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Gaussian Mixture Model with Bayesian approach for maximizing RSS-based localization in underwater Wireless Sensor Networks

Kiruthiga V, and Narmatha V

***Abstract*—Source localization is a highly challenging and complex task in underwater environments due to uncertainties and unknown sound propagation speed profiles in underwater channels, as well as increased Doppler effects and constraints on the energy sources of the sensor nodes. To address these issues, we propose an energy-efficient Joint Gaussian Mixture Model with a Bayesian approach for localization algorithms, aiming to improve Received Signal Strength (RSS) accuracy. In this article, we represent the additive noise using a Gaussian Mixture Model to calculate the maximum likelihood estimation. The Bayesian statistical approach solves the convex optimization problem to find effective globally optimal solutions. These joint methods help mitigate the underwater Doppler spread effects and improve the estimation of sensor node positions. The simulated results are analyzed, and the performance metrics show that the proposed GMM-Bayesian approach is very close to the Cramér-Rao Lower Bound and this method also outperforms other existing localization algorithms in terms of lower Root Mean Squared Error (RMSE) relative to anchor nodes and a better Cumulative Distribution Function (CDF) for localization errors. From the simulation results, it is evident that the proposed approach achieves substantial performance gains in the localization of underwater wireless sensor networks.**

***Keywords*—Underwater Wireless Sensor Networks, Localization, Received Signal Strength, Gaussian Mixture Model, Bayesian Approach, Localization Error.**

I. Introduction

UNDERWATER communication has become very important and is attracting significant attention due to its wide range of applications in underwater environments [1]. Underwater communication has become very important and is attracting more attention based on the wide range of applications in underwater environments [2]. This UASN has many applications [3], such as monitoring climatic changes, coastal monitoring, surveillance, and oil pipeline monitoring systems. Due to the channel characteristics in underwater environments [4], [5], such as limited bandwidth, high signal propagation delay, and the salinity of ocean water, there are inherent challenges for signal transmission and reception. Therefore, data transmission from effective localization [6] is becoming a challenging task. In order to overcome these issues, we need an effective localization technique for identifying the location of the sensor nodes.

Several localization methods are used to optimize the placement of sensor nodes and facilitate data forwarding through effective beaconing of signals [7]. Some of the methods include energy-based localization and Received Signal Strength (RSS) difference-based schemes [8]. Most UWSN sensor nodes utilize energy-based localization methods, which depend on the underwater environment and specific applications [9]. Similarly, some localization schemes use RSS-based localization to achieve better estimation through Maximum Likelihood (ML) estimators to determine the positions of target nodes, from which signals or information need to be transmitted or received between nodes [10], [11]. This problem leads to a non-linear and non-convex optimization challenge for RSS-based localization.

A. Related Works

Many researchers have proposed various localization techniques. This section discusses some of the most important and recent works. This localization strategy is achieved through measurements that include Time of Arrival (TOA), Time Difference of Arrival (TDoA), Angle of Arrival (AoA), Received Signal Strength (RSS), and differences in RSS. Detecting received signals based on RSS values is always a challenging task due to the characteristics of underwater channels [12]. The localization problems based on RSS measurements are generally formulated using techniques such as Least Squares Estimation and Convex Optimization. Existing methods [13] are generally classified into three types: semidefinite programming-based, Taylor series expansion-based, and least squares method-based [14]. SDP techniques also use convex optimization methods with certain relaxations. The major limitation of this method is the computational complexity involved in solving the optimization problem [15]. In [16] poses a two-step, iteratively reweighted least squares (LS) algorithm. In the first step, the algorithm calculates the initial estimates of position and velocity. In the second step, the algorithm refines the RSS measurements through an iterative process.

The Taylor series expansion is frequently used to optimize problems related to the estimation of velocity and position

Kiruthiga V, Research Scholar, Department of computer & Information Science, Annamalai university, Chidamabaram, India. (e-mail: kiruthivaradhan@gmail.com).
Narmatha V, Assistant Professor, Department of computer & Information Science, Annamalai university, Chidamabaram, India. (e-mail: balaji.narmatha8@gmail.com).

for targeted nodes. The major limitation of this work is that selecting the initial values makes it very difficult to find the local solutions. To effectively address such local optimizations, SDP techniques are proposed. These SDP techniques provide a relaxation of these non-convex problems into convex optimization problems [17]. This method has very good accuracy, but it involves tight optimization and high computational complexity. For effective position estimation and velocity values, the authors propose a multiple and robust version of the two-step weighted least squares method [18]. The main issue with these methods is that they are not suitable for practical and realistic underwater environments. In [19], the authors propose a three-step TS-WLS algorithm that jointly estimates location information based on the coordinates. In this method, the estimation of Doppler spread values is omitted from the optimization function. In [20], a TOA measurements-based SDP localization algorithm is proposed. This approach also suffers from issues such as clock synchronization and computational complexity.

Some algebraic localization methods [21] have been proposed by researchers for solving optimization functions using their position and speed uncertainties with different measurement models. The limitation is that underwater environments remain unpredictable, and localization error comparisons are not very accurate due to uncertainties in underwater sound propagation profiles. In [22], a TDoA-based underwater positioning system (UPS) is proposed for determining the localization of sensor nodes. This method collects positioning information, and the aggregated data will be transferred to the respective anchor positions after calculating the range differences. In some papers [23], [24] genetic-based localization algorithms are proposed using surface beaconing signals. Similarly, in [25] modified genetic algorithms are investigated using effective time-of-arrival techniques. In these technique [26], the Doppler effect is also considered to determine localization. This method is based entirely on the communication or received signal strength values between the sensors and anchor nodes [27]. There are some range-free localization algorithms [28], [29] that rely on beacon signals, topology, and position information based on their area of coverage. These algorithms consume more energy than range-based algorithms.

In this article, we focus on proposing an effective solution to the localization problem by jointly estimating Gaussian Mixture Model (GMM) with a Bayesian statistical approach to solve the optimization function. This proposed localization algorithm will be an effective solution for all types of realistic, harsh underwater environments. In this proposed work, we assume that the unknown sound propagation speed and its parameters are estimated using RSS measurements. This joint localization estimation algorithm, using GMM and a Bayesian approach, will yield good performance in overcoming localization errors. We have also derived and studied the equations of the localization algorithms corresponding to the Cramér-Rao Lower Bound (CRLB) to analyse their performance in terms of PDF and localization errors.

B. *Organization and Notations*

The rest of this article is organized as follows: Section II discusses the underwater system model and its preliminaries. Section III provides a detailed explanation of the proposed joint GMM-Bayesian localization algorithm. The numerical simulation results are discussed in the subsequent section. Finally, the article is concluded in the last section.

The notations used in this article are given as follows: the term R^n represents the set of n vectors, and the term S^n represents the $n \times n$ symmetric matrix. If the matrix $A > 0$, it means that the matrix A is said to be positive semidefinite. The terms l_1, l_2, and l_∞ norms of the vectors are represented as $||.||_1$, $||.||_2$, and $||.||_\infty$ respectively.

II. System Model and Preliminaries

In the underwater acoustic communication channel model, the term $\varphi_j = \begin{bmatrix}\varphi_{j1} & \varphi_{j2}\end{bmatrix}^T$ represents the unknown coordinates where j varies from 1 to M, denoting the j^{th} targeted node. Similarly, $\alpha_j = \begin{bmatrix}\alpha_{i1} & \alpha_{i2}\end{bmatrix}^T$ represents the known coordinates of the i-th anchor node, where i varies from 1 to N. Here, M and N represent the total number of target nodes and anchor nodes deployed in the acoustic channel, respectively. From [30] and [31], the total received power $P_{\text{uw}}(d)$ at the j-th target node from the i-th anchor node is given as,

$$P_{\text{uw}}(d) = P_{0\_\text{uw}} - 10 \cdot \alpha_{\text{uw}} \cdot \log_{10}\left(\frac{d_{\text{uw}}}{d_0}\right) \tag{1}$$

Here, the term $P_{\text{uw}}(d)$ represents the initial received power at the reference distance (d_0), and the term α represents the path loss exponent. Similarly, d_{uw} represents the distance at which the path loss value is calculated, and d_0 represents the reference distance.

In this underwater acoustic channel model, two related issues arise. They are:

1) The Received Signal Strength (RSS) level behaviors are depicted in the figure 1. We have tried to adopt a suitable logarithmic curve for RSS level fitting and its measurements to achieve less localization error.
2) The noise factor in this underwater acoustic channel model does not always follow a Gaussian distribution, especially in real-time application scenarios.

For RSS measurements, the Gaussian mixture model is generally used with a noise factor when modeling underwater environments [32], [33]. The comparison of empirical path loss model of underwater acoustic channel without noise factors is depicted in the Figure 2. This joint modeling will result in a conditional probability distribution function (PDF) based on the observed power factors $P_j = [P_{1,j}, P_{2,j} \cdots P_{N,j}]$ inferred at the j^{th} target is given as,

$$P[P_j|Q_j] = \prod_{i=1}^{N}\sum_{s=1}^{S}\tau_{i,s}\left[\mathcal{N}(\mu_s, \sigma_s^2)\right] \tag{2}$$

Here, $\tau_{i,s}$ represents the cluster weights with the average mean (μ) and noise variance (σ_s^2), and S represents the total number of GMM mixture components.

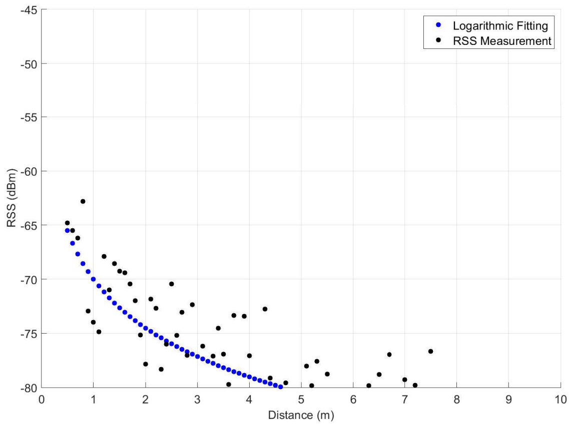

Fig. 1. Comparison of real RSS measurements with Empirical path loss model of underwater acoustic channel without noise

The path loss exponent α_{uw} value is known and fixed for the entire simulation experiments. Based on prior measurements, the values are fixed using logarithmic fitting for the underwater channels. In the simulation, $P = -70$ dB, the α_{uw} value is 1.5, and the reference distance d_0 is fixed for multiple iterations of the algorithms.

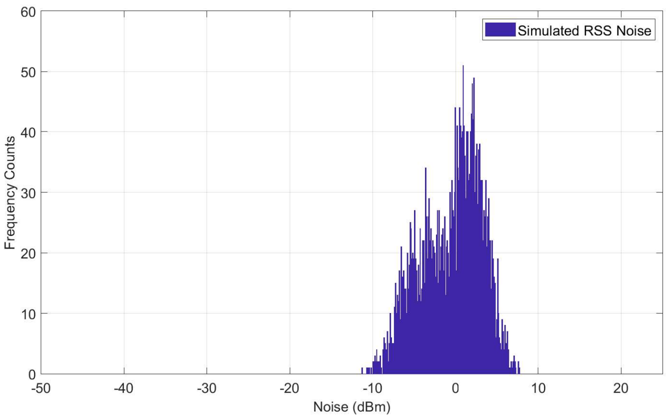

Fig. 2. Histogram of RSS measurement noise versus frequency

III. Proposed localization method

This section give the detailed explanation for optimization problem formulation and the proposed localization algorithm.

A. Problem formulation

In real-time scenarios, the values of μ_s, σ_s^2, and $\tau_{i,s}$ variables need to be estimated to calculate the localization of sensor nodes [34]. As referred to in [35], according to the Expected Conditional Maximization (ECM) criterion, the position ϕ is updated and estimated using the μ_s^η, σ_s^η, and η be the iteration values. The $\tau_{i,s}^\eta$ values are generally not updated. To overcome this joint optimization problem, the proposed Gaussian mixture model, along with appropriate relaxation techniques, is employed. This method is widely used to obtain the Maximum Likelihood (ML) estimate ϕ for finding the $\tau_{i,s}$. After estimating the ML values, the equation can be formulated as follows,

$$\max_{\phi_j,\tau} \Phi(\phi_j,\tau) \cdot s.t \cdots C1, C2, C3 \tag{3}$$

Where the term Φ represents the log-likelihood function, which exhibits the conditional probability density function (pdf) as given in equation 2. Here, the value of the Gaussian mixture weight is constrained by the sum of the values being equal to 1. The equation for $C1, C2 \& C3$ is given as,

$$C1 : \sum_{s=1}^{S} \tau_{i,s} = 1 \tag{4}$$

$$C2 : 0 \leq \tau_{i,s} \leq 1 \tag{5}$$

$$C3 : d(\phi_j, \alpha_i) \neq 0 \tag{6}$$

Due to these inherent conditions and the combinatorial behavior, this will lead to NP-hard problems. To reduce computational complexity, the joint optimization problem is reformulated and approximated to find a lower bound of the non-convex objective function. Such an objective function is solved by using suitable relaxation techniques to achieve an effective optimal solution. The joint optimization problem is formulated as follows,

$$\max_{\phi_j} \sum_{i=1}^{N} \sum_{s=1}^{S} \tau_{i,s} ln\big[\mathcal{N}(\mu_s, \sigma_s^2)\big] \tag{7}$$

Here, the term (ϕ_j,τ) represents the sub-optimal pairs for this primary subsumed problem.

B. Proposed GMM with Bayesian Approach

In order to reduce the computation complexity of the exisiting GMM-SDP model, the new objective function is reformulated by the suboptimal function. The optimal value of ϕ_j is expressed as,

$$\Phi^* = sub\big[\Phi(\phi_j,\tau)\big] | C1, C2, C3 \tag{8}$$

The lower bound for the given primary non-convex weighted objective function which will give the effective and feasible solution for this optimization problems. The ojective function is formulated using Jensen's inequality [36] and is expressed as follows,

$$\Phi(\phi_j,\tau) \geq \sum_{i=1}^{N} \sum_{s=1}^{S} \tau_{i,s} ln\big[\mathcal{N}(\mu_s, \sigma_s^2)\big] = \Phi_1(\phi_j,\tau) \tag{9}$$

The term $\Phi(\phi_j,\tau)$ will be sereves as a lower bounds for the maximum likelihood function exist based on parameters such as ϕ_j and τ. Due to the existence of £, the log-likelihood function will always be sub-optimal. The objective function is rewritten as follows,

$$\min_{\phi_j,\tau} \Phi_2(\phi_j,\tau) | C1, C2, C3 \tag{10}$$

where,

$$\Phi_2(\phi_j,\tau) = -\Phi_1(\phi_j,\tau) = \sum_{i=1}^{N}\sum_{s=1}^{S}\tau_{i,s}\left[ln\sqrt{2\pi}\sigma_s + \frac{(n_{i,j}-\mu_s)^2}{2\sigma_s^2}\right] \tag{11}$$

The above equation clearly shows that the objective exhibits non-convexity, even though the ϕ_j values are discontinuous. Hence, it is always difficult to calculate the global optimal solutions for this problem. In this article, we propose using a Bayesian approach combined with a Gaussian mixture model to achieve this convex estimator [37]. In this work, the localization of target nodes in underwater wireless sensor networks is determined using a Gaussian mixture model combined with a Bayesian approach. Such Bayesian inference is widely used to estimate the posterior distribution of the target nodes' positions based on received signal strength measurements. This proposed joint Gaussian Mixture Model and Bayesian approach enables updating the target positions based on the observed data. Such Bayesian inference is based on Bayes' theorem, which is given by,

$$p(\phi|P) = \frac{(p(P|\phi).p(\phi)}{p(P)} \tag{12}$$

Here, ϕ represents the position of the target node, P denotes the RSS measurement vector, and $p(\phi|P)$ is the posterior distribution of the target node's position. $p(P|\phi)$ is the likelihood function, and $p(\phi)$ is the prior distribution of the target node's position. Similarly, $p(P)$ is the marginal distribution of the RSS measurements, and it represents the normalization constant. By using this RSS measurements-based localization model, the measurements will follow the modified Gaussian Mixture Model (GMM) which is given as,

$$P_m = P_0 - 10\beta log_{10}\left(\frac{d}{d_0}\right) + \eta \tag{13}$$

Here, P_m represents the total power received by the target node, d is the distance between the target and anchor nodes, $d-0$ is the reference distance, P_0 is the reference power level, β is the path loss exponent, and η represents the Gaussian noise in the RSS localization model. The maximum likelihood estimation for each Gaussian component is given by,

$$p(P \mid \phi) = \prod_{i=1}^{N}\sum_{s=1}^{S}\tau_s\cdot\mathcal{N}\left((P_i \mid P_0) - 10\cdot\beta\cdot\log_{10}\left(\frac{d_i}{d_0}\right), \sigma_s^2\right) \tag{14}$$

Similarly, τ_i represents the weights of the Gaussian components, and $\mathcal{N}$ denotes the normal distribution with specified mean and variance.

The pseudocode for the GMM-Bayesian approach-based localization algorithm is explained in 1. This algorithm helps to estimate the positions of the targeted nodes by using a Gaussian mixture model with a Bayesian approach. At first, the inputs collected are the power levels of RSS, path loss exponents, reference distance, GMM mean vectors, weights of GMM components, targeted nodes, anchor nodes, and positioning information. In the initial step of this algorithm, the RMSE array is created for the given number of anchor nodes. The estimated positions and true position values are

Algorithm 1 Bayesian-GMM Localization

Input: P_0: Reference power level, β: Path loss exponent, d_0: Reference distance, μ: Mean vectors of GMM components, sigmasq: Covariance matrices of GMM components, τ: Weights of GMM components, side: Length of the area side, n_{Nodes}: Number of target nodes, $n_{\text{AnchorsList}}$: List of anchor counts, ϕ_{Index}: Index of the target node to estimate.
Output: RMSE values for different anchors
for each $n_{\text{AnchorsList}}$ **do**
  Display ϕ_{Hat} and ϕ_{True}
  for i=1: 100 **do**
    Generate ϕ and α
    $\phi \sim \text{Uniform}([0, \text{side}], [0, \text{side}])$
    α = predefined positions based on n_{Anchors}
    Compute RSS measurements P based on ϕ and α:
    Compute distances between i and j:
    $d(i,j) = \sqrt{(\phi_{0,i} - \alpha_{0,j})^2 + (\phi_{1,i} - \alpha_{1,j})^2}$
    $P(i,j) = P_0 - 10 \cdot \beta \cdot \log_{10}(d(i,j)/d_0) + n(i,j)$
    where $n(i,j) \sim \text{Gaussian}(\mu, \text{sigmasq})$
    Target position estiamation by Bayesian inference
    for i=n+1 **do**
      for each $i_m = 1 : m$ **do**
        Propose a new target position:
        $\phi_{\text{prop}} = \phi_{\text{curr}} + \text{random\_perturbation}$
        Compute the equation 14
        Compute acceptance probability:
        $\alpha = \min\left(1, \frac{p(P|\phi_{\text{prop}})}{p(P|\phi_{\text{curr}})}\right)$
        Accept or reject of position based on α:
        if random(i) $\leq \alpha$ **then**
          $\phi_{\text{curr}} = \phi_{\text{prop}}$
        end if
      end for
      Compute the estimated position:
      $\phi_{\text{hat}} = \text{mean}(\phi_{\text{samples}})$
    end for
    Store in ϕ_{Hat}
    Store in ϕ_{True}
  end for
  $dsq = \sum(\phi_{\text{True}} - \phi_{\text{Hat}})^2$
  RMSE $= \sqrt{\text{mean}(dsq)}$
end for
Save RMSE results

Algorithm 2 findRSS

Compute distances between each target and each anchor:
$d(i,j) = \sqrt{(\phi_{0,i} - \alpha_{0,j})^2 + (\phi_{1,i} - \alpha_{1,j})^2}$
Generate noise samples from the GMM:
$n(i,j) \sim \text{Gaussian}(\mu, \text{sigmasq})$
Compute RSS measurements based on the path loss model and add noise:
$P(i,j) = P_0 - 10 \cdot \beta \cdot \log_{10}(d(i,j)/d_0) + n(i,j)$

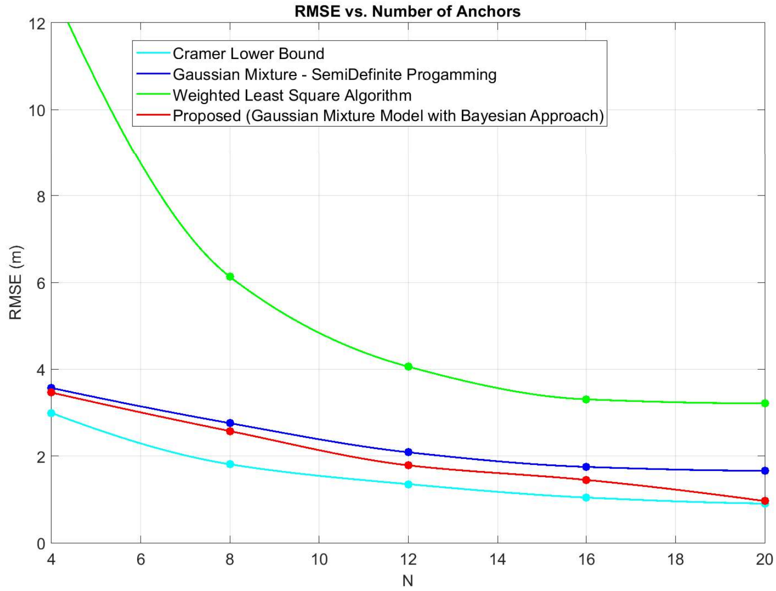

Fig. 3. RMSE versus the number of anchor nodes

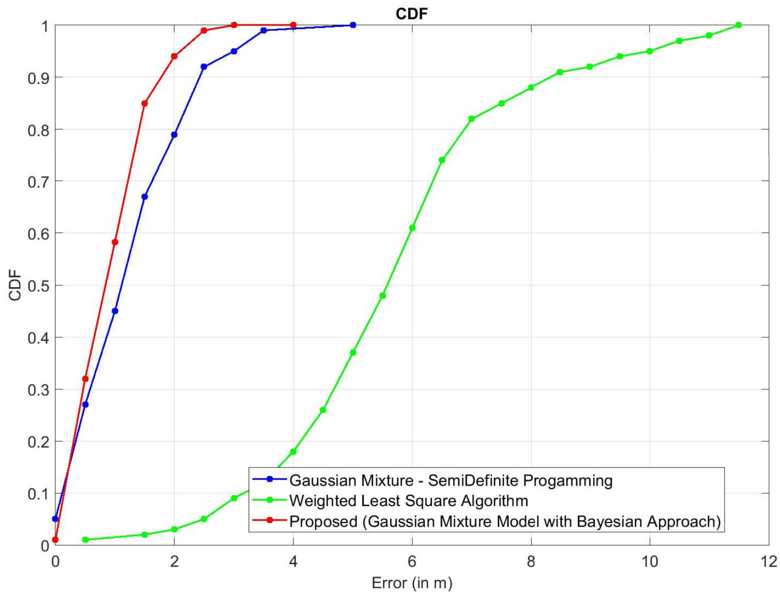

Fig. 4. CDF of localization errors for the different algorithms

initialized to determine the positions of the anchor nodes. We increased the value of n varies from 1 to 100 to run Monte Carlo simulations and compute the RSS measurements. These RSS measurements are calculated based on the positions using the path loss model and added Gaussian noise. The target positions are estimated using Bayesian inference and the Markov Chain Monte Carlo (MCMC) sampling process. The MCMC sampling process provides the positions and helps compute the likelihood of these positions using the RSS values and GMM components.

The algorithm 2 provides a detailed explanation of how to find the RSS values. First, the Euclidean distance is calculated using the squared distance between the targeted and anchor nodes. The noise samples are generated and added to the GMM parameters based on the randomly deployed target nodes within the predefined area. Bayesian inference is effectively utilized to find the unknown target positions based on the observed RSS measurement data.

IV. Simulated Results and Discussions

This section provides a detailed explanation of the proposed Gaussian Mixture Model with a Bayesian approach (GMM-Bayesian) algorithm and compares it with other existing localization algorithms. The performance metrics are evaluated through Monte Carlo simulations, and the simulated results are compared with the Cramer-Rao Lower Bound (CRLB), the Weighted Least Squares (LS) algorithm, and the Gaussian Mixture Model with Semidefinite Programming algorithms. Recently developed algorithms are taken into account for comparison of RMSE and localization error.

A. System Model

In this simulation trial, the sound propagation speed in underwater environments is varied within the range of 1400 to 1600 m/s. A numerical simulation is conducted to evaluate and analyse the proposed GMM-Bayesian algorithm. For the simulation, a square region with dimensions of 15 x 15 square meters is considered. For the UWSN, a total of 120 sensor nodes are considered, and 20 anchor nodes are deployed throughout the entire communication region. For an effective evaluation, we assume two modes of the Gaussian Mixture Distribution Models with values. Using this Gaussian Mixture Model, the mean and variance values of each mixture component are replicated, which helps us to better understand the actual indoor noise measurements of UWSN channels. The performance of the proposed GMM-Bayesian algorithm will be evaluated by computing the Root Mean Squared Error (RMSE) and Cumulative Distributuion Function (CDF) values using 100 Monte Carlo simulations, with the number of anchor nodes varied from 4 to 20.

B. RMSE versus Number of anchor nodes

The RMSE is the most important performance metric for evaluating the accuracy of the system's ability to estimate the positions of sensor nodes in localization models. The RMSE is the square root of the average squared differences between the actual and predicted positions of the nodes. It is expressed as,

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2} \tag{15}$$

Where, the term y_i is the actual position of the sensors, $\hat{y}_i$ is the predicted position, and the term n is the total number of sensor nodes in the network.

The figure 3 illustrates the RMSE performance of the proposed GMM-Bayesian algorithm as the number of anchor nodes increases, compared to other existing methods. These results are achieved because the GMM-Bayesian approach provides more positioning information from the anchor nodes based on the estimation of RSS measured values. This method provides an RMSE value nearly equivalent to the CRLB because the proposed model requires fewer mixture components to predict the localization of all sensor and anchor nodes with improved accuracy. The results show that the proposed GMM-Bayesian approach outperforms all other existing localization algorithms, such as GMM-SDP and WLS algorithm, and is nearly as close to the ideal CRLB algorithm in underwater environments.

C. CDF of localization errors

The term Cumulative Distribution Function (CDF) is defined as a statistical tool used to analyze the distribution of errors in localization systems. The CDF is a function that shows the probability that localization errors will be less than or equal to a defined threshold value, illustrating how the errors are distributed across different magnitudes. The CDF helps us understand the likelihood of errors falling within a specific range. For any error value x, the CDF is expressed as

$$F(x) = P(E \leq x) \tag{16}$$

where, the terms E and P is the error and the probalbility of the localization function respectively.

The figure 4 illustrates a comparison of the impacts on the CDF with respect to localization errors. It clearly shows that the proposed GMM-Bayesian algorithm provides a better CDF for localization errors compared to the GMM-SDP and WLS methods. An increase in the CDF curve indicates that the localization model has fewer errors and that the errors are more evenly distributed. This is achieved because this model handles more complex error distributions compared to other models. By incorporating prior information about the underwater sensors and anchor nodes, the efficiency of the algorithm is improved. This joint estimation using the GMM-Bayesian approach will be computationally effective for various types of high-dimensional data with mixture components, provided that Gaussian components and prior information parameters are carefully tuned to achieve better localization.

V. Conclusion

In this article, we propose a joint Gaussian Mixture Model with a Bayesian approach-based localization algorithm to improve RSS estimation accuracy in underwater environments. Due to the high Doppler spread and unpredictable characteristics of the underwater environment, the measured RSS values of sensor node parameters are not accurate and are also affected by Gaussian-distributed errors. The proposed joint approach-based localization algorithm effectively estimates the maximum likelihood of node locations, solving optimization problems with globally optimal solutions based on the RSS measurements. The simulated results clearly show that the proposed GMM-Bayesian approach-based localization algorithm achieves CRLB accuracy and provides better performance compared to other existing localization methods. In the future, we plan to implement it in highly sparse and Time Difference of Arrival (TDoA) based schemes in real underwater acoustic environments.

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