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Overview of Solutions in Quantum Information Technologies, part 3

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***Abstract*—This paper presents the third installment in a series reviewing contemporary solutions in quantum information technologies. Seven thematic areas are surveyed: (1) quantum control engineering, covering theoretical foundations, open-loop and feedback architectures, and industrial optimization use cases; (2) quantum radar, examining quantum illumination principles alongside the fundamental power and decoherence barriers to practical deployment; (3–4) the integration of quantum technologies with embedded systems and the Internet of Things, including quantum random number generators, nitrogen-vacancy magnetometers, miniaturized atomic clocks, and post-quantum cryptographic protocols such as QKD; (5) quantum simulation, contrasting analog and digital approaches with an emphasis on recent large-scale experiments demonstrating quantum utility; (6) quantum haptic interfaces for education, molecular simulation, and research visualization; and (7) quantum entanglement theory, tracing the path from the EPR paradox through Bell's theorem to the 2022 Nobel Prize. Across these domains, the paper identifies recurring engineering challenges — particularly decoherence, scalability, and the transition from laboratory demonstrations to industrial deployment — and highlights the growing convergence of quantum physics with control theory, embedded computing, and applied engineering.**

***Keywords*—quantum information technologies, quantum control engineering, quantum illumination, quantum key distribution, quantum simulation, quantum sensing, quantum entanglement**

I. Quantum Control Engineering: From Foundational Principles to Technological Realization

A. *Introduction: The Evolution of Control Systems*

The trajectory of industrial civilization is defined by the precision with which we manipulate energy and matter. In 1788, James Watt's centrifugal governor provided the macroscopic feedback loop necessary to stabilize the steam engine, marking the birth of classical control engineering. For over two centuries, this paradigm sufficed for systems governed by Newtonian mechanics. However, as Richard Feynman articulated in his seminal 1959 lecture, "Plenty of Room at the Bottom," the next frontier involves the manipulation of individual atoms.

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In the 21st century, the limits of classical scaling are manifesting in the semiconductor industry; with feature sizes approaching the 20 nm threshold, Moore's Law is colliding with the quantum noise floor. The transition from quantum science to quantum technology requires more than just understanding wave functions — it requires active, robust control. As Dowling and Milburn (2003) asserted, quantum control is the "essential task" for the second quantum revolution. Without it, high-value resources like entanglement and superposition remain fragile, decohering before they can perform useful work. To move these technologies from the laboratory to the industrial floor, we must adopt an engineering paradigm grounded in non-commutative logic and stochastic estimation within Hilbert spaces.

B. *Theoretical Foundations: The Quantum Probability Framework*

For a systems engineer, the shift to quantum mechanics is fundamentally a shift in the underlying probability framework. Classical control assumes a commutative σ-algebra of events where state estimation is a matter of refining probability densities. In the quantum domain, observables are represented by self-adjoint operators on a Hilbert space H that do not necessarily commute. This non-commutativity is the origin of the Heisenberg uncertainty principle, rooted in the canonical relation $[Q, P] = i\hbar I$, which dictates that $\Delta Q \Delta P \geq \frac{\hbar}{2}$.

The following postulates define the operational constraints for any quantum control system [1]:

1) **States:** Summarized by a density matrix ρ, a positive, normalized operator ($Tr[\rho] = 1$) that serves as the non-commutative counterpart to a probability density.
2) **Observables:** Physical quantities correspond to self-adjoint operators. The expectation is given by the functional $Tr[\rho A]$.
3) **Measurement:** Numerical outcomes are restricted to the spectrum spec(A). The probability of observing outcome a is $Tr[\rho P_a]$, where P_a is the projection operator onto the relevant eigenspace.
4) **Conditioning:** Post-measurement, the state is updated via the projection postulate to account for the information gain:

$$\rho \to \rho'[a] = \frac{P_a \rho P_a}{Tr[\rho P_a]} \qquad (1)$$

5) **Evolution:** Isolated systems evolve unitarily via the Schrödinger equation: $i\hbar\frac{d}{dt}U(t) = H(t)U(t)$.

C. Open-loop approach [2]

In quantum engineering, the act of "seeing" a system introduces back-action. It is not possible to measure a system's exact state without destroying its quantum state, thus stripping the system of all its desired features. One of the solutions to this problem is open-loop control.

First, the probabilistic model is constructed. Then, an artificial closed-loop is simulated using, for example, the Lyapunov-based control methodology. Although this approach has achieved some successes, it is specifically vulnerable to any model uncertainties and external or internal disturbances. That is why engineers keep working on more robust methods.

D. Feedback Architectures: Measurement-Based vs. Coherent Control

The design of feedback loops [2] in NISQ-era devices requires choosing between architectural philosophies based on the flow of information.

1) **Estimation-Based Feedback (weak measurements):** instead of the system itself, its surroundings are measured. System X interacts with a probe M. A critical engineering requirement is that for the conditional expectation to be well-defined, the system observables must belong to the **commutant** $\mathcal{Y}'$ of the measurement algebra. Specifically, after the interaction U, we ensure:

$$[U^*(X \otimes I)U, U^*(I \otimes M)U] = 0 \tag{2}$$

It allows information transfer without destroying the underlying coherence. There are several methods of weak measurements, one of them being deep reinforcement learning [3].

2) **Coherent Feedback (Non-Measurement):** the controller is itself a quantum system. Information flows directly as quantum fields (e.g., optical or microwave) between the system and the controller. This maintains the "quantumness" of the signal, avoiding the measurement-induced noise floor. One of the first examples of such system is presented in [4], where authors used just nine atoms to create high accuracy predictive quantum neural network.

Strategic selection between these depends on the decoherence rates; coherent feedback represents the frontier for 21st-century engineering as it allows for the manipulation of entanglement without the classical "bottleneck."

E. Industrial Use Cases: Process Optimization

The shift from the lab to industry is evidenced by the adoption of quantum optimization in logistics and manufacturing.

- **Volkswagen (Traffic Flow):** During the 2019 Web Summit in Lisbon, VW utilized quantum annealing to process real-time GPS data from bus fleets, optimizing urban mobility in near real-time.
- **ExxonMobil (Maritime Logistics):** Managing a global LNG fleet involves optimizing routes for 50,000 ships. Using IBM's gate-based processors, ExxonMobil evaluated the Quantum Approximate Optimization Algorithm (QAOA) and the Variational Quantum Eigensolver (VQE). Their engineering finding: while QAOA scales with sample size, VQE demonstrated higher success rates with limited samples, a critical factor for early NISQ hardware integration.

F. Regulatory and Regional Strategy

The Regulatory Horizons Council (RHC) 2024 report advocates for "pro-innovation" governance. Key recommendations include the creation of application-specific regulatory frameworks and the use of regulatory sandboxes to test quantum-classical hybrid systems. In the United States, South Carolina's industrial hub—specifically in aerospace, communications, and textiles — is a prime candidate for early quantum control adoption. Professional teams must move beyond the hype, building "quantum literacy" to evaluate QUBO and VQE use cases for competitive advantage.

G. Conclusion: The Path to Quantum Supremacy in Engineering

The convergence of quantum mechanics and systems engineering defines the limit of technological realization. As Dowling and Milburn noted, quantum control is the "essential task" for the transition from science to utility. With the integration of AI for real-time error correction and improved computation, the role of the Senior Quantum Systems Engineer is to master this non-commutative landscape to stabilize the next generation of industrial processes.

II. Quantum Radar

A. Introduction and Typology

Quantum radar technology encompasses a variety of theoretical and experimental approaches designed to enhance target detection capabilities beyond classical limits. These systems can generally be classified into three distinct architectures. The first category utilizes a quantum transmitter, which emits non-classical states of light, such as single photons, without maintaining any entanglement with the base station. The second category relies on a quantum receiver; in this setup, the system broadcasts standard classical radar waves but employs highly sensitive quantum sensors to measure the returning echoes with extreme precision [5]. The third and most heavily researched paradigm is quantum illumination [6]. This method relies on generating a quantum state that is initially entangled with the receiver, allowing the system to distinguish the true returning signal from environmental interference [5], [6].

B. Fundamental Challenges in Radar Implementation

1) Detection Mechanics and Economic Viability: Transitioning quantum illumination from a theoretical concept to a practical radar system presents significant physical and economic barriers. When a signal strikes a target, the nature of the reflection heavily impacts the amount of energy returned to the sensor. Furthermore, the financial cost of deploying optimal quantum radar systems is much higher than that of currently existing real-world classical radars operating in the X-band [7].

2) The Power Deficit: A critical limitation of quantum radar systems is power generation. Standard classical radars routinely operate with transmitter powers around 1 MW, at an approximate cost of one US dollar per watt. In stark contrast, experimental quantum transmitters operate at microscopic power levels, roughly -102 dBm. Scaling quantum power up to classical levels is currently unfeasible, as generating 1 W of power in a quantum system is estimated to be ten billion times more expensive. Even when factoring in a theoretical 6 dB quantum advantage, engineers are still left with an insurmountable power deficit of 186 dB [7].

3) Frequency Regimes and Thermal Decoherence: The choice of operational frequency introduces an environmental dilemma. Operating in the optical regime (often referred to as quantum LIDAR) offers the distinct advantage of negligible thermal background noise ($n_B \approx 0$), which is an ideal environment for preserving fragile quantum entanglement. However, optical frequencies are severely limited by atmospheric conditions, experiencing intense attenuation and scattering [8].

Conversely, operating in the microwave regime is necessary for effectively penetrating atmospheric obstacles like fog and clouds. Unfortunately, this spectrum is plagued by massive thermal background noise at room temperature. Empirical constraints dictate that the quantum signal power must be kept incredibly low, approximately 0.7 photons per mode, to maintain strong quantum correlations. Meanwhile, the ambient thermal noise introduces around 610 photons per mode. This overwhelming noise ratio causes the transmitted quantum signal to undergo instantaneous thermal decoherence, destroying the very entanglement the system relies upon [8].

C. Applications of Quantum Illumination

Given the severe atmospheric attenuation in the optical regime and the thermal decoherence in the microwave regime, long-range aerospace quantum radar is currently impractical [7], [8]. However, the core principles of quantum illumination show immense promise in highly controlled, close-range environments. Promising applications include non-invasive scanning of highly sensitive electronic components, where classical high-power signals might cause damage. Furthermore, quantum illumination holds significant potential in the medical and biological fields, particularly for biomedical imaging and protein spectroscopy, where the ability to detect exceptionally weak resonance signals is paramount [6].

III. Quantum Hardware for Edge Devices

A. Introduction and the Quantum 2.0 Era

The rapid development of the Internet of Things (IoT) ecosystem imposes rigorous design constraints on the architecture of embedded systems, commonly referred to as the SWaP-C (Size, Weight, Power, and Cost) paradigm. Edge devices operate in stringent energy regimes of single milliwatts, possessing limited memory resources. Concurrently, the field of information technologies is undergoing a transformation into the era known as Quantum 2.0, in which single quantum systems, such as photons, electrons, or atoms, are precisely controlled [9].

The utilization of fundamental quantum mechanical phenomena, particularly state superposition and entanglement, represents a new technological resource that enables the realization of unprecedented computational and measurement functionalities. The main barrier to integrating quantum systems with IoT infrastructure remains the phenomena of decoherence and the environmental operating requirements of quantum systems, which often dictate the need for cryogenic cooling and electromagnetic shielding. The solution to this problem lies in the miniaturization of specific technologies capable of operating at room temperature and the implementation of hybrid architectures.

B. Quantum Random Number Generators (QRNG) as a Hardware Root of Trust

In distributed IoT systems, secure device authorization and the distribution of cryptographic keys are of critical importance [10]. Traditional pseudorandom generators, which rely on deterministic algorithms, can present a significant vulnerability to predictive attacks by advanced computer systems.

Quantum Random Number Generators (QRNG) provide a physical solution to this problem, drawing entropy directly from the probabilistic nature of quantum phenomena. A significant breakthrough from the perspective of embedded systems design is the radical miniaturization of these circuits. Commercially available System-on-Chip (SoC) devices with a built-in QRNG generator currently reach dimensions on the order of 2.5 by 2.5 millimeters [11]. They are characterized by very low power consumption and are adapted for direct communication with microcontrollers (MCUs) using buses such as SPI or I2C [12], providing a hard, hardware-based cryptographic root of trust.

C. Quantum Sensing in Edge Devices

Quantum phenomena are characterized by extreme sensitivity to external environmental disturbances [13]. A leading sensing technology with high application potential in IoT devices is magnetometers based on Nitrogen-Vacancy (NV) color centers in synthetic diamonds [14].

Crucially for embedded devices, this process is stable at room temperature [15]. The miniaturization of diamond circuits opens up possibilities for application in autonomous navigation systems in environments lacking GNSS signals through vector analysis of the Earth's natural magnetic field [16].

Another key component is miniaturized atomic clocks in the CSAC format. CSAC systems utilize the absolutely constant resonance of trapped rubidium or cesium atoms [17], [18]. These modules operate with low power consumption, enabling implementation directly in edge nodes and providing them with sub-microsecond time autonomy [19].

D. Predictive Maintenance (PdM) and Embedded Quantum Machine Learning (EQML)

The implementation of quantum sensors completely redefines the approach to Predictive Maintenance. The use of ultrasensitive quantum accelerometers allows for the identification of the early stages of wear before vibration signatures measurable by classical MEMS transducers appear.

Due to the limitations of microcontrollers, EQML operates in a hybrid model [20]. A typical IoT node is responsible for data acquisition, whereas complex optimization problems are handed over (offloaded) to specialized quantum coprocessors in the cloud [20]

IV. Quantum-Safe IoT Security

The Internet of Things (IoT) is a very quickly growing segment of technology, achieving 21 billion devices connected to the cloud, as of late 2025. It is estimated that the number of IoT devices will nearly double to 39 billion by 2034. Such a massive amount of devices introduces many vulnerabilities that could be exploited. IoT devices use different standard and thus lack standardization, including lack of standardized security protocols. Vulnerable data can be leaked, or compromised devices might be used to further perform malicious attacks.

The rapid advancement of quantum computing threatens to break current cryptography. Such an event is estimated to happen by the year 2040. However, the "Store Now, Decrypt Later" strategy necessitates moving to quantum-resistant algorithms as quickly as possible, since malicious third parties could be gathering sensitive data right now, with the intent of deciphering it years down the line. Most of contemporary cryptography is key-based, relying on computational intensity of problems such as integer factorization, which is difficult to solve by contemporary computers in a timely manner. However, quantum computers would enable for example performing the quantum Shor's algorithm, which can solve this problem many times faster than a classical computer [21]. While symmetric cryptography, like the Advanced Encryption Standard (AES) is generally considered safe, a significant amount of devices in the IoT use Rivest–Shamir–Adleman (RSA) or Elliptic-curve cryptography (ECC) based cryptography, which are threatened to be broken by quantum computers. However, to be quantum-resistant, even the more resistant AES would require to double its key size, from AES-128 to AES-256. While relatively simple, it's important to keep the narrow bandwidth of IoT devices in mind.

Seeing as a lot of IoT devices, such as smart locks, cars or medical devices transfer sensitive data, it introduces the need for quantum-resistant cryptography, allowing for safe data transfer between devices and the cloud . Such concept falls under the term of Post-Quantum Cryptography, which describes algorithms that are believed to be secure against attacks performed even by very powerful quantum computers.

Quantum Key Distribution (QKD) is a concept which enables secure exchange of keys using quantum technologies. One of the most prominent examples is the BB84 protocol, developed by Charles Bennett and Gilles Brassard in 1984 [22], in which photons are utilized as qubits. The cryptographic key is sent as a stream of photons, whose polarization matches either 1 in 0, allowing to send data as a stream of bits. However, when emitted, a photon is in a state of superposition. Even if an eavesdropper intercepted the message, The no-cloning theorem makes it impossible to copy a superposition. An attempt to measure the superposition by the eavesdropper would alter its state. Thus, any interference via third-party would be discovered, ensuring that the exchanged key is uncompromised. Succesful implementation of such protocols would make IoT networks and devices significantly more resistant to cyber-attacks, and even quantum-resistant to those performed by quantum computers.

Another field of implementing quantum principles in the internet of things are Quantum sensors, which utilize quantum principles, such as entanglement, superposition or spin, to measure certain physical quantities with very high accuracy. That makes them ideal for use cases demanding high precision [23]. Quantum sensors utilizing precise magnetometers can enable navigation in situations when GPS is unavailable, such as underwater or in areas surrounded by impenetrable objects. Quantum sensors can also find great use in healthcare, in neuro-imaging or monitoring in the human body. Precise atomic clocks can serve as time-keeping reference in a multitude of devices, allowing for precise synchronization in industrial processes. Such sensors can be used for real-time environmental monitoring, which is one of the most common use cases of the internet of things.

However, implementation of quantum technologies in the internet of things is met with significant challenges. Implementation of QKD would need either fiber optic cables or satellite communication, which wouldn't be economically viable for most of the nodes connected to the cloud. Nodes are designed to have lowest possible energy consumption, while quantum algorithms consume a high amount of processing power. Implementation of quantum technologies would need to design algorithms that utilize low amounts of energy, while providing sufficient security. One solution could be a more centralized approach, moving some of the more demanding operations to the cloud.

V. Quantum Simulations

A. Introduction

Simulating quantum many-body systems is a major goal in modern physics. The main difficulty comes from the exponential growth of the Hilbert space. Representing the exact state of an N-qubit system requires tracking a massive number of complex probability amplitudes, which quickly blocks classical computers from solving these problems. To address this issue, quantum simulators use controllable quantum systems to mimic the target models. As Georgescu et al. described in their foundational work [24], these simulators are generally divided into two main types: Analog Quantum Simulation (AQS) and Digital Quantum Simulation (DQS).

B. Analog Quantum Simulation (AQS)

The general rule of operation for AQS, as defined by Georgescu, is to map the Hamiltonian of the system we want to study directly onto the physical interactions of the simulator.

The system then naturally evolves according to the laws of physics, without the need for discrete logic gates.

A great example of this approach is the 2021 experiment by Ebadi et al. [25]. They used a programmable two-dimensional array of up to 256 neutral rubidium atoms held by optical tweezers. In this setup, the hardware inherently realizes an Ising-type quantum spin model using the Rydberg blockade mechanism. When atoms are hit by a laser and transition to a high-energy Rydberg state, strong van der Waals forces prevent neighboring atoms from reaching the same state if they are too close. This physical distance limit acts as the hardware version of magnetic interactions.

By changing the laser settings and the distance between atoms, the researchers mapped different quantum phases of matter. The continuous physical evolution of the system naturally settled into complex patterns. One of the most notable results was the checkerboard phase, where excited and ground-state atoms alternate just like squares on a standard chessboard. This analog approach is highly effective for studying specific physical systems and solving graph-based optimization problems.

C. Digital Quantum Simulation (DQS)

In contrast to AQS, Digital Quantum Simulation uses a sequence of discrete operations to simulate time evolution. The central challenge in this approach arises because the total Hamiltonian $\hat{H}$ is usually a sum of individual, non-commuting parts (for example, $\hat{H} = \hat{H}_A + \hat{H}_B$). In quantum mechanics, if operators do not commute ($[\hat{H}_A, \hat{H}_B] \neq 0$), the exponential of their sum is not equal to the product of their separate exponentials:

$$e^{-i(\hat{H}_A+\hat{H}_B)t/\hbar} \neq e^{-i\hat{H}_A t/\hbar} e^{-i\hat{H}_B t/\hbar}$$

Because of this inequality, the continuous evolution of a complex system cannot be simulated in one single step. To solve this, DQS relies on Trotterization. Based on the Lie-Trotter product formula, it breaks the total evolution time t down into n small, discrete slices. For each short time slice, the simultaneous physical interactions are mathematically approximated by a sequence of simpler, individual steps:

$$e^{-i\hat{H}t/\hbar} \approx \left(\prod_j e^{-i\hat{H}_j \Delta t/\hbar} \right)^n , \tag{3}$$

where $\Delta t = t/n$. As n increases and the time slices become smaller, this approximation approaches the exact evolution of the system.

A recent study by Kim et al. [26] demonstrated DQS on a massive scale using a 127-qubit IBM processor with a heavy-hex connectivity layout. They simulated the time evolution of the transverse-field Ising model by translating the Trotter steps into alternating layers of 1- and 2-qubit gates. The experiment required up to 60 layers consisting of nearly 3000 gates. The main challenge in DQS is that hardware noise accumulates during such long gate sequences, which normally destroys the results.

To solve this, the IBM team used an error mitigation technique called Zero-Noise Extrapolation (ZNE). Instead of waiting for future error-free quantum computers, they intentionally amplified the existing hardware noise. By running the same circuit at increasing noise levels, they could calculate a mathematical trend and extrapolate backward to estimate what the result would be if there was zero noise.

D. Verification and Quantum Utility

A critical challenge in these large-scale simulations is proving the results are correct when classical supercomputers cannot double-check them. In the IBM experiment, researchers solved this by first running specific test circuits made entirely of Clifford gates. These specific gates create complex entanglement but, due to their mathematical properties, can still be perfectly simulated on a standard computer. This allowed the team to verify that their ZNE error mitigation was working correctly.

Once they trusted the mitigation method, they switched back to the standard, non-Clifford gates required for the Ising model. At this point, classical approximation methods failed to keep up with the complexity of the quantum states. By producing reliable results at a scale where classical supercomputers hit a wall, this experiment showed that current noisy quantum processors can already provide practical utility in physics.

VI. Quantum Haptic Tools

A. Introduction

Quantum physics is intrinsically unintuitive and intangible because its concepts defy everyday physical intuition. Traditionally, we rely heavily on abstract graphs and equations to describe the quantum world, which can create a steep learning curve for students and complicate advanced research. There are, however works exploring haptic interfaces to bridge this gap. By using mechanical feedback—such as targeted forces, vibrations, or motion—haptic technology translates abstract data into physical sensations. Currently, these tactile interfaces are being applied in three main areas: education, molecular simulation (such as drug design), and exploratory research.

B. Educational Haptic Tools: Quantum Dice

In the classroom, one of the most interesting developments is the use of interactive ”quantum dice” [27]. These devices serve as macroscopic stand-ins for quantum particles, giving students a tangible way to explore phenomena like superposition and entanglement. Each die uses color-coded faces to represent six different states across three measurement bases. Before it is rolled, the die simulates a state of superposition. The physical act of rolling acts as a quantum measurement, collapsing the superposition into a single definite state. Furthermore, bringing two dice together simulates entanglement; if measured in the same basis, they exhibit perfect anti-correlation, meaning their sum always equals seven. This kind of physical representation allows students to literally get a feel for complex protocols like BB84 quantum key distribution.

C. Haptic Feedback in Molecular Simulation and Drug Design

Haptics also offer a significant advantage in computational chemistry and drug design. When working at the nanoscale, researchers can use adaptive algorithms to manipulate molecular structures or rigid bodies in real time [28]. For example, a user can physically feel the resistance and interaction forces when trying to dock an inhibitor into an HIV protease. To couple these simulations with a haptic device, two main control schemes are typically used. In a Direct Force Feedback (DFF) setup, the user manually guides the molecule's position while the system constantly calculates and returns the molecular forces to the user's hand to minimize the system's energy. Alternatively, in a Force-Force (FF) coupling scheme, the user applies virtual torque and force, and the simulation dynamically updates the molecular geometry. This allows scientists to intuitively gauge both internal elastic tension and intermolecular forces.

D. Visualization of 3D Quantum Dynamics in Research

For more fundamental research, haptic tools are being paired with Web3D environments to explore quantum dynamics. Using setups like the Phantom Desktop, researchers can map potential energy surfaces (PES) to physical resistance [29]. A practical example of this is the study of the cyanamide molecule. Instead of just looking at a 3D model, a researcher can physically push against the energetic barriers involved in the molecule's inversion. Feeling the "curvature" of the energy surface provides a layer of sensory intuition that visual data alone misses. Pairing this tactile feedback with the visual evolution of 3D wavepackets gives researchers a much more grounded understanding of reaction energetics and dissipative systems.

E. Conclusion

Ultimately, quantum haptics provides a unique way to translate the microscopic world into something we can physically experience. By turning abstract math into tangible interactions—whether that means rolling quantum dice to learn cryptography or feeling the chemical repulsion in a drug docking simulation—these tools make the quantum realm significantly more accessible and intuitive.

VII. Quantum Entanglement Theory

A. Introduction: Physics on the Brink of Revolution

The development of quantum mechanics in the first half of the 20th century initiated a profound transformation of scientific thought. At the heart of this upheaval lay quantum entanglement. This phenomenon, which contradicts our daily intuition, suggests the existence of non-local connections within the very fabric of the universe. What Albert Einstein once famously dubbed "spooky action at a distance" now defines a new era of technical information science.

B. Local Realism vs. The Copenhagen School

The foundation of the dispute that shaped modern physics was the debate between Niels Bohr and Albert Einstein.

The Copenhagen School argued that microscopic reality is undefined until the moment of observation, and particles exist in a superposition of all possible states.

Albert Einstein, a proponent of local realism, opposed this vision, expressed in his famous remark: "God does not play dice." Einstein believed in a predictable and objective world. He postulated the existence of so-called "hidden variables"—properties of particles that determine the measurement outcome from the beginning, analogous to two halves of the same coin enclosed in separate boxes.

C. The EPR Paradox (1935)

In May 1935, Einstein, Podolsky, and Rosen published the paper "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?". They presented a thought experiment: if we measure the momentum of particle A in an entangled pair, we instantly know the momentum of particle B. The authors argued that since we did not disturb particle B, its state must have been an "element of reality" beforehand. They concluded that quantum mechanics was an incomplete theory [30].

D. Mathematical Formalism

Entanglement is not merely an interpretational curiosity but a measurable physical resource.

1) Werner's Definition and Separability: A mixed state ρ is called separable if it can be represented as a convex combination of product states:

$$\rho_{AB} = \sum_i p_i \rho_i^A \otimes \rho_i^B \tag{4}$$

If such a decomposition does not exist, the particles lose their individuality, and their properties are defined solely by their mutual relationships.

2) PPT Criterion and Entanglement Measures: To identify entanglement, the Peres-Horodecki (PPT) criterion is used, examining the eigenvalues of the partial transpose of the density matrix. The degree of entanglement is quantified using von Neumann entropy or Concurrence.

E. Bell's Theorem (1964)

For nearly 30 years, the dispute remained in the realm of philosophy. It was John Bell who proved that if Einstein were right, the statistical results of correlations must satisfy certain mathematical constraints (Bell's Inequalities). Quantum mechanics predicted their violation, providing physicists with a tool to finally resolve the conflicting visions of the world.

F. Experimental Milestones

1) Alain Aspect (1982): The French physicist provided the first definitive evidence of non-locality. Using a switching mechanism that changed detector settings faster than light could travel between them, he demonstrated that the world is indeed "spooky" and breaks classical constraints [31].

2) Loophole-Free Tests (2015): In 2015, groups in Delft, Vienna, and Boulder closed all experimental loopholes (locality and detection). Using entanglement over a distance of 1.3 km, it was finally confirmed that quantum mechanics survived the most rigorous test possible [32].

G. Multi-particle Entanglement: GHZ and W

For systems of three or more particles (GHZ states), the contradiction with local realism appears at the level of a single measurement. W states, on the other hand, exhibit remarkable resilience to particle loss, making them crucial for building stable networks.

H. Engineering and Resource Theory

Today, entanglement is treated like energy in thermodynamics—a resource necessary for informational work.

- **Cryptography (QKD):** Security guaranteed by the fact that the act of observing a particle destroys its state.
- **Teleportation:** Instantaneous transfer of information about a particle's structure, the foundation of the currently built Quantum Internet.
- **Quantum Repeaters:** Utilizing entanglement swapping to connect distant network nodes without transmitting matter.

I. Conclusions and the 2022 Nobel Prize

The honoring of Alain Aspect, John F. Clauser, and Anton Zeilinger with the Nobel Prize in 2022 closed the chapter on fundamental disputes. Experiments refuted local realism: the world does not consist of objects with fixed, independent properties. Despite non-locality, the theory of relativity remains safe, as entanglement does not allow for the transmission of useful information faster than light without a classical communication channel. Today's physics no longer asks "if," but "how" best to utilize entanglement to build the civilization of the 21st century.

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