

Optimal thermal placement of electronic functional blocks using symbolic algebra. A new application of Groebner Analysis

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Abstract—The paper presents a new idea and algorithm for arranging topography of functional blocks (hereinafter referred to as components) on an integrated circuit substrate, e.g. a digital processor, power circuit etc., in such a way as to minimize the mutual thermal interactions of individual components. Our analytical method finds the global optimum, which distinguishes it from numerical methods that can only find local minima. This leads to uniform temperature distribution, and therefore full use of the thermal properties of the electronic system, minimizing the maximum temperature on the substrate, and consequently allows for increasing the throughput. Presented approach is universal and allows for solving many similar global minimum search problems.

Keywords—analytical optimization; arrangement of functional blocks on integrated circuit substrate; reduction of maximum temperature of integrated circuit

I. INTRODUCTION

OUR analytical method finds the global optimum, which distinguishes it from numerical methods that can only find local minima. It is universal and allows for solving many similar global minimum search problems. Functional blocks of integrated circuits that dissipate a significant amount of heat, hereinafter referred to as components, must be placed on a substrate with good thermal conductivity, e.g., silicon, ceramic, sapphire or other, in such a way that the electrical connections are as short as possible, which ensures the appropriate speed of data processing. However, reducing the distance between power components leads to parasitic thermal coupling. Therefore, the trade-off must be considered when designing the IC layout. The optimal arrangement of power components leads to the minimization of the temperature on the substrate. It ensures that the maximum temperature on the substrate is minimized as much as possible under the given conditions [1,2] and at the same time the shortest possible delay times between the different components are achieved. In the case of a component with a certain amount of power dissipation, other criteria, such as heat transfer through the substrate, also play a role. In this paper, we focus on the optimal layout of components from the thermal perspective i.e., minimizing the temperature of each component while taking into account the geometric conditions of the substrate and the power dissipated in the individual components. Fig. 1 shows an example arrangement of the components dissipating heat on the

substrate. One observes that placement has a major influence on the temperature distribution.

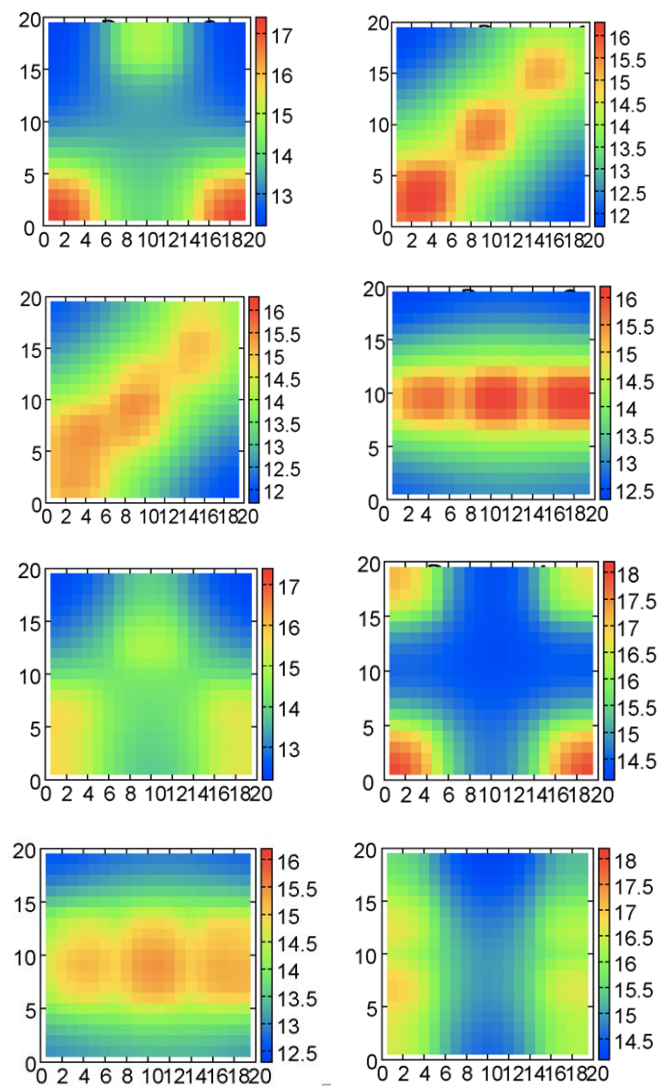


Fig. 1. Exemplary placement of components dissipating different power [2]

We consider components with varying power dissipation that cannot be cooled by powerful cooling systems (e.g., thermal Peltier pumps, forced convection by a fan, etc.). Most often, the

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cooling path is the substrate itself. However, additional cooling layers such as aluminum, copper, or ceramics that have good thermal conductivity can be very helpful from this point of view.

It has been pointed out that the substrate is not only used to guarantee the electric interconnections between the components but also serves as a cooling fin. If two or more power components are located on one substrate they should be put as far away from each other. Neither they should be placed close to the boundary. This guarantees that each power component has a certain part of the substrate available for its heat removal.

This problem has been attacked by several authors [1-10]. Most techniques used the numerical extremisation of a cost function in order to find the optimal placement of the components.

One disadvantage of a pure numerical method is that in most cases only one solution is obtained. Although these problems are nonlinear and should provide several solutions. If a solution has been obtained numerically, one is never sure that other possible solutions might be better ones. In this paper the minimization of the cost function is done using symbolic algebra techniques. One has to limit oneself to cost functions which be written as simple functions involving polynomials in x_i and y_i , where x_i and y_i are the coordinates of the middle of i -th component on the substrate. This allows us to use an elimination technique called Groebner algebra [11-17]. One obtains at the end a set of algebraic equations arranged in such a way all mathematically possible solutions are easily obtained. For every x_i or y_i an n -th degree algebraic equation is obtained. Nowadays simple numerical methods are available to find all n roots of an algebraic equation.

By combining all possible solutions, one obtains a spectrum of all the possible placements of power components on a substrate, each of them corresponding to a local minimum of the cost function. It is then quite straightforward to find among them the solution with the most extreme value of the cost function.

II. BASIC ANALYSIS

The method will be explained with the help of the particular example shown in Fig.2.

Two components have to be placed on a substrate with dimensions a and b . If (x_1, y_1) and (x_2, y_2) are the coordinates of the two components, the following cost function C is used:

$$C = \ln[x_1^2(a - x_1)^2 y_1^2(b - y_1)^2] + \ln[x_2^2(a - x_2)^2 y_2^2(b - y_2)^2] + \ln[(x_1 - x_2)^2 (y_1 - y_2)^2] \quad (1)$$

The terms x_i^2 and $(a-x_i)^2$ express the distance of the first component to the boundaries at $x=0$ and $x=a$. One can easily verify that the cost function C increases with x_i due to the term $\ln x_i^2$. Hence, we are looking for a maximum value of the cost function in this paper. Strictly speaking the logarithmic function is not necessary to construct the cost function.

The only reason it was introduced was to reduce the overall computation time for the algebraic manipulations. The term $(x_1 - x_2)^2 (y_1 - y_2)^2$ expresses the "distance" between the two components. The expression $(x_1 - x_2)^2 + (y_1 - y_2)^2$ would be more justified but gives rise to more complicated algebraic manipulations afterwards. Again $\ln[(x_1 - x_2)^2 (y_1 - y_2)^2]$ was used to reduce the CPU time. The other terms in (1) can be easily interpreted in a similar way.

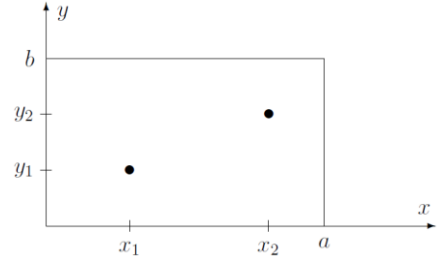


Fig. 2. Two components to be placed on a rectangular substrate

If the second component dissipates more power than the first one, coefficients can be inserted into (1) in order to stress the fact that the second component requires more substrate area for its cooling:

$$C = \ln[x_1^2(a - x_1)^2 y_1^2(b - y_1)^2] + \alpha \ln[x_2^2(a - x_2)^2 y_2^2(b - y_2)^2] + \ln[(x_1 - x_2)^2 (y_1 - y_2)^2] \quad (2)$$

where $\alpha > 1$.

In order to find the optimum value of C , one has to solve the following set of equations:

$$\frac{\partial C}{\partial x_1} = \frac{\partial C}{\partial x_2} = \frac{\partial C}{\partial y_1} = \frac{\partial C}{\partial y_2} = 0 \quad (3)$$

In the next section, it will be shown how all the possible solutions of a set like (3) can be obtained using the so called Groebner analysis, developed by Buchberger [14-17]. The computer algebra software package REDUCE has been used to which a Groebner analysis package has been added.

Now, most computer algebra software packages offer a Groebner analysis package.

III. GROEBNER ANALYSIS

The Groebner analysis will be outlined in detail for a simple situation: two components have to be placed on one axis in the interval $[0,1]$ i.e. a one-dimensional substrate. The cost function reads:

$$C = \ln(x_1 - x_2)^2 + \ln[x_1^2(1 - x_1)^2] + \ln[x_2^2(1 - x_2)^2] \quad (4)$$

The derivative of C with respect to x_i turns out to be:

$$\frac{\partial C}{\partial x_1} = \frac{2}{x_1 - x_2} + \frac{2}{x_1} - \frac{2}{1 - x_1} = \frac{2x_1 - x_2 + 2x_1x_2 - 3x_1^2}{(x_1 - x_2)x_1(1 - x_1)} = 0 \quad (5)$$

A similar relation is found for the derivative with respect to x_2 . Hence, one has to solve following nonlinear algebraic set:

$$3x_1^2 - 2x_1x_2 - 2x_1 + x_2 = 0 \quad (6)$$

$$3x_2^2 - 2x_1x_2 - 2x_2 + x_1 = 0 \quad (7)$$

This is a 4-th order algebraic set so that 4 solutions are expected. The principle of the Groebner algebra is quite complicated, but roughly speaking this analysis gives always an equivalent algebraic set which is much easier to handle. One could say that it is some kind of generalised elimination method. The results are:

$$x_1 - 10x_2^3 + 15x_2^2 - 6x_2 = 0 \quad (8)$$

$$(5x_2^2 - 5x_2 + 1)(x_2 - 1)x_2 = 0 \quad (9)$$

The second equation (9) is a 4th order algebraic equation in x_2 . For this particular case the REDUCE software performed already a factorial decomposition, so that two roots are found by inspection. But in the most general case, one has to determine all roots numerically, which is no longer a problem from a numerical point of view. For each solution x_2 , this value can be inserted into equation (8) to find the corresponding value x_1 . Therefore, it was shown that using Groebner analysis it is possible to find all possible solutions. In this simple example, the calculation can also be performed by hand by simply subtracting (6) and (7).

It is quite easy to obtain all the solutions of (9):

$$x_2 = 0 ; x_2 = 1 ; x_2 = 0.7236 ; x_2 = 0.2763 \quad (10)$$

It is clear that the first two solutions $x_2 = 0$ and $x_2 = 1$ provide an extremum value for C but not a maximum. Hence, only the last two solutions of (10) are acceptable from physical point of view. Inserting the solutions (10) in (8) yield the corresponding x_1 values. An overview is shown in table I.

TABLE I
TWO COMPONENTS ON AN INTERVAL [0,1]

Solution	x_1	x_2	C
(1)	0	0	$-\infty$
(2)	1	1	$-\infty$
(3)	0.2763	0.7236	-3.4938
(4)	0.7236	0.2763	-3.4938

The corresponding values of the cost function C has been displayed as well. Strictly speaking only one acceptable solution is found, the second one being found by interchanging x_1 and x_2 , i.e. the symmetrical solution.

IV. EXAMPLES

A. Two Components on a Line with Different Weight Factors

We consider now the same case as the previous one, but the first component is dissipating more heat than the other one. Hence a coefficient $\alpha=2$ is inserted into the cost function:

$$C = \ln(x_1 - x_2)^2 + 2 \ln[x_1^2(1 - x_1)^2] + \ln[x_2^2(1 - x_2)^2] \quad (11)$$

The Groebner analysis gives rise to the following polynomial equations:

$$5x_1 - 42x_1^3 + 63x_1^2 - 26x_2 = 0 \quad (12)$$

and

$$(21x_2^2 - 26x_2 + 4)(x_2 - 1)x_2 = 0 \quad (13)$$

The solutions are listed in table II.

TABLE II
TWO COMPONENTS ON AN INTERVAL [0,1], DIFFERENT HEAT DISSIPATION

Solution	x_1	x_2	C
(1)	0	0	$-\infty$
(2)	1	1	$-\infty$
(3)	0.6463	0.2560	-4.8213
(4)	0.3536	0.7439	-4.8213

Again two unacceptable solutions are corresponding to $C = -\infty$. It is clearly seen that more space is now provided for the first element. The solution 4 cannot be found from solution 3 by interchanging the coordinates x_1 and x_2 . However both solutions are equivalent from thermal point of view because they give rise to the same value of the cost function and symmetry, i.e. $x_i(3) = (1-x_i)(4)$.

B. Three Elements on a Line

If three elements with equal power dissipations are put one on a single line in the interval [0,1], the following cost function is used:

$$C = \ln[(x_1 - x_2)^2(x_1 - x_3)^2(x_2 - x_3)^2] + \ln[x_1^2(1 - x_1)^2] + \ln[x_2^2(1 - x_2)^2] + \ln[x_3^2(1 - x_3)^2] \quad (14)$$

Extremisation of C gives rise to a 12th order algebraic system. The Groebner analysis provides the following polynomial equations:

$$(x_1 - x_2 - x_3)(x_1 - 1)x_1 + (2x_1 - 1)(x_1 - x_2)(x_1 - x_3) = 0 \quad (15)$$

$$(x_1 + x_2 - 2x_3)(x_3 - 1)x_3 + (x_1 - x_3)(x_2 - x_3)(2x_3 - 1) = 0 \quad (16)$$

$$(x_2 - x_3)(7x_3^2 - 7x_3 + 1)(4x_3 - 1)(4x_3 - 3)(2x_3 - 1)(x_3 - 1)^2 x_3^2 = 0 \quad (17)$$

Remark that the last equation (17) gives all the possible solutions for the unknown x_3 .

This time 6 unacceptable solutions are observed. The remaining ones are permutations so that only one single solution is found here as shown in table III.

TABLE III
THREE COMPONENTS ON AN INTERVAL [0,1]

Solution	x_1	x_2	x_3	C
(1)	0.75	0	0	$-\infty$
(2)	0	0.75	0	$-\infty$
(3)	0	0	0.75	$-\infty$
(4)	0.25	1	1	$-\infty$
(5)	1	0.25	1	$-\infty$
(6)	1	1	0.25	$-\infty$
(7)	0.8274	0.1726	0.5	-6.8925
(8)	0.1726	0.8273	0.5	-6.8925
(9)	0.8273	0.5	0.1726	-6.8925
(10)	0.1726	0.5	0.8273	-6.8925
(11)	0.5	0.8273	0.1726	-6.8925
(12)	0.5	0.1726	0.8273	-6.8925

C. Two components on a square substrate

If two components with coordinates (x_1, y_1) and (x_2, y_2) have to be positioned on a square substrate with unit side length, the following cost function has been presented:

$$C = \ln[(x_1 - x_2)^2(y_1 - y_2)^2] + \ln[x_1^2(1 - x_1)^2 y_1^2(1 - y_1)^2] + \ln[x_2^2(1 - x_2)^2 y_2^2(1 - y_2)^2] \quad (18)$$

By minimising the cost functions, one obtains the following algebraic equations after Groebner analysis:

$$\begin{aligned}
x_1 - 10x_1^2 + 15x_2^2 - 6x_2 &= 0 \\
(5x_2^2 - 5x_2 + 1)(x_2 - 1)x_2 &= 0 \\
y_1 - 10y_1^2 + 15y_2^2 - 6y_2 &= 0 \\
(5y_2^2 - 5y_2 + 1)(y_2 - 1)y_2 &= 0
\end{aligned} \quad (19)$$

The acceptable solutions are shown in table IV.

TABLE IV
TWO COMPONENTS ON AN A SQUARE SUBSTRATE

Solution	x_1	y_1	x_2	y_2	C
(1)	0.7636	0.7236	0.2763	0.2763	-6.9897
(2)	0.2763	0.2763	0.7636	0.7636	-6.9897
(3)	0.2763	0.7636	0.7636	0.2763	-6.9897
(4)	0.7636	0.2763	0.7636	0.2763	-6.9897

The 12 unacceptable solutions have not been included in table 4. Again the 4 solutions are all equivalent, they can be obtained from each other by simple permutations

D. Two components with different weight factors on a square substrate

Two different cases will be considered here. In the first one a coefficient $\alpha=2$ is inserted in the cost function:

$$C = \ln[(x_1 - x_2)^2(y_1 - y_2)^2] + 2 \ln[x_1^2(1 - x_1)^2y_1^2(1 - y_1)^2] + \ln[x_2^2(1 - x_2)^2y_2^2(1 - y_2)^2] \quad (20)$$

It is clear from (20) that the first component (x_1, y_1) dissipates more heat. The acceptable solutions are all listed in table V.

TABLE V
TWO COMPONENTS ON AN A SQUARE SUBSTRATE.
DIFFERENT HEAT DISSIPATION ($\alpha=2$)

Solution	x_1	y_1	x_2	y_2	C
(1)	0.6463	0.6463	0.2560	0.2560	-9.6426
(2)	0.3536	0.3536	0.7439	0.7439	-9.6426
(3)	0.3536	0.6463	0.7439	0.2560	-9.6426
(4)	0.6463	0.3536	0.2560	0.7439	-9.6426

Again, the 4 solutions can be derived from each other by a simple permutation of the coordinates. Thermally speaking, these 4 solution are equivalent, they all correspond to the same value of the cost function (-0.964).

By comparing these results with those of example 3, one observes that the first component (x_1, y_1) is positioned closer to the centre.

In other words, this component has more space available to release its heat by thermal conduction through the substrate.

In order to see the influence of the power dissipation, a high value has been used in the second case for the parameter $\alpha=10$:

$$C = \ln[(x_1 - x_2)^2(y_1 - y_2)^2] + 10 \ln[x_1^2(1 - x_1)^2y_1^2(1 - y_1)^2] + \ln[x_2^2(1 - x_2)^2y_2^2(1 - y_2)^2] \quad (21)$$

The results are listed in table VI.

TABLE VI
TWO COMPONENTS ON AN A SQUARE SUBSTRATE.
DIFFERENT HEAT DISSIPATION ($\alpha=10$)

Solution	x_1	y_1	x_2	y_2	C
(1)	0.5394	0.5394	0.2242	0.2242	-29.2347
(2)	0.4606	0.4606	0.7758	0.7758	-29.2347
(3)	0.4606	0.5394	0.7758	0.2242	-29.2347
(4)	0.5394	0.4606	0.2242	0.7758	-29.2347

It is observed that the hottest component (x_1, y_1) is now almost positioned in the centre of the substrate. In fig. 3 the first results of tables IV, V and VI are presented graphically.

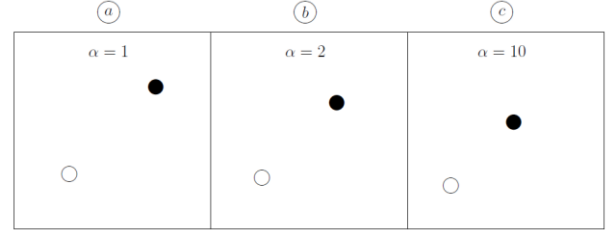


Fig. 3. Two components on a square substrate (1x1). • = (x_1, y_1) , ◦ = (x_2, y_2) with (a) $\alpha=1$, (b) $\alpha=2$ and (c) $\alpha=10$.

One clearly observes that in fig.3a both components are placed symmetrically whereas in fig.3b ($\alpha=2$) and fig.3c ($\alpha=10$) the first component moves towards the centre of the substrate.

It should be noted here that for both cases shown in table V and table VI again 12 unacceptable solutions corresponding to $C = -\infty$ have been obtained too.

E. Three components on a square substrate

With 3 components on a square substrate, the following cost function is used:

$$C = \ln[(x_1 - x_2)^2(x_1 - x_3)^2(x_2 - x_3)^2(y_1 - y_2)^2(y_1 - y_3)^2(y_2 - y_3)^2] + \ln[(x_1 - 1)^2(y_1 - 1)^2x_1^2y_1^2] + \ln[(x_2 - 1)^2(y_2 - 1)^2x_2^2y_2^2] + \ln[(x_3 - 1)^2(y_3 - 1)^2x_3^2y_3^2] \quad (22)$$

36 acceptable solutions and 8 unacceptable solutions are obtained this time. All acceptable solutions give rise to the same value of the cost function ($C = -13.7851$). Hence, from thermal point of view all acceptable solutions can be considered as equivalent.

In Fig. 4 two typical solutions are presented. Remark again that all 36 solutions can be derived from each other by permutations of the coordinates. They all give rise to the same value of the cost function $C = -13.7851$.

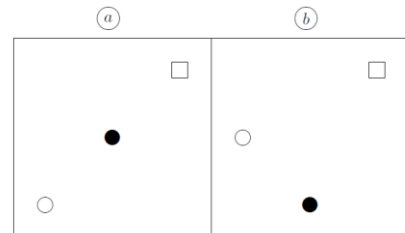


Fig. 4. Two possible solutions for 3 components placed on a square substrate. (a) and (b) correspond to the same value of the cost function $C = -13.7851$

F. Three components on a rectangular substrate with different heat dissipation

As a final example, 3 components have to be placed on a rectangular substrate ($a=2$, $b=1$). The first component (x_1, y_1) dissipates the highest amount of power. The other two have the same power dissipation. The cost function reads:

$$C = \ln[(x_1 - x_2)^2(x_1 - x_3)^2(x_2 - x_3)^2(y_1 - y_2)^2(y_1 - y_3)^2(y_2 - y_3)^2] + 10 \ln[(x_1 - 2)^2(y_1 - 1)^2x_1^2y_1^2] + \ln[(x_2 - 2)^2(y_2 - 1)^2x_2^2y_2^2] + \ln[(x_3 - 2)^2(y_3 - 1)^2x_3^2y_3^2] \quad (23)$$

The Groebner analysis gives rise to 36 acceptable solutions, 6 of them are listed in table 7. This time one gets 3 groups of

solutions corresponding to the obtained C -value. Only the results (1), (2), (5), (6), (21) and (22) are displayed in table VII.

The first group corresponds to the highest value $C = -19.2037$. With the other values obtained for C (-20.1820 and -21.1604), two sets of 16 solutions are found. The solutions in each set are all equivalent because the coordinates can be obtained from each other by permutations or symmetry operations like mirroring with respect to a central axis e.g. Strictly speaking only 3 acceptable solutions independent solutions are found here. This example illustrates very well the features of our approach: namely that 3 possible solutions are obtained, each corresponding to another value of the cost function.

TABLE VII
THREE COMPONENTS ON AN A RECTANGULAR SUBSTRATE.
THE FIRST COMPONENTS DISSIPATES MORE

	x_1	y_1	x_2	y_2	x_3	y_3	C
(1)	1.0	0.5	1.655	0.827	0.345	0.173	-19.2037
(2)	1.0	0.5	0.345	0.173	1.655	0.827	-19.2037
(5)	1.0	0.413	1.655	0.864	0.345	0.614	-20.1820
(6)	1.0	0.413	0.345	0.614	1.655	0.864	-20.1820
(21)	1.174	0.587	0.771	0.386	0.271	0.136	-21.1604
(22)	1.174	0.587	0.271	0.136	0.771	0.386	-21.1604

In Fig. 5 the solutions labelled 1, 5 and 21 in table VII are presented graphically. It must be noted again that fig. 5a corresponds to the highest value of the cost function so that this is the best solution from a thermal point of view i.e. to put all the components as far as possible from each other and from the boundaries.

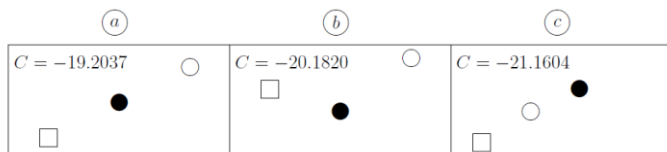


Fig. 5. Three components on a rectangular substrate. • = (x_1, y_1) , ◦ = (x_2, y_2) and ◻ = (x_3, y_3) . The cost functions are (a) $C = -19.2037$, (b) $C = -20.1820$ and (c) $C = -21.1604$.

Remark that the most dissipating component is always placed near the center of the substrate.

V. DISCUSSION

Different authors should discuss the results and how they can be interpreted from the perspective of previous studies and of the working hypotheses. The findings and their implications should be discussed in the broadest context possible. Future research directions may also be highlighted.

VI. CONCLUSIONS

The analytical method finds the global optimum, which distinguishes it from numerical methods that can only find local minimums. It is universal and allows for solving many similar global minimum search problems.

The paper presents the optimization of the integrated circuit layout with regard to the thermal aspect. This means that it is proposed to arrange the functional blocks on the substrate in such a way that the mutual thermal interference of the blocks is minimal. This leads to the minimization of the maximum substrate temperature, and thus allows for the increase of the throughput of the considered integrated circuit. The proposed analytical method, unlike numerical methods, leads to obtaining a global minimum. The paper presents analytical formulas for three heat sources. Of course, it is possible to extend it to a larger number of functional blocks in a similar way. The proposed method can be useful in the design of integrated circuits, in which it is important to limit the mutual thermal interactions i.e., in the design of systems with significant power dissipation, for example, the topography of numerical processors.

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