

# Subcarrier Filtering For Spectrally Efficient Multicarrier Modulation Schemes and Its Impact on PAPR: A Unified Approach

Kiran V. Shanbhag, and Dayakshini Sathish

**Abstract**—Multicarrier modulation (MCM) based schemes have been a major contributing factor in revolutionizing cellular networks due to their ability to overcome fading. One of the popular scheme orthogonal frequency division multiple access (OFDMA), having been part of 4G, is also adapted as part of 5G enhanced mobile broadband (eMBB). Though it has several advantages, spectral efficiency (SE) and peak to average power ratio (PAPR) have been two major concerns which have attracted lot of attention resulting in proposals of several other MCM schemes. But most of these studies have treated the two issues independently. This paper in particular studies the subcarrier filtering approach to improve the spectral efficiency of MCM scheme and its impact on the overall PAPR of such schemes. The analysis shows that the PAPR improvement is also achieved by such filters meant for spectral confinement and the simulation results validate the same provoking.

**Keywords**—OFDMA; PAPR; subcarrier filtering; MCM scheme; spectral efficiency; prototype filter

## I. INTRODUCTION

THE ability to effectively nullify the impact of fading in wireless channel and the ease of implementation has made fast Fourier transform (FFT) based multicarrier modulation scheme orthogonal frequency division multiplexing (OFDM), a preferred waveform from 4G onwards. OFDM is the multiple access version of OFDM which involves use of inverse FFT (IFFT) to convert a single high speed data stream into several low speed data streams which are orthogonal to each other. OFDM has several advantages such as ease of implementation due to VLSI technology, ease of equalization, compatibility with MIMO, to name a few [1]. But poor sideband suppression characteristics reduce the spectral efficiency of OFDMA resulting in adjacent channel interference which has been a concern. This led to further investigation towards improvement in spectral characteristics the MCM waveforms resulting in proposals of several new waveform carriers. Techniques namely filtered OFDM (f-OFDM), filter bank multicarrier (FBMC), universal filtered multicarrier (UFMC) have been proposed and extensively researched while they contended as preferred waveform for 5G [2]. The common factor among these three techniques is the filtering of the subcarriers, be it applied

entirely or group of subcarriers with an aim to have desired spectral response.

Another disadvantage of OFDM which has been extensively investigated is the issue of high PAPR which results in either poor power amplifier efficiency or errors due to non linearity. Several techniques such as peak clipping and filtering, selective mapping, PTS have been proposed to tackle the issue [3]. Not only did the studies try all the techniques for OFDM, but same techniques have been tried out for even filtered OFDM, FBMC and UFMC [4] but interestingly none of these PAPR reduction techniques have been employed as a standard, neither in 4G nor in 5G as of now and the DFT spread version of OFDMA called single carrier OFDMA (SC-FDMA) is still the preferred waveform for low PAPR uplink transmission partially due to the prohibitory nature of complex overhead operations [5].

This paper explores a unified approach of filtering of subcarriers to achieve both, a decent spectral efficiency and a fair PAPR improvement, especially for the FFT based MCM schemes, so that the additional filtering step adds dual advantage making the effort worth it. The fact that the current 5G eMBB waveform carrier being OFDMA, a FFT based MCM scheme and the non orthogonal schemes being considered for 5G mMTC, URLLC such as RSMA, SCMA, IDMA etc are also FFT based multicarrier, makes the whole study more relevant[6-9]. Section II presents a brief survey on the approaches pursued so far with respect to the PAPR reduction and improving SE of MCM schemes. Section III discusses few of the filter based MCM approaches to improve SE and generalizes the impact of such filtering on PAPR. Section IV presents the simulation results which justify the motivation. Section V concludes the paper.

## II. BACKGROUND AND MOTIVATION

The study on reduction of PAPR in OFDM has led to several research outputs and several schemes have been proposed. Along with the amount of PAPR reduction, the important design consideration among the schemes has been the additional computational overhead, the amount of side information affecting bandwidth efficiency and the impact of the measure on

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overall BER performance. While few schemes like Partial Transmit Sequence (PTS), Selective Mapping (SLM) etc. either introduce additional bandwidth requirement as they need to carry substantial amount of side information thus reducing bandwidth efficiency[10][11], other schemes like clipping, peak filtering, companding etc need very little side information but result in poor BER performance[12][13][14]. As of now, SC-FDMA has been adapted for low PAPR uplink transmission.

On the other hand, spectral confinement of baseline OFDM has resulted in research proposing several subcarrier filtering based waveforms with f-OFDM, UFMC, FBMC being the prominent contenders and all these have been found to suppress the sidebands effectively enough with little or no BER degradation[15][16][17]. What has likely kept these schemes being adapted as the preferred waveform in 5G revolution as against OFDM are the factors like relative ease of implementation, their adaptability into a multiple access scheme and compatibility with multiple input multiple output(MIMO) systems etc which are still being researched[18].

Even these MCM schemes, have had PAPR issues which have been addressed by several researchers by employing basically the very same approaches that were employed for OFDM such as PTS, SLM, companding etc [19-23]. Few even suggested the DFT spread FBMC, UFMC which mimic the action of SC-FDMA on these MCM schemes [24]. An already computationally extensive MCM scheme burdened with filtering task, further subjected to additional overhead related to the PAPR reduction method is the concern which the paper addresses. The intention of this study is to analyze the PAPR reduction capabilities of the subcarrier filters, so that the possibility of a well designed filter which not only suppresses sidebands but also reduces PAPR is explored further by the research community.

### III. FILTER BASED MCM SCHEMES

The intention for the filtering OFDM is the poor sideband suppression characteristics of OFDM as shown in figure 1, which is the power spectral density (PSD) of wave thus generated. As seen in figure, a difference of mere 40 dB between main lobe and the side lobes is the concern.

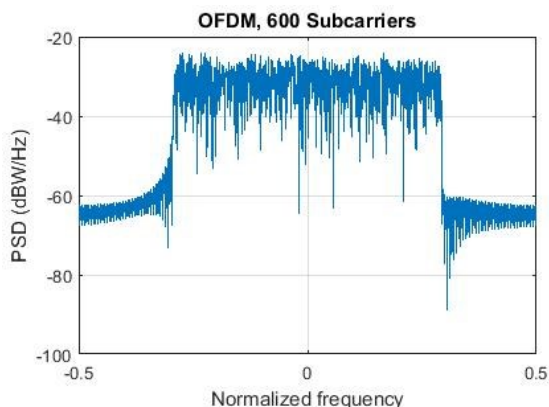


Fig.1 Normalized PSD of a sample baseline OFDM scheme

Fig.2 a, b and c depict the popular versions of modified OFDM with improved spectral characteristics namely f-OFDM, UFMC and FBMC respectively which involve filtering to improve OFDM spectral efficiency as indicated by the blue outlined box.

While f-OFDM and UFMC perform the filtering after the IFFT stage, FBMC does effective filtering before the IFFT stage as depicted in fig.2 a,b,c respectively.

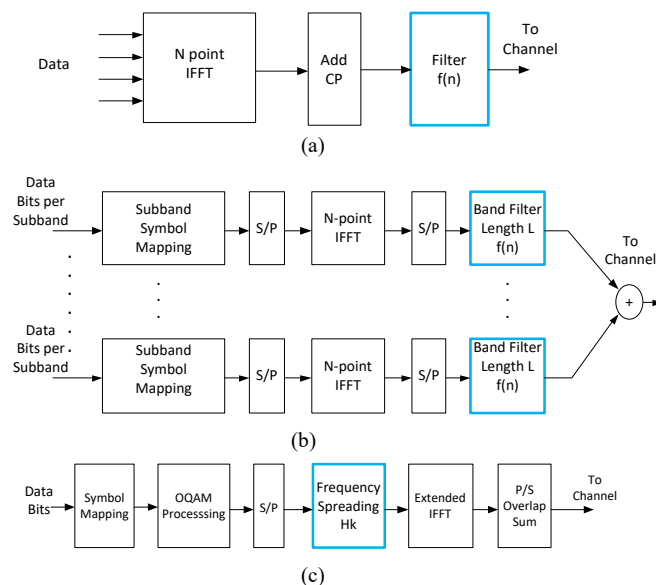


Fig.2 The basic block diagrams of the schemes (a) F-OFDM (b) UFMC (c) FBMC

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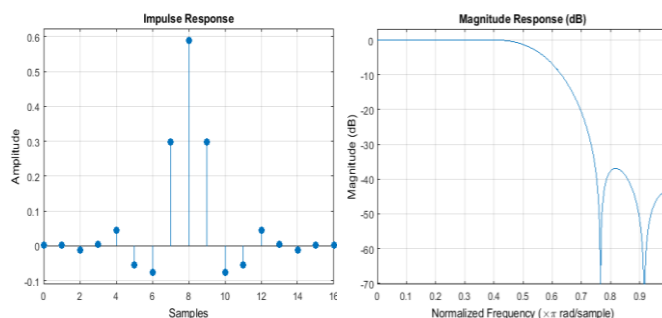


Fig.3 (a) F-OFDM prototype filter impulse response (b) Frequency Response (c) PSD of resulting F-OFDM

*Prototype filter in f-OFDM and its impact:* As per the [15] f-OFDM, the prototype filter should satisfy the criteria such as “a flat passband over the subcarriers in the subband, a sharp transition band to minimize guard-bands, sufficient stop-band

attenuation". Accordingly the filter  $h(t)$  given in equation below is used.

$$h(t) = \text{sincB}(t)w(t) \quad (1)$$

where  $B$  is the bandwidth of the sinc impulse response and  $w(t)$  is windowing for smooth transitions to avoid the frequency spillover in the truncated filter, given by:

$$w(t) = \begin{cases} 0.5 [1 + \cos(2\pi t/T_w)]^\alpha, & |t| \leq T_w/2 \\ 0 & |t| > T_w/2 \end{cases} \quad (2)$$

where  $\alpha$  is the roll-off factor used to control the shape of the window, and  $T_w$  is the filter length. The plot of the impulse response of the filter along with the frequency response and PSD of f-OFDM is as shown in figures 3 a,b and c respectively.

*Prototype filter in UPMC and its impact:* The Dolph-Chebyshev window  $w(n)$ , which forms the UPMC prototype filter [16] is computed as the inverse DFT of  $W(\omega_k)$  given by

$$W(\omega_k) = \frac{\cos\{M \cos^{-1}[\beta \cos(\frac{\pi k}{M})]\}}{\cosh[M \cosh^{-1}(\beta)]}, \quad k = 0, 1, 2, \dots, M - 1 \quad (3)$$

where

$$\beta = \cosh\left[\frac{1}{M} \cosh^{-1}(10^\alpha)\right], \quad \alpha \approx 2, 3, 4 \quad (4)$$

The parameter  $\alpha$  controls the side-lobe level,  $M$  is the window length. Thus,  $\alpha = 2$  gives side-lobes which are 40 dB below the main-lobe peak. The plot of the impulse response of the filter  $w(n)$  along with the frequency response and PSD of thus filtered UPMC is as shown in figures 4 a,b and c respectively.

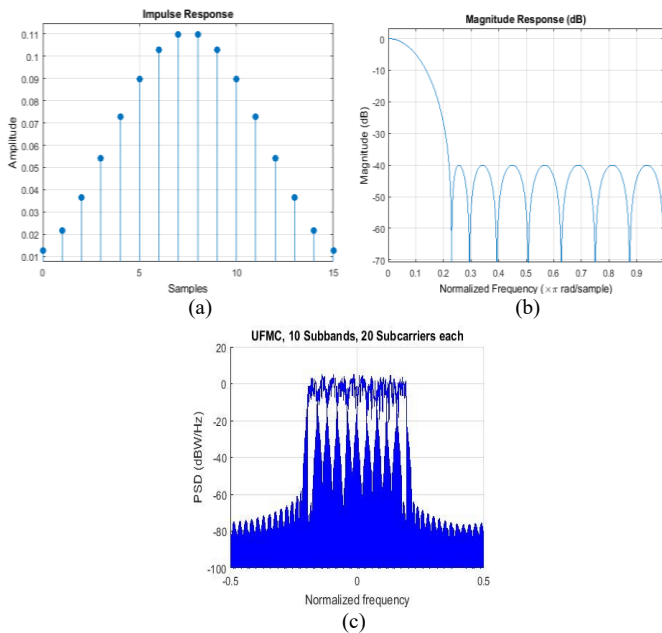


Fig.4 (a) UPMC prototype filter impulse response (b) Frequency Response (c) PSD of resulting UPMC

*Prototype filter in FBMC and its impact:* The filtering in FBMC as compared f-OFDM, UPMC, is carried out before taking IFFT, which is equivalent to frequency domain. In the frequency domain, the filter response consists of  $2K-1$  pulses, as per equation 5 and value in table I. The continuous frequency response, also shown in Fig.5, is obtained from the frequency coefficients through the interpolation formula given in equation 5. [17]

$$H(f) = \sum_{k=-(K-1)}^{K-1} H_k \frac{\sin(\pi(f - \frac{k}{MK})MK)}{MK \sin(\pi(f - \frac{k}{MK}))} \dots \dots \dots (5)$$

Here  $K$  is the overlapping factor and  $M$  is no. of FFT points. Figures 5 (a), (b) and (c) show the impulse response, frequency response and the PSD of thus resulting FBMC

TABLE I  
FREQUENCY DOMAIN PROTOTYPE FILTER COEFFICIENTS

K	H <sub>0</sub>	H <sub>1</sub>	H <sub>2</sub>	H <sub>3</sub>	$\sigma^2$ (dB)
2	1	$\sqrt{2}/2$	-	-	-35
3	1	0.911438	0.411438	-	-44
4	1	0.971960	$\sqrt{2}/2$	0.235147	-65

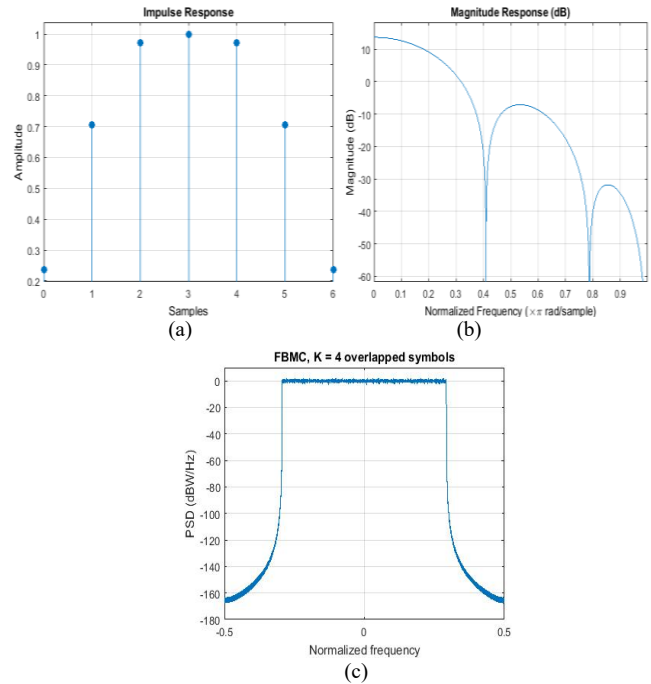


Fig.5 (a) FBMC prototype filter (a) Impulse response (b) Frequency Response (c) PSD of resulting FBMC

Having noted all the above techniques broadly, the following observations can be made

1. The filters chosen are having a bell shaped impulse response, resulting in flat frequency response in the passband and an abrupt or tapered decay towards stopband.
2. Irrespective whether filtering is carried out before or after IFFT, its important to note that the whole additional computational complexity resulting from the filtering action is justified for achieving better SE in most MCM contenders.
3. All the above techniques have been studied for further PAPR improvement employing the very same popular schemes which were tried for OFDM, independent of the existing filtering complexity, adding to additional computational overhead.

In this paper we show that such filters as discussed above are capable of not only improving SE but can also provide a decent PAPR reduction. This paper demonstrates that PAPR

capabilities of the study on the impact of filtering on PAPR characteristics, for these prototype filters exhibiting considerable improvement in PAPR and generalizes the action for any filter response, especially after IFFT stage.

#### Impact of filtering on PAPR

The PAPR for any complex MCM segment  $x[n]$  which is obtained after the  $N$  point IFFT stage, where  $N$  is the number of subcarriers, is evaluated using the following expression

$$\text{PAPR} = \frac{\max|x[n]|^2}{E\{|x[n]|^2\}} \quad (6)$$

For any filtering action that takes place after an  $N$  point IFFT stage, the worst case scenario that can happen, though very rarely, would be the occurrence of an impulse of strength  $N$ , either at origin in case of DC or a shifted impulse at any other instance  $m$  i.e.  $x[n] = N\delta(n - m)$ , [25], resulting in the maximum possible PAPR given by

$$\text{PAPR}_{\max} = \frac{\max|x[n]|^2}{E\{|x[n]|^2\}} = \frac{N^2}{N} = N \quad (7)$$

Filtering such an IFFT subcarrier set  $x[n]$  which is nothing but a shifted and scaled impulse, using any filter with impulse response  $h[n]$  will result in output  $y[n]$  which is impulse response itself though shifted to instant ' $m$ ' and scaled by factor  $N$

$$y[n] = x[n] * h[n] = N\delta[n - m] * h[n] = Nh[n - m]..(8)$$

So the peak power in such case would be the square of the maximum amplitude of impulse response scaled by factor  $N$  i.e.  $N \cdot \max|h[n]|$  and the resulting PAPR for such a filtered sequence PAPR' will be

$$\text{PAPR}'_{\max} = N \left[ \frac{\text{papr}(h[n])}{\text{length}(h[n])} \right] \quad (9)$$

Above equation thus is the generalized expression for maximum possible PAPR that can occur after filtering the IFFT coefficients, by filter with response  $h[n]$ . Compared to equation 7, the derived equation implies a further PAPR reduction which depends directly on the PAPR of a cleverly designed filter impulse response and its length. The result in expression, though very optimistic, is achieved very rarely. But the expression reveals that filtering action which has been treated as of now with purely spectral confinement measure, can also serve the purpose of PAPR reduction.

#### IV. SIMULATION RESULTS

In this section the demonstration on the impact of filtering on the PAPR performance is presented via simulation results. Matlab is the simulation tool used. The comparisons have been made between baseline OFDM and OFDM filtered with different types of prototype filters after the IFFT stage as shown in fig.2(a). Number of subcarriers chosen is  $N=1024$ . 16-QAM modulation is used for simulations. A Dolph Chebyshev prototype with attenuation 40 dB, Kaiser prototype with beta 4, Gaussian prototype with alpha 2.5 and also a simple moving average filter respectively were used for demonstration purpose[26][27]. Figure 6 shows the PAPR comparison of

regular OFDM waveform along with the said filters, all of length 30. The PAPR is characterized by its complementary cumulative distribution function (CCDF)  $P_c$ , which is expressed as

$$P_c = \Pr(\text{PAPR} > \text{PAPR}_0) \quad (10)$$

where  $P_c$  is the probability that of PAPR exceeds a particular value of  $\text{PAPR}_0$ . The figure clearly shows a PAPR improvement of about 1 dB in most of the filtered OFDM schemes employing above quoted filters, except in moving average filter which is about 0.8 dB.

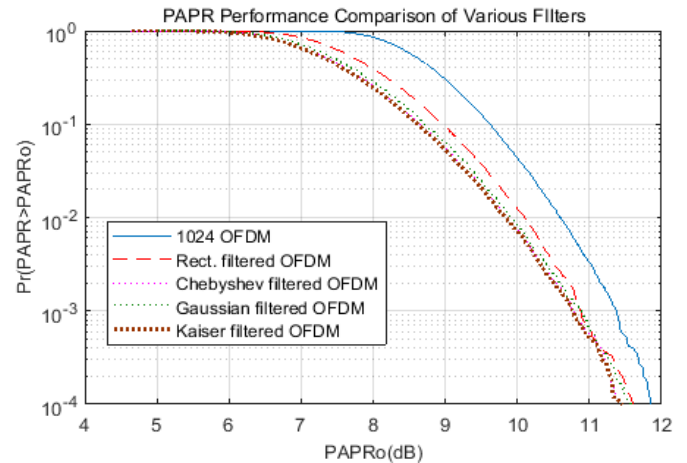


Fig.6. PAPR performance comparison of baseline OFDM with OFDM filtered with few prototype filters clearly showing PAPR improvement with filtering action

Figures 7 and 8 show the impact of filter length and the PAPR of the impulse response of prototype filter, on the overall PAPR performance as indicated by equation 9. For a Chebyshev filter with side lobe attenuation of 40dB, it can be observed that the PAPR gets better by 0.75 dB, 1.1 dB, 1.7 dB respectively for filter lengths of 20, 40 and 75 as shown in fig.7. For a Chebyshev filter with length 45, it can be observed that as the PAPR of the filter impulse response is reduced, which in turn is directly dependent on the lobewidth and inversely on attenuation as shown in figure 8(a), the PAPR performance of the filtered OFDM gets relatively better as shown in figure 8(b).

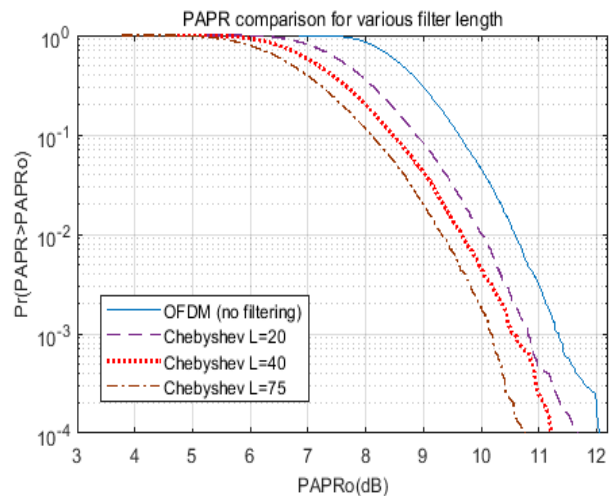


Fig.7 PAPR performance comparison of baseline OFDM with OFDM filtered with Dolph-Chebyshev prototype with different filter lengths, which improves with increasing length

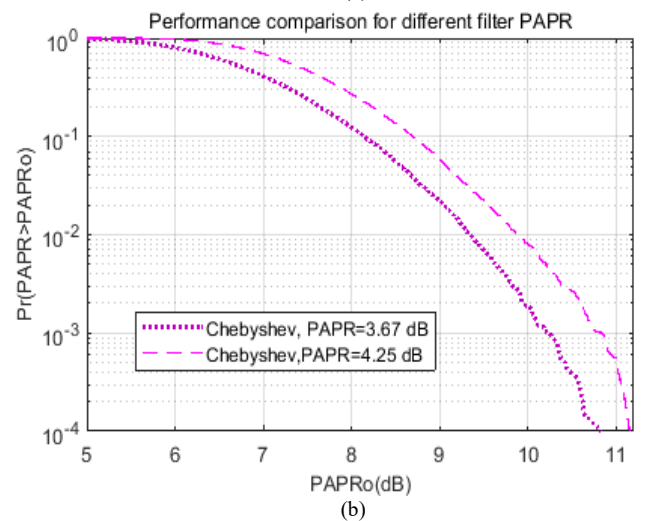
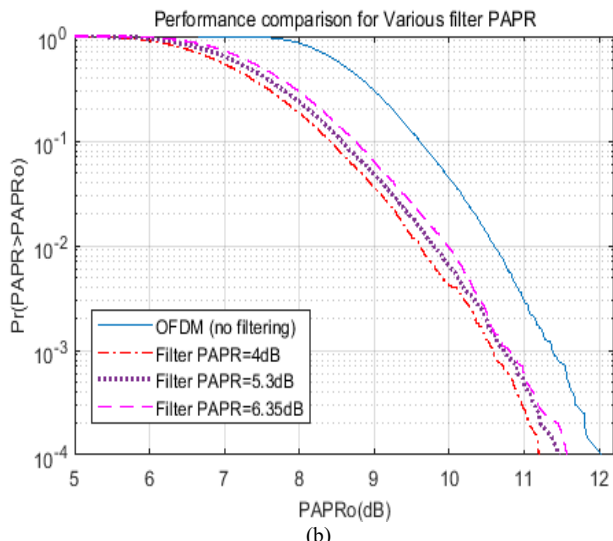
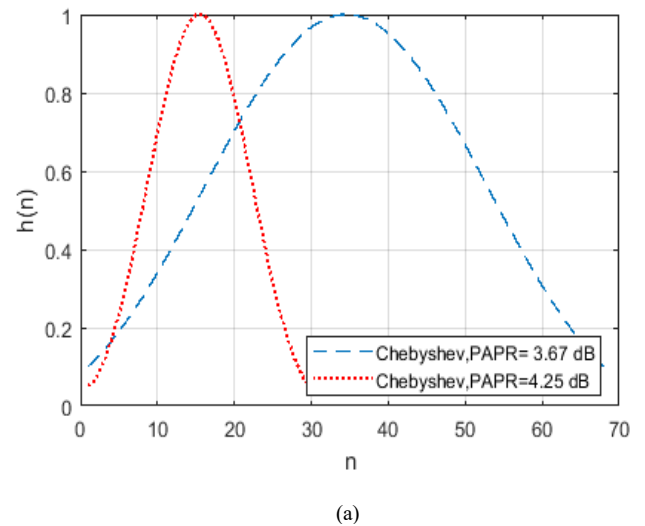
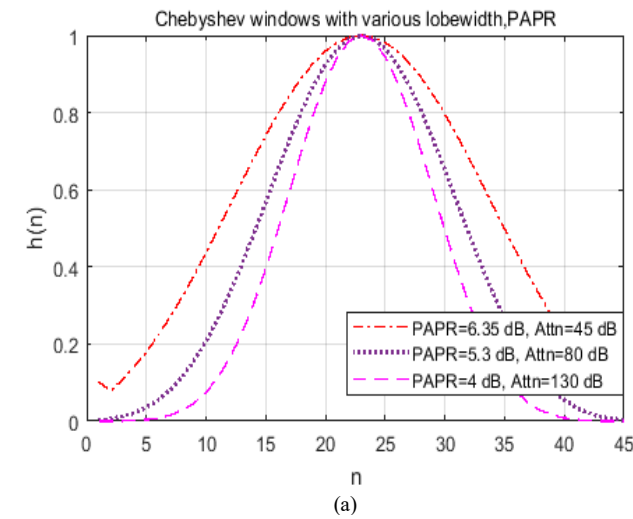


Fig.8 (a) Various Dolph-Chebyshev filters of equal length but different PAPR (b) Corresponding PAPR performance which improves with reduction in filter PAPR values

Fig. 9 Performance comparison of combined impact of filter length, PAPR on overall MCM PAPR performance (a) Chebyshev filters of different length, PAPR (b) Corresponding filtered OFDM PAPR performance comparison

Figure 9 gives an overall comparison where it can be observed that two impulse responses differing in both length as well as sidelobe attenuation result in different PAPR responses such that the filter with low PAPR of 3.67dB and higher length of 70 as in fig.9(a) results in a MCM scheme with better PAPR performance as shown in fig.9 (b) as against a filter with PAPR of 4.25dB and length 30.

But this PAPR improvement comes at the cost of decay in spectral efficiency since the filters with wider lobe width which tend to reduce PAPR also end up affecting spectral confinement properties which has been depicted in fig.10 as in the case of a UFMC system. A sample UFMC system with 10 subbands of 20 subcarriers each, as depicted in fig. 5 was used better demonstration of impact of filtering on sideband attenuation, where same filters were used for all subband filtering. 10000 samples of size 512 each with 512 point FFT and Dolph Chebyshev (DC) prototype filter were used for simulations.

Two DC filter impulse responses with PAPRs of 2.35 dB, 2.92 dB are shown in figure 10 (a) where the filter with wider lobe is exhibiting lower PAPR.

Figure 10 (b) shows the PAPR performance corresponding UFMC system, where the system filtered with DC filter of low PAPR ie 2.35 dB performs better compared to system filtered with DC filter of bit higher PAPR of 2.92 dB. But their impact on spectrum is otherwise opposite. Figures 11 (a) and (b) show the spectra of UFMC systems implemented using these prototype filters, where it can be clearly seen that the sidelobe attenuation of the system employing filter with higher PAPR is about 80 dB but that of lower PAPR is about 50 dB. This indicates a tradeoff that exists between out of band suppression and PAPR improvement capabilities of prototype filters. Thus the challenge would be the design of filters that would not only improve spectral performance but also reduce overall PAPR.

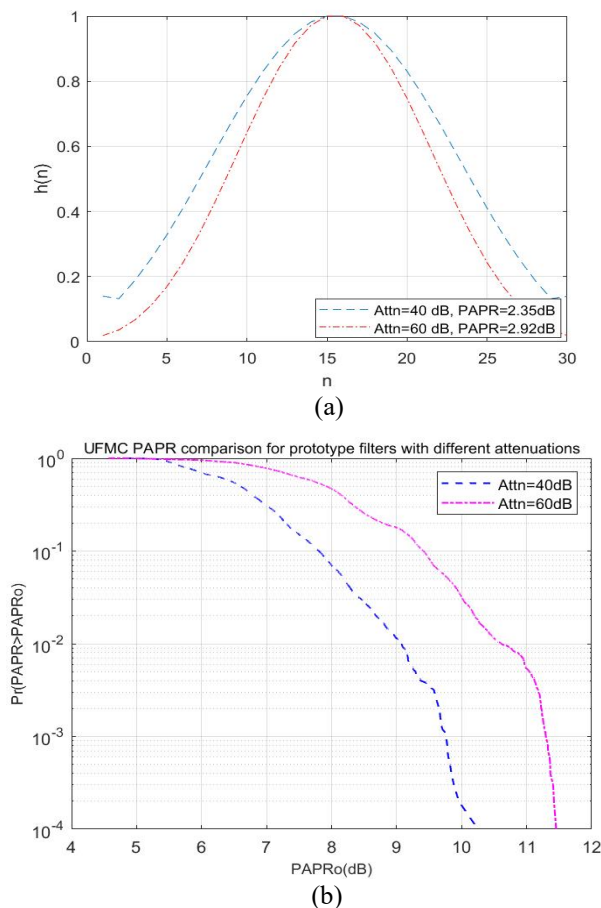


Fig. 10 (a) Chebyshev filter prototypes of different PAPR, side lobe attenuation (b) Corresponding UPMC PAPR performance comparison where the one filtered with low PAPR prototype filter performs better

## V. CONCLUSION

Due to poor sideband suppression characteristics of OFDMA, several subcarrier filtering based MCM schemes have been proposed as the likely solutions. Moreover, the high PAPR in baseline OFDMA too has garnered attention of researchers resulting in several independent approaches to reduction of the same. This paper briefly summarized the popular filter based MCM schemes that have been proposed so far to reduce OOB emission, justifying the additional task of filtering. A generalized expression for maximum PAPR reduction capability of such filtering action on OFDM after the IFFT stage was derived. Simulation results demonstrated the ability of the filtering action to not only improve the spectral efficiency but also improve PAPR characteristics at the same time, based on various parameters of the filter impulse response. A unified investigation of efficient prototype filter design to achieve optimum spectral properties along with PAPR improvement capability can be studied in future along their impact when applied to various orthogonal, non-orthogonal MCM schemes for 5G.

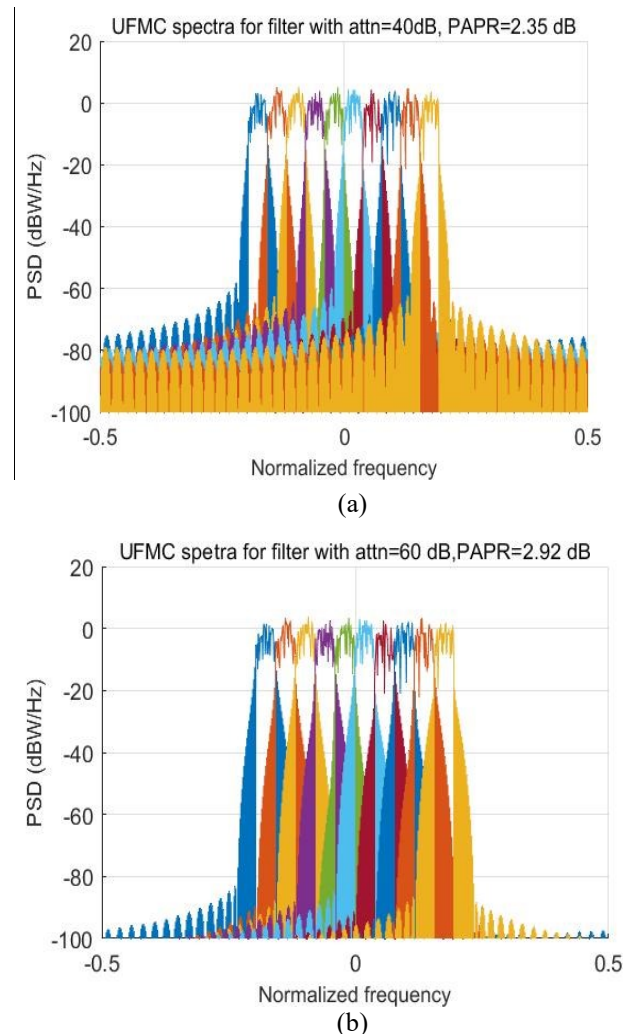


Fig. 11 (a) Spectrum corresponding to filter with lower PAPR but showing poor sideband attenuation (b) Spectrum corresponding to filter with higher PAPR, showing relatively better sideband attenuation

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