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Wavelet Neural Network based adaptive event triggered control scheme for a class of nonlinear systems with uncertain dynamics

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***Abstract*—In this paper, an adaptive event-triggered control approach for a class of unknown dynamics networked stringent feedback nonlinear systems is developed. The approximation of system uncertainties by a wavelet neural network (WNN) frequently presents a significant obstacle in the development of a precise control strategy. In order to guarantee the specified system performance and Zeno-free behaviour of networked control systems, we build an adaptive event triggering mechanism that is enhanced with WNN and outfitted with predetermined event triggering circumstances. In order to ensure the uniform ultimate boundedness (UUB) of all closed loop signals, the controller works to reduce the amount of information exchanged between the sensor and the controller. We offer numerical simulations to demonstrate the efficiency of the suggested plan.**

***Keywords*—Networked control systems; Event triggered control; Zeno behaviour; Wavelet neural network**

I. Introduction

NETWORKED control systems can be considered as one of the notable dimensions of contemporary control systems. In network control systems information exchange between various components of the closed loop control system is carried out through a communication network. This realm of the control systems allows the control of various tele operated dynamical systems like mining robots etc. The extension of this idea can be viewed in terms of cooperative control systems, cyber physical systems, IIOT etc.

Control over a communication network has enhanced the applicability areas of control systems but is also associated with issues like reliability, security, limited bandwidth, latency, and others. These issues of concern often demand modified control strategies instead of the classical control laws [1].

Limited channel bandwidth often detunes the system performance and sometimes even leads to system instability.

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Event triggered control schemes can be viewed as one effective strategy to deal with the constraints imposed by limited network resources such as channel bandwidth. In event control schemes, system dynamics-based event mechanisms are designed for the allocation of network resources and whenever the system conditions violate some prescribed performance criteria, event, command signals are updated by allowing the information exchange between the sensor and controller. Thus, the information is transmitted only discrete instants and between two consecutive instants the channel resources are free. This technique is particularly useful in the case of network resource sharing and optimum utilization of resources [2-6].

Event triggering mechanisms are required to be carefully designed in order to avoid Zeno behaviour which denotes the infinite number of triggering in finite time. Zeno behaviour is a devastating phenomenon for practical systems as it requires the system components to undergo high frequency switching resulting in phenomenon like chattering and even device breakdown and so is required to be avoided. The presence of complicacies like unmodelled dynamics, external disturbances make the system dynamics highly sensitive to Zeno behaviour and triggering mechanisms for such systems are required to guarantee the avoidance of Zeno behaviour. [2-11]

Effectiveness of the control law mainly relies on the accuracy of the mathematical model of system. However, it appears a non-trivial task to obtain mathematical model of a real time dynamical system with a prescribed level of accuracy when such systems are associated with complicated system dynamics which are difficult to be derived mathematically. There always exists a discrepancy between the real time system and a model developed under these conditions. These discrepancies are usually reflected in the performance of real time system controlled by using the control law developed by using the mathematical model. Semi generic approach of control law development has been proved highly effective under these circumstances, these control terms are augmented with approximation tools like neural networks, wavelet networks which are used to model the system uncertainties accurately. These tools work on the principle of Weierstrass approximation theorem which provides an analytical statement about the capability of polynomial families to approximate any continuous nonlinear function and can be considered as the truncated version of the analytical functions. Wavelet families illustrate the accurate approximation capability due to distinguished characteristics of orthonormality and multiresolution. Wavelet networks have been successfully used as approximation tools in several adaptive control schemes by appropriately aggregating the baseline control with wavelet network. These adaptive control terms are not only useful under the conditions of uncertainties, but also expand the applicability of the control law for the systems with similar dynamics.

Due to universal approximation capability and parsimonious structure, this work utilizes wavelet neural network for function approximation. [12-19].

The major contribution of this paper is the Zeno free adaptive event triggered control scheme for effective control of networked nonlinear system with unmodelled dynamics and limited network resources.

The rest of the paper is structured as follows: Section II presents the system preliminaries, mathematical model of the system, approximation aspects of wavelet networks are discussed in this section. Section III presents controller design along with event triggering mechanism designed to incorporate with control scheme. Section IV details the stability aspects of the closed loop system. Section V verifies the existence of lower bound between two consecutive triggering. Section VI illustrates a simulation study whereas section VII presents a conclusion.

II. SYSTEM PRELIMINARIES

A. Mathematical model of the System

Consider the following form of strict feedback nonlinear system with uncertain dynamics

$$\begin{cases} \dot{x}_i = x_{i+1} \quad ; i = 1, n-1 \\ \dot{x}_n = f(x) + u \\ y = x_1 \end{cases} \tag{1}$$

where $x = [x_1 \quad \cdots \quad x_n]^T \in R^n$ are the system states, $u \in R$ is the control input, $y \in R$ is the system output while the nonlinear function $f(x): R^n \to R$ represents the system uncertainty satisfying Assumption 1.

Strict feedback nonlinear model is an effective way of nonlinear system modelling, several nonlinear systems like flexible link robots, inverted pendulum, ball and beam can be modelled in this form. A remarkable feature of this modelling is the insurance of input to state stabilizability and existence of stabilizing feedback control law.

Objective is to design an adaptive event triggered control with event triggering conditions so that the system error dynamics converge to a compact set $\Omega_r \subset R$ including origin and at the same time system behaviour is Zeno free.

Assumption 1: System uncertainty is Lipschitz continuous on every compact set $S_x \subset R^n$ satisfying the following property

$$|f(x(t)) - f(x(t_k))| \le L\{\textstyle\sum_{i=1}^{n} |x_i(t) - x_i(t_k)|\} \tag{2}$$

where $L > 0$ is the Lipschitz constant.

Assumption 2: Reference signal $y_d(t) \in R$ and its derivatives up to $(n-1)^{th}$ order $\{y_d(t), \dot{y}_d(t), \ddot{y}_d(t), \ldots\}$ are bounded and known.

B. Wavelet Network

Wavelet networks can be viewed as effective and parsimonious network structures for function approximation. One of the appealing features of wavelet networks is the use of orthonormal wavelet functions which satisfy the norms of multiresolution analysis as activation functions. In this work, wavelet neural network is used to estimate the uncertain square integrable functions $f(x) \in L^2(R)$ defined on a compact set of state trajectories $S_x \subset R^n$. Wavelet network representation of any signal $f(x)$ can be expressed as

$$\hat{f}(x) = \textstyle\sum_{k=1,2,\ldots}^{K_{J_0}} \alpha_{J_{0,k}} \varphi_{J_{0,k}}(x) + \sum_{j=J_0,}^{J} \sum_{k=1,2\ldots}^{K_j} \beta_{j,k} \psi_{j,k}(x) \tag{3}$$

where $[J_0, J] \in Z^2$ represent the coarsest and finest resolution level, $K_j \in Z$ is the number of translates at a given resolution level.

While $\alpha_{J_{0,k}}$ and $\beta_{j,k}$ are the weights of scaling function $\varphi_{J_{0,k}}(x)$ and wavelet function $\psi_{j,k}(x)$ respectively. Any wavelet function satisfying the norms of multiresolution analysis and its associated scaling function can be used.

Lemma1: There exist an unknown but finite value of J and K_j such that any unknown nonlinear function $f(x) \in L^2(R)$ defined on a compact set $S_x \subset R^n$ can be approximated with prescribed accuracy.

Thus, for a given constant $0 < \epsilon < |\epsilon_m|$ there exist optimum weight values $\alpha^*_{J_{0,k}}$ and $\beta^*_{j,k}$ such that

$$\sup_{S_x \subset R^n} \left| f(x) - \left\{ \textstyle\sum_{k=1,2,\ldots}^{K_{J_0}} \alpha^*_{J_{0,k}} \varphi_{J_{0,k}}(x) + \sum_{j=J_0,}^{J} \sum_{k=1,2\ldots}^{K_j} \beta^*_{j,k} \psi_{j,k}(x) \right\} \right| \le \epsilon \tag{4}$$

Considering $\hat{\alpha}_{J_{0,k}}$ and $\hat{\beta}_{j,k}$ as the estimates of $\alpha^*_{J_{0,k}}$ and $\beta^*_{j,k}$, wavelet network estimate of the function $f(x)$ can be expressed as

$$\hat{f}(x) = \sum_{k=1,2,\ldots}^{K_{J_0}} \hat{\alpha}_{J_{0,k}} \varphi_{J_{0,k}}(x) + \sum_{j=J_0,}^{J} \sum_{k=1,2\ldots}^{K_j} \hat{\beta}_{j,k} \psi_{j,k}(x) \tag{5}$$

With estimation error defined as

$$\tilde{f}(x) = f(x) - \hat{f}(x) = \textstyle\sum_{k=1,2,\ldots}^{K_{J_0}} \tilde{\alpha}_{J_{0,k}} \varphi_{J_{0,k}}(x) + \sum_{j=J_0,}^{J} \sum_{k=1,2\ldots}^{K_j} \tilde{\beta}_{j,k} \psi_{j,k}(x) + \epsilon = \tilde{\alpha}^T \varphi(x) + \tilde{\beta}^T \psi(x) + \epsilon \tag{6}$$

where $\tilde{\alpha}_{J_{0,k}} = \alpha^*_{J_{0,k}} - \hat{\alpha}_{J_{0,k}}$ and $\tilde{\beta}_{j,k} = \beta^*_{j,k} - \hat{\beta}_{j,k}$

By developing appropriate update laws for weight value updation weight estimation error can be reduced to arbitrarily small value.So due to the universal approximation property of wavelet networks, with enough resolutions and translates, the estimation error can be confined to a compact set so that $|\tilde{f}(x)| < F$ for $\forall x \in S_x \subset R^n$[15,17].

For multidimensional cases, wavelet and scaling functions are obtained by performing the tensor product of single dimensional scaling and wavelet functions in different dimensions, for example [15] $\psi_{j,k}(x) = \prod_{i=1}^{n} \psi_{j,k}(x_i);\psi_{j,k}(x) = \prod_{i=1}^{n} \psi_{j,k}(x_i)$

III. Adaptive Controller Design

This section details the designing of adaptive controller, weight update laws and event triggering conditions for effective implementation of closed loop system.

A. Controller Design

Defining error terms for the system of form (1)

$$\begin{cases} e_1 = x_1 - y_d \\ e_2 = x_2 - \dot{y_d} \\ e_n = x_n - \overset{n-1}{y}_d \end{cases} \tag{7}$$

Differentiation of these error terms along the system trajectories leads to the following error dynamics

$$\begin{cases} \dot{e_1} = e_2 \\ \dot{e_2} = e_3 \\ \vdots \\ \dot{e_n} = f(x) + u - \overset{n}{y}_d \end{cases} \tag{8}$$

Defining the filtered tracking error as

$$r = k_1 e_1 + k_2 e_2 + \cdots + e_n \tag{9}$$

where $k_i, (i = 1,2, \dots n - 1)$ are positive constants

Differentiation of (8) and subsequent substitutions ultimately leads to the control law of the form (10)

$$\dot{r} = k_1 e_2 + k_2 e_3 + \cdots + f(x) + u - \overset{n}{y}_d \tag{10}$$

$$u = -\left\{k_1 e_2 + k_2 e_3 + \cdots + k_{n-1} e_n + \hat{f}(x) - \overset{n}{y}_d + kr\right\} \tag{11}$$

here k is a positive constant whereas $\hat{f}(x)$ (5) is the wavelet network estimate of the uncertain term $f(x)$.

The control term (11) updates itself in a continuous way and its application in networked system will result in continuous allocation of network resources. This control scheme can be considered optimum from the point of view of limited network resources and resource sharing. Next subsection details the event triggering based modifications in (11) so that the updates are carried out only at stipulated instants known as triggering instants [5].

B. Event Triggering

In order to implement the event triggering strategy, control term (11) is redefined as

$$v(t) = u(t_k) t \in [t_k, t_{k+1}]; k \in Z \tag{12}$$

here $t_k and t_{k+1}$ can be viewed as the current and next triggering instants and between these two instants control term is held constant. Thus $v(t)$ (12) can be viewed as the event triggered version of control term $u(t)$ (11).

Control term (12) be defined by considering the control term (11) at the update instant t_k i.e. at $t = t_k$

$$v(t) = u(t_k) = -\left\{k_1 e_2(t_k) + k_2 e_3(t_k) + \cdots + k_{n-1} e_n(t_k) + \hat{f}\big(x(t_k)\big) - \overset{n}{y}_d(t_k) + kr(t_k)\right\} \tag{13}$$

The term is held constant until a predefined event triggered criteria is not violated. All the components of control term (13) are updated only at the triggering instants so the weights of the wavelet estimation term $\hat{f}\big(x(t_k)\big)$ are also updated at these instants only.

The rest of this subsection describes the development of triggering criterions to control the update instants of the control term (12). These designs mainly emphasize on system stability and prescribed performance accuracy [5].

Assuming that the last update was carried out at instant t_k and next update will be carried out at t_{k+1}. Defining an error term of the form

$$e_\Delta(t) = \sum_{i=1}^{n} |e_i(t) - e_i(t_k)| \; \forall t \in [t_k, t_{k+1})$$

The term reflects the dispersion between values used in control laws and the values which should have been used. Whenever the dispersion exceeds a specific limit updation will occur,

$$e_\Delta(t) \geq m_1 \tag{14}$$

where m_1 is the threshold for updation. Thus, triggering instant can be defined as

$$t'_{k+1} = inf\{t \geq t_k; e_\Delta \geq m_1\} \tag{15}$$

Similarly, another criterion considered is

$$e_\theta(t) = \textstyle\sum_{i=1}^{n} |x_i(t) - x_i(t_k)| \; \forall t \in [t_k, t_{k+1}) \tag{16}$$

with, triggering condition and update instants defined as

$$e_\theta(t) \geq m_2 \tag{17}$$

$$t''_{k+1} = inf\{t \geq t_k; e_\theta \geq m_2\} \tag{18}$$

With these update conditions, next triggering instant will be defined as

$$t_{k+1} = inf\{t'_{k+1}, t''_{k+1}\} \tag{19}$$

Next section establishes the stability of closed loop system.

IV. Stability Analysis

This section analyzes the stability issues of the closed loop system. Analysis is carried out at update instant and for the duration between two consecutive updates [5].

A. Stability Analysis at update instant

Considering the update instant $t = t_k$, at this instant all the control term components and subsequently the control term (12) is updated as

$$v(t) = u(t_k) = -\Big\{k_1 e_2(t_k) + k_2 e_3(t_k) + \cdots + k_{n-1} e_n(t_k) + \hat{f}\big(x(t_k)\big) - \overset{n}{y}_d(t_k) + kr(t_k)\Big\}$$

Consider a Lyapunov function of the form

$$V = \frac{1}{2}\{r^2 + \tilde{\alpha}^T\tilde{\alpha} + \tilde{\beta}^T\tilde{\beta}\} \tag{20}$$

Differentiating it along the trajectories of the system

$$\dot{V} = r\dot{r} + \tilde{\alpha}^T\dot{\tilde{\alpha}} + \tilde{\beta}^T\dot{\tilde{\beta}} \tag{21}$$

Substitution of error dynamics (9) and control term (12) results in

$$\dot{V} = r(t_k)\{\tilde{f}\big(x(t_k)\big) - kr(t_k)\} + \tilde{\alpha}^T\dot{\tilde{\alpha}} + \tilde{\beta}^T\dot{\tilde{\beta}}$$

$$\dot{V} = r(t_k)\{\tilde{\alpha}^T\varphi\big(x(t_k)\big) + \tilde{\beta}^T\psi\big(x(t_k)\big) + \epsilon - kr(t_k)\} + \tilde{\alpha}^T\dot{\tilde{\alpha}} + \tilde{\beta}^T\dot{\tilde{\beta}} \tag{22}$$

With the following update rules, above equation becomes

$$\begin{cases} \dot{\tilde{\alpha}} = -r(t_k)\varphi\big(x(t_k)\big) \\ \dot{\tilde{\beta}} = -r(t_k)\psi\big(x(t_k)\big) \end{cases} \tag{24}$$

$$\dot{V} = -kr^2(t_k) + \epsilon r(t_k) \tag{25}$$

Thus, at the update instant all the closed loop signals are bounded and $\dot{V}$ is negative outside a compact set defined as

$$\Omega_r = \Big\{r(t_k) \big| |r(t_k)| \le \frac{\epsilon}{k}\Big\} \tag{26}$$

B. Stability Analysis between two consecutive update instants

This subsection displays the stability condition at an instant lying between two consecutive triggering instants. In this duration control term and all its components are held constant.

Consider a Lyapunov function defined at some instant $t \in (t_k, t_{k+1})$

$$V = \frac{1}{2}r^2(t) \tag{27}$$

Differentiating (27) and carrying out subsequent substitution

$$\dot{V} = r(t)\dot{r}(t)$$

$$\dot{V} = r(t)\Big\{k_1 e_2(t) + k_2 e_3(t) + \cdots + f\big(x(t)\big) + -\overset{n}{y}_d(t)\Big\} \tag{28}$$

At this instant, control term is equal to its last updated value i.e. $u(t_k)$. Substituting the control term (12)

$$\begin{aligned}\dot{V} = r(t)&\Big\{k_1 e_2(t) + k_2 e_3(t) + \cdots + f\big(x(t)\big) \pm \overset{n}{y}_d(t)\Big\} \\ &- r(t)\Big\{k_1 e_2(t_k) + k_2 e_3(t_k) + \cdots + k_{n-1}e_n(t_k) + \hat{f}\big(x(t_k)\big) - \overset{n}{y}_d(t_k) + kr(t_k)\Big\}\end{aligned}$$

Inserting the term $\{kr(t) - kr(t)\}$ in above equation and rearranging the terms

$$\begin{aligned}\dot{V} = r(t)\Bigg\{&-kr(t) + kk_1\big(e_1(t) - e_1(t_k)\big) \\ &+ \sum_{i=2}^{n-1}(kk_i + k_{i-1})\big(e_i(t) - e_i(t_k)\big) \\ &+ (k + k_{n-1})\big(e_n(t) - e_n(t_k)\big) \\ &+ \Big(f\big(x(t)\big) - \hat{f}\big(x(t_k)\big)\Big) \\ &+ \Big(\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t)\Big)\Bigg\}\end{aligned}$$

Considering the following mathematical substitutions for transforming the above equation into a conclusive form

$$\begin{aligned}\Big(f\big(x(t)\big) - \hat{f}\big(x(t_k)\big)\Big) &= \Big(f\big(x(t)\big) - f\big(x(t_k)\big)\Big) + \Big(f\big(x(t_k)\big) - \hat{f}\big(x(t_k)\big)\Big) \\ &= \Big(f\big(x(t)\big) - f\big(x(t_k)\big)\Big) + \tilde{f}\big(x(t_k)\big)\end{aligned}$$

and

$$\rho = sup\,\{kk_1, (kk_2 + k_1), (kk_3 + k_2), \ldots, (k + k_{n-1})\}$$

$$\begin{aligned}\dot{V} \le r(t)\Bigg\{&-kr(t) + \rho\sum_{i=1}^{n}\big(e_i(t) - e_i(t_k)\big) \\ &+ \Big(f\big(x(t)\big) - f\big(x(t_k)\big)\Big) + \tilde{f}\big(x(t_k)\big) \\ &+ \Big(\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t)\Big)\Bigg\}\end{aligned}$$

$$\dot{V} \le r(t)\left\{-kr(t) + \rho\sum_{i=1}^{n}\left|\left(e_i(t) - e_i(t_k)\right)\right| + \left|\left(f(x(t)) - f(x(t_k))\right)\right| + \left|\tilde{f}(x(t_k))\right| + \left|\left(\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t)\right)\right|\right\}$$

$$\dot{V} \le r(t)\left\{-kr(t) + \rho\sum_{i=1}^{n}\left|\left(e_i(t) - e_i(t_k)\right)\right| + L\sum_{i=1}^{n}|x_i(t) - x_i(t_k)| + \left|\tilde{f}(x(t_k))\right| + \left|\left(\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t)\right)\right|\right\} \quad (29)$$

Event triggering conditions considered in this work imposes an upper bound on some components of above equation

$$\sum_{i=1}^{n}\left|\left(e_i(t) - e_i(t_k)\right)\right| \le m_1 \quad (30)$$

$$\sum_{i=1}^{n}|x_i(t) - x_i(t_k)| \le m_2 \quad (31)$$

Also, as per the assumption 2 desired trajectory and its derivatives are bounded so

$$\left|\left(\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t)\right)\right| \le m_3 \quad (32)$$

The term $\tilde{f}(x(t_k))$ represents the estimation error of the wavelet network at the instant of updation and proved in pervious subsection all the closed loop signals are bounded at the update instants and so it is justifiable to assume that

$$\left|\tilde{f}(x(t_k))\right| \le m_4 \quad (33)$$

where m_3 and m_4 are positive constants.
These justifications, thus lead to following equation

$$\dot{V} \le r(t)\{-kr(t) + \rho m_1 + Lm_2 + m_4 + m_3\} \quad (34)$$

Thus, $\dot{V}$ is negative outside a compact set defined as

$$\Omega_r = \left\{r(t_k)\big||r(t_k)| \le \frac{\vartheta}{k}\right\} \quad (35)$$

where $\vartheta = \rho m_1 + Lm_2 + m_4 + m_3$. By optimally setting the constants this set can be reduced to an arbitrarily small value.

Thus, for the closed loop system containing the dynamics (1), event triggered control strategy (12) with wavelet estimator and update laws (24), all the closed loop signals are ultimate upper bounded. Boundedness of the closed loop signals under the action of event triggered strategy is ensured at update instant as well as for the duration between the two consecutives updates.

Also, $\dot{V} \in L_\infty$ at $t = t_k\,; k \in Z^+$ implies that the considered Lyapunov function is bounded and continuous over the entire time span i.e.

$$\{V(t)|V(t) \in C; V(t) \le V(0)\}\,; \forall t \in [0, \infty) \quad (36)$$

Next section explores the issue of the existence of finite inter-execution time.

V. INTER-EXECUTION TIME

This section presents the analytical proof for the existence of finite time duration between two consecutive update instants.

For the closed loop system under consideration, for a compact set $S_x \subset R^n$ there exist a positive constant Δt such that inter-execution time is lower bounded by this value i.e.

$$(t_{k+1} - t_k) \ge \Delta t \quad (37)$$

Considering the event triggering mechanisms (30, 31)

$$e_\Delta(t) = \sum_{i=1}^{n}|e_i(t) - e_i(t_k)|$$

$$e_\theta(t) = \sum_{i=1}^{n}|x_i(t) - x_i(t_k)| \; \forall t \in [t_k, t_{k+1})$$

Their differentiation results in

$$\dot{e}_\Delta(t) = \sum_{i=1}^{n}\dot{e}_i(t)sgn(e_i(t) - e_i(t_k) \le \sum_{i=1}^{n}|\dot{e}_i(t)|$$

$$\dot{e}_\theta(t) = \sum_{i=1}^{n}\dot{x}_i(t)sgn\left(x_i(t) - x_i(t_k)\right) \le \sum_{i=1}^{n}|\dot{x}_i(t)|$$

As all the closed loop signals, system nonlinearities are bounded and continuous (36), it implies that

$$\dot{e}_\Delta(t) \in L_\infty \quad (38)$$

$$\dot{e}_\theta(t) \in L_\infty \quad (39)$$

Thus, it requires a finite time for these triggering mechanisms to make transition to next update state.

Let there exist positive constants d_1 and d_2 such that

$$\{t_{k+1} - t_k\} \ge min\left\{\frac{m_1}{d_1}, \frac{m_2}{d_2}\right\} \quad (40)$$

Thus, there exists a positive lower bound on inter-execution time and so Zeno behaviour can be successfully avoided [5-7].

Next section will illustrate the effectiveness of the control design with the help of simulation study.

VI. SIMULATION

In this section, a simulation study is demonstrated to verify the effectiveness of the event triggered control scheme. Simulation is carried out using following system dynamics

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x) + u \end{cases} \tag{41}$$

where $f(x) = 1.25\sin(x_1 x_2) + 1.1x_1^2(1 - x_2)$ is the nonlinear dynamics which is considered as system uncertainty and approximated by a wavelet network.

Considered dynamics belongs to the class of strict feedback systems (1) considered for controller development (12,13). Proposed event triggered control scheme (12) and wavelet network tuning laws (24) are applied to the system (41) to solve the tracking and regulation problem of the system.

The Daubechies wavelet (db2) with $n = 2$ is utilised to build the wavelet network that serves as the estimator. The coarsest and finest resolution levels are chosen to be 1 and 3, respectively. Three translations are performed at the coarsest resolution level, and whenever the resolution is increased by one, the number of translations is doubled.

Network weight adjustment is carried out using tunning laws (26 and 43) and initial setting for parameters is taken as zero. Updates are carried out only at triggering instants.

Two simulation cases are studied. The first case of the simulation displays the tracking performance of the system whereas second case illustrates the regulatory performance. For case one, simulation is conducted with following initial condition and gain settings

$x(0) = [0.3 \quad 0]^T$; $k1 = 3.42$; $k = 2.42$

Desired trajectory is taken as

$$y_d = 1.5\sin(t) \tag{42}$$

Results of the simulation are shown in Fig. 1 and 2. Figures reveal the system performance and quality of the control input. As per the event triggered methodology, control updates are carried out at discrete instants on the violation of triggering mechanisms. Following (14), triggering threshold is selected as $m_1 = 0.95$. As clear from the figure, system state closely tracks the desired trajectory, and the tracking error is bounded within the small bounds. As far as the control policy is concerned, acceptable control quality with successful avoidance of Zeno behaviour is achieved. Fig. 2 depicts the zoom of control input with clear display of triggering instants. Also, as clear from the figure, event triggered control term evolved here is free from Zeno behaviour with a minimum and maximum inter-execution interval about $0.09\,sec$ and $0.442\,sec$ respectively with average number of update instants around 9 over a duration of 2 sec.

Second case of the simulation displays the regulatory performance of the system (41) with control law (12, 14), following initial condition and gain settings are taken for simulation

$x(0) = [0.3, \quad 0]^T$; $k1 = 1.3$; $k = 2.2$

Triggering threshold is selected as $m_1 = 0.65$.

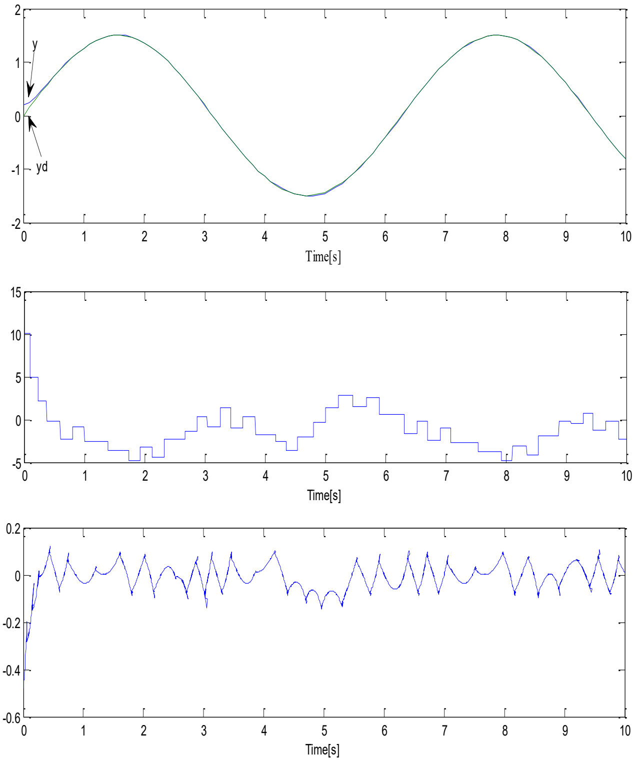

Fig. 1. Trajectory tracking performance (a) State and desired trajectories, (b) control effort, (c) tracking error

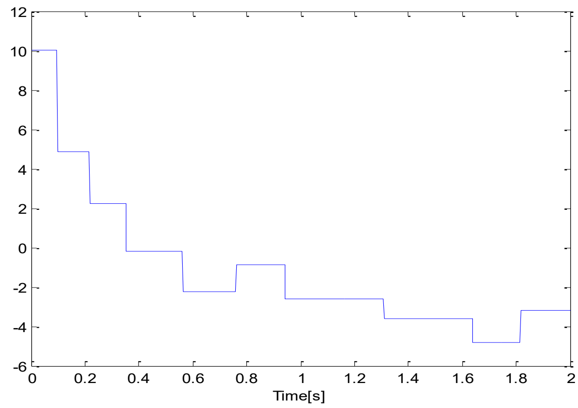

Fig.2. Extended view of control effortfor trajectory tracking

As revealed from Fig. 3, effective regulation is achieved with error converging to the bounds of the order 0.15. Also, the control term shown (Fig. 3 and Fig. 4) displays a Zeno free behaviour with a minimum inter-execution time of $0.08\,sec$ and a maximum value of $0.25\,sec$. Average number of update instants around 13 over a duration of $2\,sec$. Comparison of two cases displays the effect of threshold magnitude on system performance, higher threshold proves greater inter execution time however at the expense of system detuning. Thus, there is a trade-off between the system performance and optimal utility of network resources.

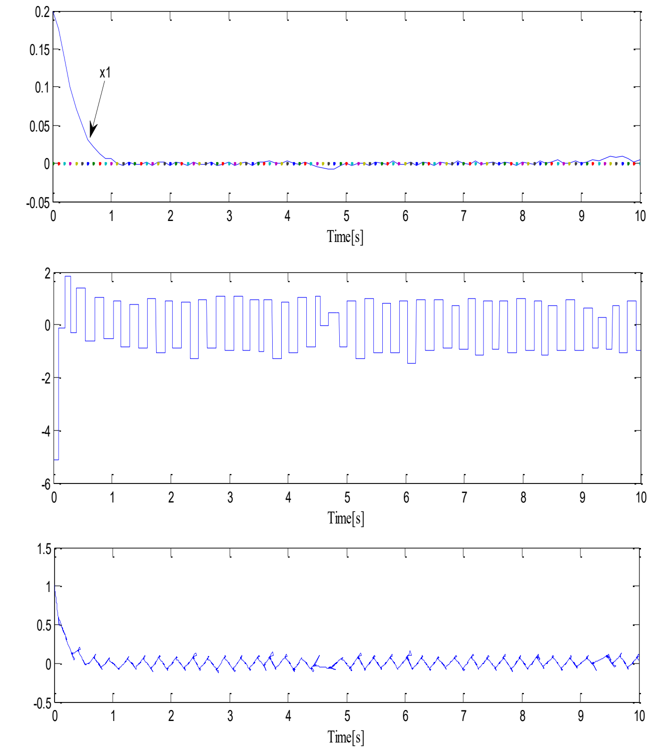

Fig. 3. Regulatory performance (a) State trajectory, (b) Control signal, (c) state error

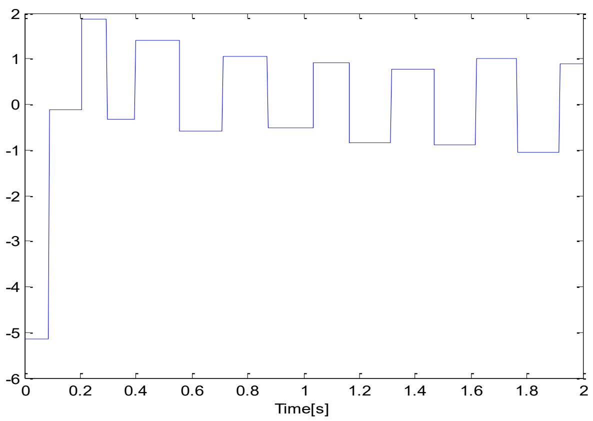

Fig. 4. Extended view of control effortfor regulation

Finally, system performance and control quality clearly observed in both the cases reflects the acceptability of the event triggered control scheme (12, 14) for the strict feedback nonlinear systems (41) configured in closed loop over a communication channel with finite resources.

VII. CONCLUSION

This paper presents an event triggered adaptive control scheme for strict feedback uncertain systems. To implement the concept of event triggering, event triggering mechanisms are designed which ensure prescribed system performance with Zeno free behaviour. There exist a nonzero value of minimum inter-execution time ensuring the feasibility of the control term. Unknown nonlinear dynamics are approximated by using wavelet neural network which uses wavelets as basis. Wavelet functions display enhanced approximation capabilities due to orthonormality of wavelet basis functions and paves a systematic network construction methodology due to the norms of multiresolution analysis. The weights of these functions are tuned online using the tuning laws developed in this work and the tuning is carried out only at the update instants. Simulation results showed the evolution of an achievable control policy with control performance within the acceptable bounds.

CONFLICTS OF INTEREST

"No conflict of interest has been disclosed by the authors." By writing this statement, each author confirms that no conflict of interest or other personal considerations could have an improper effect on the presentation or interpretation of the research findings.

AUTHOR CONTRIBUTIONS

For this article, individual contributions of the authors is stated in following paragraph: “Conceptualization, and formulation: Ajay Kulkarni and Sachin Puntambekar, analytical development: Ajay Kulkarni and Neha Kapil; simulation and validation: Neha Kapil and Nitesh Soni; literature review: Neha Kapil, Nitesh Soni and Ajay Kulkarni; writing—original draft preparation, Neha Kapil and Sachin Puntambekar; review and editing: Nitesh Soni and A. Kulkarni ,supervision: A. Kulkarni, and Sachin Puntambekar; project administration: Ajay Kulkarni.

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