N)\overset{\Delta}{=}A_n-P_nK_n(N), \quad \forall n\in N_y,$$

   which depends on the length of the optimization interval N. This dependence is specified by the presence of a matrix of coefficients $K_n(N)$, which is uniquely determined by the fundamental matrix of solutions φ(n+N,n).
3. A closed loop is affected by a signal that represents the predicted values, averaging with a certain weight over the entire sliding optimization interval.
4. The trajectory of the closed loop system (32) and its forecast on the sliding optimization interval (30) for arbitrary $n\in N_y$ and N≠0 have a contact at not lower than the first order $\bar{y}_{n+1}^0=A_n\bar{y}_n^0+B_n\bar{v}_n^0+W_n\bar{\omega}_n$ and $\bar{x}_{n+1}=A_n\bar{x}_n+B_n\bar{u}_n+W_n\bar{\omega}_n$ with regard to conditions $\bar{y}_{K=n}^0=\bar{x}_n$ and $\bar{u}_n=\bar{v}_{K=n}^0$, it follows that $\forall n\in N_y$ identically satisfies $\bar{x}_{n+1}\equiv\bar{y}_{n+1}^0$, although for arbitrary $K\in N_n$ and K≠n+1, the trajectory (32) and the forecast of its optimal value (30) generally do not coincide. In other words, the trajectory of a closed loop system is a trajectory that envelopes up to a set of optimal forecasts.

V. Conclusion

The solution of the control synthesis problem is received taking into account the length of the sliding interval of optimization of a class of discrete linear multidimensional objects with a quadratic quality criterion. The equation of motion of a closed multidimensional discrete system in the general nonstationary case is derived, taking into account the length of the optimization interval and their basic properties.

The main feature of systems with optimization on a sliding interval of constant finite length with constant values of the parameters of the object model and functional is that the closed loop control system obtained in this case has time-invariant parameters that depend only on the length of the optimization interval N, unlike the case when the finite optimization interval has a fixed end in time. This makes it possible to obtain the equation of motion of a closed multidimensional system in the general non-stationary case, taking into account the length of the sliding optimization interval.

REFERENCES

[1] V.P. Safronova, and M.V. Yakovleva, “Microprocessor Controllers of Automatic Systems and Automatic Control Systems,” in proc. 2019 IEEE Conference of Russian Young Researchers in Electrical and Electronic Engineering (EIConRus), pp. 319-321, 2019. https://doi.org/10.1109/EIConRus.2019.8656677

[2] E. P. Ivanova, “Differential-difference equations with incommensurable shifts of arguments,” in Proceedings of the Voronezh Spring Mathematical School "Modern methods in the theory of boundary value problems. Pontryagin readings-XXX", Voronezh, May 3-9, vol. 191, no. 0, pp. 92-100, 2021. https://doi.org/10.36535/0233-6723-2021-191-92-100

[3] V. N. Panovskiy, A. V. Panteleev, “Meta-heuristic interval methods of search of optimal in average control of nonlinear determinate systems with incomplete information about its parameters,” Journal of Computer and Systems Sciences International, vol. 56, no. 1, pp. 52-63, 2017. https://doi.org/10.1134/S1064230717010117

[4] A. S. Antipov, D. V. Krasnov, A. V. Utkin, “Decomposition synthesis of the control system of electromechanical objects in conditions of incomplete information,” Mechanics of Solids, vol. 54, no. 5, pp. 669-682, 2019. https://doi.org/10.3103/S0025654419050042

[5] F. A. Aliev et al., “Larin Parameterization to Solve the Problem of Analytical Construction of the Optimal Regulator of Oscillatory Systems with Liquid Dampers,” Journal of Applied and Computational Mechanics, vol. 6, special issue, pp. 1426-1430, 2020. https://doi.org/10.22055/JACM.2020.34950.2548

[6] R. F. Stengel, “Flight dynamics,” Princeton University press, 2022.

[7] A. T. Saidmamatov, “Theory of Optimal Design of Construction,” Eurasian Journal of Engineering and Technology, no. 11, pp.43-48, 2022. https://www.geniusjournals.org/index.php/ejet/article/view/2325

[8] X. H. Chang, R. R. Liu, and J. H. Park, “A Further Study on Output Feedback H∞ Control for Discrete-Time Systems,” IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 67, no. 2, pp. 305-309, 2019. https://doi.org/10.1109/TCSII.2019.2904320

[9] E. Isufi, A. Loukas, N. Perraudin, and G. Leus, “Forecasting Time Series With VARMA Recursions on Graphs,” IEEE Transactions on Signal Processing, vol. 67, no. 18, pp. 4870-4885, 2019. https://doi.org/10.1109/TSP.2019.2929930

[10] N. V. Balashevich, R. Gabasov, and F. M. Kirillova, “Numerical methods for open-loop and closed-loop optimization of linear control systems,” Computational mathematics and mathematical physics, vol. 40, no. 6, pp. 799-819, 2000. https://www.mathnet.ru/eng/zvmmf1260

[11] N. A. Dyakonov, and O. S. Logunova, “Process control systems based on predictive analytics: design,” Electrotechnical systems and complexes, no. 1, vol. 50, pp. 58-64, 2021.

[12] O. Vasilevskyi, P. Kulakov, D. Kompanets, O. M. Lysenko, V. Prysyazhnyuk, W. Wójcik, and D. Baitussupov, “A new approach to assessing the dynamic uncertainty of measuring devices,” in Proc. SPIE, vol. 10808, pp. 728-735, 2018. https://doi.org/10.1117/12.2501578

[13] I. V. Gogol, O. A. Remizova, V. V. Syrokvashin, and A. L. Fokin, “Adaptive-robust control of technological processes with delay on control,” in: Kravets, A., Bolshakov, A., Shcherbakov, M. (eds) Cyber-Physical Systems: Industry 4.0 Challenges. Studies in Systems, Decision and Control, vol 260, Springer, Cham, 2020. https://doi.org/10.1007/978-3-030-32648-7_11

[14] A. P. Pandey, and M. C. de Oliveira, “Discrete-time H∞ control of linear parameter-varying systems,” International Journal of Control, vol. 12, no. 92, pp. 2750-2760, 2019. https://doi.org/10.1080/00207179.2018.1459855