

Diagnosis methods — or perhaps methods for solving inverse problems?

On ambiguity groups and measurement accuracy in the diagnosis of linear analog circuits

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Abstract—This article theoretically discusses the impact of the ambiguity groups and the measurement accuracy on the diagnostic capabilities of a selected class of verification methods for diagnosing parametric faults in analog circuits. At the same time it points out that what constitutes a limitation in the diagnostics can be used constructively in the design process (which is in its essence the inverse problem) with regard to the selection of the parameters of the circuit components in terms of the circuit's intended functionality.

Keywords—linear circuit diagnosis; test equation; ambiguity groups; inverse problem

I. INTRODUCTION

MOST generally, the faults of analog circuits can be divided into catastrophic and parametric ones [1], [2], [3]. The catastrophic faults (hard faults) change the circuit topology. These are short circuit and open circuit faults. The parametric faults (soft faults) involve changes in the parameters of the circuit components which are over tolerance limits. Many methods have been developed to diagnose analog circuits. They are generally divided into two groups: Simulation Before Test methods (SBT methods) and Simulation After Test methods (SAT methods). The determining factor for classification into a given class of methods is the ratio of the computation time required before and after the circuit test has been performed. SBT methods primarily include dictionary methods [4], which are used in the diagnosis of single catastrophic faults. However, there are also solutions for single parametric methods [5]. In the case of dictionary methods, neural networks are usually used as a classifier [6], and the fault dictionary typically stores data processed using various transforms, such as for example Fourier transform, or wavelet transform. The SAT methods these are primarily: the identification methods, the verification methods, and the optimization methods [7]. All of these methods involve finding solutions to diagnostic (test) equations for the values of circuit component parameters or for the deviations of these parameters from their nominal values. Identification

methods involve identifying all the parameters of the circuit elements, making them applicable only to circuits with a small number of elements. In the cases of the verification methods and the optimization methods the number of simultaneously diagnosed faults (usually limited by convergence problems or singularities of the relevant matrices) is reduced, because the diagnostic equations are solved for selected sets (reduced sets) of circuit component parameters, assuming the remaining elements remain unfaulty. The considerations presented in this article are based mainly on works of which the author of this article is the co-author or the author, and which concern a method of the parametric faults diagnosis based on the SAT approach [8], [9], [10]. The diagnostic method presented in [10] allows for taking into account the tolerance of circuit element parameters in the diagnostic process. The main challenges in conducting the diagnostic process are the multiple solutions and the accuracy of the measurements. Two types of multiple solutions in the diagnosis process can be distinguished. The first type involves multiple solutions of the single diagnostic equation with respect to the considered fixed subset of the parameters of the circuit components [11]. The second type involves multiple solutions of the diagnosis process for different subsets of parameters satisfying the verification criteria, which is related to the occurrence of so-called ambiguity groups [8], [9], [10], [11]. The ambiguity group with respect to the given measurement signal is such a set of the parameters of the circuit components for which, after solving reduced test equations built with using this signal as the reference one, it is not possible to unambiguously indicate using verification criteria which parameters of the set have values which differ from the nominal ones, see Section II. The aim of this article is to demonstrate that ambiguity groups can be used constructively in the process of selecting the parameters of the circuit components based on design assumptions. Measurement accuracy, in turn, is irrelevant at the design stage because the signals in relation to which the parameters of the circuit components are selected are software-generated. Therefore, it seems to be interesting to consider using the framework of the diagnosis method described in [10] as a tool for solving the so-called inverse problems, which are discussed for example in the literature [12], [13].

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II. AMBIGUITY GROUPS

Let us consider a diagnostic equation of the form

$$\mathbf{u} = \mathbf{f}(\mathbf{x}), \quad (1)$$

where $\mathbf{u}$ is a measurement vector, $\mathbf{x}$ is a vector of parameters of the circuit components, $\mathbf{f}(\mathbf{x})$ is a nonlinear function of the elements of the vector $\mathbf{x}$. The solution of the equation can be given in absolute values, i.e. as the calculated vector $\mathbf{x}$ or as a vector of deviations of the parameters of the circuit components from the nominal values $\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}_{nom}$, where $\mathbf{x}_{nom}$ is a vector of the nominal values of the parameters of the circuit components. At this point, it is irrelevant at how many measurement points the measurements are performed or whether they are performed in the time domain, in the frequency domain or in the domain of changing values of the source element parameters (which applies to DC systems, both linear and nonlinear). What is important is the nonlinear dependence of the measurement vector on the elements of the vector of the parameters of the circuit components. Let's assume that the measurement vector has the same number of elements as the vector of the parameters of the circuit components. The number of equations in the equation (1) is equal to the number of unknowns. If the Jacobian matrix of the equation (1) is nonsingular, meaning it has a nonzero determinant

$$\det(\mathbf{J}(\mathbf{f}(\mathbf{x}))) \neq 0 \quad (2)$$

it is possible to identify all the circuit's element parameters. We are therefore dealing with an identification method. However, it should be remembered that due to the nonlinear nature of the equation (1) it may have multiple solutions [11]. These are multiple solutions with respect to the fixed subset of parameters of the circuit components. In the case at hand, with respect to the set of parameters of all circuit components. However, alternative solutions may not satisfy other design assumptions unrelated to the measurement vector. It is conceivable that in a circuit with parameters other than the nominal values resulting from the design, overvoltages or overcurrents may occur in selected circuit branches. As mentioned in the introduction, the ability to identify all parameters of the circuit components is usually possible in the case of a very small number of components, which is related to the singularity of the Jacobian matrix (2). For this reason, we usually deal with a diagnostic equation of the form

$$\mathbf{u} = \mathbf{f}(\mathbf{x}', \mathbf{x}_{nom}''), \quad (3)$$

where $\mathbf{x}'$ – the vector of the parameters of the potentially faulty circuit components, $\mathbf{x}_{nom}''$ – the vector of the parameters of the unfaulty circuit components. The Jacobian matrix of the equation (3) is calculated only with respect to the elements of the vector $\mathbf{x}'$ and if it is non-singular

$$\det(\mathbf{J}(\mathbf{f}(\mathbf{x}', \mathbf{x}_{nom}''))) \neq 0 \quad (4)$$

it is possible to identify the parameters of all potentially faulty components of the circuit. Equations (1) and (3) can be solved using various methods, including optimization methods, hence the term "optimization methods" in the context of SAT methods. In the case of linear systems, however, the Jacobian matrix (4) for the selected vector $\mathbf{x}'$ is still most often

singular, and one must settle for diagnosing a smaller number of the circuit components. To identify faulty elements, one must solve an appropriate number of the reduced diagnostic equations.

$$\mathbf{u} = \mathbf{f}(\mathbf{x}'_{(j)}, \mathbf{x}_{(j)nom}''), \quad (5)$$

for reduced vectors $\mathbf{x}'_{(j)}$ where $j = 1, \dots, J$. $\mathbf{x}'_{(j)}$ is the vector with the reduced number of elements. The element(s) removed from the vector $\mathbf{x}'_{(j)}$ is (are) moved to the vector $\mathbf{x}_{(j)nom}''$. Then, the obtained results should be selected (verified for compliance with specific criteria - hence the verification method in the context of SAT methods) in order to determine which components are faulty and what are the values of their parameters. Let us consider specifically the case of a circuit for which the parameter vector (with dimension five) is given as $\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T$ and for which the Jacobi matrix of the form (2) is singular. The diagnostic equation (1) cannot be solved, so it is impossible to uniquely determine, for this set, which of the parameters deviate from their nominal values. The parameters that are elements of the vector $\mathbf{x}$ constitute an ambiguity group. We will therefore reduce the dimension of the vector $\mathbf{x}$ by one and create subsequent reduced diagnostic equations

$$\mathbf{u} = \mathbf{f}(\mathbf{x}'_{(1)}, \mathbf{x}_{(1)nom}''), \quad (6)$$

$$\mathbf{u} = \mathbf{f}(\mathbf{x}'_{(2)}, \mathbf{x}_{(2)nom}''), \quad (7)$$

$$\mathbf{u} = \mathbf{f}(\mathbf{x}'_{(3)}, \mathbf{x}_{(3)nom}''), \quad (8)$$

$$\mathbf{u} = \mathbf{f}(\mathbf{x}'_{(4)}, \mathbf{x}_{(4)nom}''), \quad (9)$$

$$\mathbf{u} = \mathbf{f}(\mathbf{x}'_{(5)}, \mathbf{x}_{(5)nom}''), \quad (10)$$

for reduced vectors

$$\mathbf{x}_{(1)} = [x_1 \ x_2 \ x_3 \ x_4]^T, \quad (11)$$

$$\mathbf{x}_{(2)} = [x_1 \ x_2 \ x_3 \ x_5]^T, \quad (12)$$

$$\mathbf{x}_{(3)} = [x_1 \ x_2 \ x_4 \ x_5]^T, \quad (13)$$

$$\mathbf{x}_{(4)} = [x_1 \ x_3 \ x_4 \ x_5]^T, \quad (14)$$

$$\mathbf{x}_{(5)} = [x_2 \ x_3 \ x_4 \ x_5]^T. \quad (15)$$

The vectors of parameters of unfaulty circuit components are as follows $\mathbf{x}_{(1)nom} = [x_5]$, $\mathbf{x}_{(2)nom} = [x_4]$, $\mathbf{x}_{(3)nom} = [x_3]$, $\mathbf{x}_{(4)nom} = [x_2]$, $\mathbf{x}_{(5)nom} = [x_1]$. The vectors of deviations from the nominal values corresponding to the vectors (11)–(15) are given as

$$\Delta\mathbf{x}_{(1)} = [\Delta x_1 \ \Delta x_2 \ \Delta x_3 \ \Delta x_4]^T, \quad (16)$$

$$\Delta\mathbf{x}_{(2)} = [\Delta x_1 \ \Delta x_2 \ \Delta x_3 \ \Delta x_5]^T, \quad (17)$$

$$\Delta\mathbf{x}_{(3)} = [\Delta x_1 \ \Delta x_2 \ \Delta x_4 \ \Delta x_5]^T, \quad (18)$$

$$\Delta\mathbf{x}_{(4)} = [\Delta x_1 \ \Delta x_3 \ \Delta x_4 \ \Delta x_5]^T, \quad (19)$$

$$\Delta\mathbf{x}_{(5)} = [\Delta x_2 \ \Delta x_3 \ \Delta x_4 \ \Delta x_5]^T. \quad (20)$$

If the Jacobian matrix of each of the equations (6)–(10) is non-singular, it is possible to diagnose three faulty circuit elements by verifying the results of solutions — of the form (16)–(20) — of the subsequent equations (6)–(10). As a verification criterion — in the case of circuit diagnosis without taking into account the tolerances of the parameters of the circuit components — it is assumed that deviations

from the nominal values of the parameters of the unfaulty circuit components in the solutions to the subsequent reduced diagnostic equations are sufficiently close to zero. Assuming that the circuit components with parameters x_1, x_2, x_3 are faulty, confirmation of this fact based on the verification criterion is obtained by checking that parameter Δx_4 in vector $\Delta \mathbf{x}_{(1)}$ and parameter Δx_5 in vector $\Delta \mathbf{x}_{(2)}$ have values sufficiently close to zero. Let us assume, in turn, that the reduced diagnostic equation for vector $\mathbf{x}_{(1)}$ has a singular Jacobi matrix. In this case, the set of parameters that are elements of the vector $\Delta \mathbf{x}_{(1)}$ constitutes the ambiguity group. The solution of the reduced diagnostic equation cannot be determined, and in the case of deviations from the nominal values of any three parameters from the parameters forming the four-element ambiguity group $\{x_1, x_2, x_3, x_4\}$, it is not possible to determine which parameters actually deviate from the nominal values using the verification criterion, because the parameter x_5 will be sufficient close to zero in each of the remaining vectors $\Delta \mathbf{x}_{(2)}, \Delta \mathbf{x}_{(3)}, \Delta \mathbf{x}_{(4)}, \Delta \mathbf{x}_{(5)}$. For the diagnostic process to yield a unique solution, a further reduction in the dimension of the vector of the parameters of the potentially faulty circuit components should be performed. The ten reduced diagnostic equations for the ten reduced vectors of the parameters of potentially faulty circuit components with dimension three should be considered in this case. This allows to uniquely diagnose double faults in the circuit if all ten reduced diagnostic equations had non-singular Jacobi matrices. If they did not, one would have to settle for diagnosing single faults, of course, if all reduced Jacobi matrices have rank of 2. In the case of linear circuits, such a situation occurs very often, which is why diagnostic methods of linear circuits often allow for the diagnosis of only single faults and therefore the concept of a vector of parameters of potentially faulty circuit components was introduced. When the tolerances of the parameters of the circuit component are taken into account, the verification criterion — which states that the deviation of a parameter of a unfaulty component must be sufficiently close to zero — changes to a criterion according to which this deviation must fall within the estimated range — see Section IV. It is easy to see that, in the case of circuit diagnosis the ambiguity groups are the obstacles to the diagnostic process. However, they can be used constructively when one wants to achieve a desired design effect in a situation where constraints are imposed on some parameters (on a parameter) of the circuit. If such parameters (such a parameter) constitute(s) an ambiguity group with other parameters with respect to the signal for which they are optimized, the required constraints can be imposed on their values (on its value), which will be compensated by the calculated values of the remaining parameters in that group. This leads to the identification of the ranges in which the parameters of the elements constituting the ambiguity group with the parameter(s) with imposed constraints should be located, which is equivalent to solving the diagnostic problem taking into account the tolerances of the parameters of the circuit components. Let us consider the simplest example of a dynamic circuit. A series RC circuit driven by a DC voltage source u_s . If we were to optimize the selection of resistance R and capacitance C in order to obtain the voltage waveform across the capacitor $u(t)$ after the circuit

is turned on

$$u(t) = u_s - u_s \cdot e^{-\frac{t}{RC}}, \quad (21)$$

which matches the desired waveform, then for any given fixed value of capacitance C , we can select a value of resistance R that ensures 100% waveform matching. If the constraints of $0.9 \cdot C_{nom} < C < 1.1 \cdot C_{nom}$ are imposed on the selection of capacitance, then the resistance would fall within the range $0.9091 \cdot R_{nom} < R < 1.1111 \cdot R_{nom}$, where C_{nom} and R_{nom} are the values of capacitance and resistance that ensure perfect agreement with the specified waveform. This is because the parameters R and C constitute the ambiguity group in this case. If, for a change, for a given constant supply voltage u_s , we optimized the selection of resistance R and capacitance C in order to obtain a match between the current waveform of capacitor $i(t)$

$$i(t) = \frac{u_s}{R} \cdot e^{-\frac{t}{RC}}, \quad (22)$$

with the desired current waveform, there would no longer be such freedom in the selection of parameters, because in the case of the current function $i(t)$, the parameters R and C no longer constitute an ambiguity group. However, we cannot arbitrarily choose the parameters R and C based on equation (21) because the overcurrent at time instant $t = 0$ s may occur according to the equation (22). Therefore, additional constraints must be imposed on the parameters. The knowledge of ambiguity groups provides a certain degree of flexibility in selecting parameters, but this flexibility must be controlled. Taking above into account it is clear that knowledge of the ambiguity groups associated with the considered signal (with the considered set of signals) is of fundamental importance in the process of selecting the parameters of the circuit components. In order to confirm that a given set of parameters is in fact an ambiguity group, not only the singularity of the Jacobi matrix should be confirmed, but also a diagnostic process should be carried out on the reduced vectors containing the parameters constituting the ambiguity group in order to confirm the convergence of the solutions of the associated reduced diagnostic equations.

III. DIAGNOSTIC EQUATIONS AND THEIR SOLUTIONS

In Section II, considerations about ambiguity groups are carried out using the general form of the diagnostic equation, which is independent of the type of system being diagnosed and the domain in which the measurements are performed. Below, the methods of formulating diagnostic equations for linear DC circuits, linear dynamic circuits analyzed in the time domain, and linear dynamic circuits analyzed in the frequency domain, as well as method for solving them are presented [9], [10].

1) *Diagnostic equation of a linear DC circuit:* The description of the linear DC circuit with using the modified node voltage method may be written as

$$\mathbf{0} = \mathbf{M}(\mathbf{x})\mathbf{z} + \mathbf{P}(\mathbf{x})\mathbf{w}, \quad (23)$$

$$\mathbf{y} = \mathbf{N}(\mathbf{x})\mathbf{z} + \mathbf{Q}(\mathbf{x})\mathbf{w}, \quad (24)$$

where: $\mathbf{x} = [(\mathbf{x}')^T (\mathbf{x}''_{nom})^T]^T$ — the vector of the parameters of the circuit components, $\mathbf{x}'$ — the vector of the parameters of the potentially faulty circuit components, $\mathbf{x}''_{nom}$ — the

vector of the parameters of the unfaulty circuit components, $\mathbf{z} = [z_1 \dots z_m]^T$ – the vector of the node voltages, and the currents of the elements which are not accepted by the standard node voltage method, $\mathbf{w} = [w_1 \dots w_p]^T$ – the vector of the input signals, $\mathbf{y} = [y_1 \dots y_n]^T$ – the vector of the output signals. The circuit matrices $\mathbf{M}(\mathbf{x})$, $\mathbf{P}(\mathbf{x})$, $\mathbf{N}(\mathbf{x})$, $\mathbf{Q}(\mathbf{x})$ have the dimensions respectively $m \times m$, $m \times p$, $n \times m$, $n \times p$. The diagnosis process of locating and identifying single and multiple parametric faults is based on an appropriate measurement test. During the test, the circuit is excited with input signals, which are the elements of the vector $\mathbf{w}$, and the output signals, which are the elements of the vector $\mathbf{y}$, are measured. q of different excitation vectors $\mathbf{w}^{\hat{q}}$ will be considered, where $\hat{q} = 1, \dots, q$, and q of the corresponding output signal vectors $\mathbf{y}^{\hat{q}}$ respectively. As a result, the equation can be formulated

$$\mathbf{y}^{\hat{q}} = \mathbf{N}(\mathbf{x})\mathbf{z}^{\hat{q}} + \mathbf{Q}(\mathbf{x})\mathbf{w}^{\hat{q}}. \quad (25)$$

This equation can be written in a general form

$$\mathbf{u} = \mathbf{f}(\mathbf{x}', \mathbf{x}''_{nom}), \quad (26)$$

where:

$$\mathbf{u} = [(\mathbf{y}^1)^T \dots (\mathbf{y}^q)^T]^T, \quad (27)$$

$$\mathbf{f}(\mathbf{x}', \mathbf{x}''_{nom}) = \begin{bmatrix} \mathbf{N}(\mathbf{x})\mathbf{z}^1 + \mathbf{Q}(\mathbf{x})\mathbf{w}^1 \\ \vdots \\ \mathbf{N}(\mathbf{x})\mathbf{z}^q + \mathbf{Q}(\mathbf{x})\mathbf{w}^q \end{bmatrix}. \quad (28)$$

2) *Diagnostic equation of a linear dynamic circuit tested in the time domain:* The diagnostic process of linear dynamic circuits can be performed in the time domain. A modified node voltage method is used to describe the circuit, which allows us to write the equations

$$\mathbf{K} \frac{d\mathbf{z}(t)}{dt} = \mathbf{M}(\mathbf{x})\mathbf{z}(t) + \mathbf{P}(\mathbf{x})\mathbf{w}(t), \quad (29)$$

$$\mathbf{y}(t) = \mathbf{g}(\mathbf{x}, \mathbf{z}(t), \mathbf{w}(t)), \quad (30)$$

where: $\mathbf{x} = [(\mathbf{x}')^T (\mathbf{x}''_{nom})^T]^T$ – the vector of the parameters of the circuit components, $\mathbf{x}'$ – the vector of the parameters of the potentially faulty circuit components, $\mathbf{x}''_{nom}$ – the vector of the parameters of the unfaulty circuit components, $\mathbf{z}(t) = [z_1(t) \dots z_m(t)]^T$ – the vector of the node voltages, and the currents of the elements which are not accepted by the standard node voltage method and also the capacitor currents, $\mathbf{w}(t) = [w_1(t) \dots w_p(t)]^T$ – the vector of input signals, $\mathbf{y}(t) = [y_1(t) \dots y_n(t)]^T$ – the vector of the output signals. The circuit matrices $\mathbf{K}$, $\mathbf{M}(\mathbf{x})$, $\mathbf{P}(\mathbf{x})$ have the dimensions respectively $m \times m$, $m \times m$, $m \times p$. $\mathbf{g}(\mathbf{x}, \mathbf{z}(t), \mathbf{w}(t)) = [g_1(\mathbf{x}, \mathbf{z}(t), \mathbf{w}(t)) \dots g_n(\mathbf{x}, \mathbf{z}(t), \mathbf{w}(t))]^T$. This equation is considered at time instants $t_{\hat{\lambda}}$, where $\hat{\lambda} = 1, \dots, \lambda$. The circuit being diagnosed can be driven by one or more sources, and the output signals are measured at the above specified time instants. As a result, a system of $n\lambda$ diagnostic equations is obtained in the standard form

$$\mathbf{u} = \mathbf{f}(\mathbf{x}', \mathbf{x}''_{nom}). \quad (31)$$

In the case when a measurement of the one output signal is performed at $\lambda = 10$ time instants we obtain

$$\mathbf{u} = [y(t_1) \dots y(t_{10})]^T \quad (32)$$

$$\mathbf{f}(\mathbf{x}', \mathbf{x}''_{nom}) = [y(t_1, \mathbf{x}', \mathbf{x}''_{nom}) \dots y(t_{10}, \mathbf{x}', \mathbf{x}''_{nom})]^T \quad (33)$$

3) *Diagnostic equation of a linear dynamic circuit tested in the frequency domain:* Linear dynamic circuits driven by sinusoidal signals can be diagnosed in steady state. In such a case, the symbolic method (phasor concept) is used and the diagnostic process is performed in the frequency domain. The modified node voltage method is used to describe the circuit, which allows us to write the equations

$$\mathbf{0} = \mathbf{M}(\omega, \mathbf{x})\mathbf{Z}(\omega) + \mathbf{P}(\omega, \mathbf{x})\mathbf{W}(\omega), \quad (34)$$

$$\mathbf{Y}(\omega) = \mathbf{N}(\omega, \mathbf{x})\mathbf{Z}(\omega) + \mathbf{Q}(\omega, \mathbf{x})\mathbf{W}(\omega), \quad (35)$$

where: ω – the angular frequency, $\mathbf{x} = [(\mathbf{x}')^T (\mathbf{x}''_{nom})^T]^T$ – the vector of the parameters of the circuit components, $\mathbf{x}'$ – the vector of the parameters of the potentially faulty circuit components, $\mathbf{x}''$ – the vector of the parameters of the unfaulty circuit components, $\mathbf{Z}(\omega) = [Z_1(\omega) \dots Z_m(\omega)]^T$ – the vector of the node voltages, and the currents of the elements which are not accepted by the standard node voltage method, $\mathbf{W}(\omega) = [W_1(\omega) \dots W_p(\omega)]^T$ – the vector of the input signals, $\mathbf{Y}(\omega) = [Y_1(\omega) \dots Y_n(\omega)]^T$ – the vector of the output signals. The circuit matrices $\mathbf{M}(\omega, \mathbf{x})$, $\mathbf{P}(\omega, \mathbf{x})$, $\mathbf{N}(\omega, \mathbf{x})$, $\mathbf{Q}(\omega, \mathbf{x})$ have the dimensions respectively $m \times m$, $m \times p$, $n \times m$, $n \times p$. In order to obtain more information about the circuit, responses to excitations with different angular frequencies $\omega_{\hat{\rho}}$ are considered

$$\mathbf{Y}(\omega_{\hat{\rho}}) = \mathbf{N}(\omega_{\hat{\rho}}, \mathbf{x})\mathbf{Z}(\omega_{\hat{\rho}}) + \mathbf{Q}(\omega_{\hat{\rho}}, \mathbf{x})\mathbf{W}(\omega_{\hat{\rho}}), \quad (36)$$

where $\hat{\rho} = 1, \dots, \rho$, which leads to ρ output equations, where $\mathbf{Y}(\omega_{\hat{\rho}})$ is the vector created from the symbolic values (phasors) of the measured output signals at the angular frequencies $\omega = \omega_{\hat{\rho}}$. Each of the above equations is then decomposed into two equations

$$\begin{bmatrix} \text{Re}\{\mathbf{Y}(\omega_{\hat{\rho}})\} \\ \text{Im}\{\mathbf{Y}(\omega_{\hat{\rho}})\} \end{bmatrix} = \begin{bmatrix} \text{Re}\{\mathbf{N}(\omega_{\hat{\rho}}, \mathbf{x})\mathbf{Z}(\omega_{\hat{\rho}}) + \mathbf{Q}(\omega_{\hat{\rho}}, \mathbf{x})\mathbf{W}(\omega_{\hat{\rho}})\} \\ \text{Im}\{\mathbf{N}(\omega_{\hat{\rho}}, \mathbf{x})\mathbf{Z}(\omega_{\hat{\rho}}) + \mathbf{Q}(\omega_{\hat{\rho}}, \mathbf{x})\mathbf{W}(\omega_{\hat{\rho}})\} \end{bmatrix}. \quad (37)$$

This system of equations can be written concisely in the form of a diagnostic equation

$$\mathbf{u} = \mathbf{f}(\mathbf{x}', \mathbf{x}''_{nom}), \quad (38)$$

where:

$$\mathbf{u} = [\text{Re}\{(\mathbf{Y}(\omega_1))^T\} \dots \text{Re}\{(\mathbf{Y}(\omega_{\rho}))^T\} \text{Im}\{(\mathbf{Y}(\omega_1))^T\} \dots \text{Im}\{(\mathbf{Y}(\omega_{\rho}))^T\}]^T, \quad (39)$$

$$\mathbf{f}(\mathbf{x}', \mathbf{x}''_{nom}) = \begin{bmatrix} \text{Re}\{\mathbf{N}(\omega_1, \mathbf{x})\mathbf{Z}(\omega_1) + \mathbf{Q}(\omega_1, \mathbf{x})\mathbf{W}(\omega_1)\} \\ \vdots \\ \text{Re}\{\mathbf{N}(\omega_{\rho}, \mathbf{x})\mathbf{Z}(\omega_{\rho}) + \mathbf{Q}(\omega_{\rho}, \mathbf{x})\mathbf{W}(\omega_{\rho})\} \\ \text{Im}\{\mathbf{N}(\omega_1, \mathbf{x})\mathbf{Z}(\omega_1) + \mathbf{Q}(\omega_1, \mathbf{x})\mathbf{W}(\omega_1)\} \\ \vdots \\ \text{Im}\{\mathbf{N}(\omega_{\rho}, \mathbf{x})\mathbf{Z}(\omega_{\rho}) + \mathbf{Q}(\omega_{\rho}, \mathbf{x})\mathbf{W}(\omega_{\rho})\} \end{bmatrix}. \quad (40)$$

4) *Solving the diagnostic equation:* The equations (26, 31, 38) can be written in reduced form

$$\mathbf{u} = \mathbf{f}(\mathbf{x}'_{(\hat{j})}, \mathbf{x}''_{(\hat{j})nom}), \quad (41)$$

for $\hat{j} = 1, \dots, j$. In order to solve the equation (41) with respect to $\mathbf{x}'_{(\hat{j})}$ with given $\mathbf{x}''_{(\hat{j})nom}$, its right-hand side will

be expanded into a Taylor series in the neighborhood of the point $\mathbf{x}_{(j)nom}$. Omitting higher-order terms, one can create an iterative formula in the form

$$\mathbf{u} - \mathbf{f}(\mathbf{x}_{(j)}^{(i-1)}, \mathbf{x}_{(j)nom}'') = \frac{\partial \mathbf{f}(\mathbf{x}_{(j)}', \mathbf{x}_{(j)nom}'')}{\partial \mathbf{x}_{(j)}'} \bigg|_{\mathbf{x}_{(j)}^{(i-1)}} (\mathbf{x}_{(j)}^{(i)} - \mathbf{x}_{(j)}^{(i-1)}). \quad (42)$$

The above equation can be rewritten in an equivalent form

$$\mathbf{u} - \mathbf{f}(\mathbf{x}_{(j)}^{(i-1)}, \mathbf{x}_{(j)nom}'') = \mathbf{J}(\mathbf{x}_{(j)}^{(i-1)}, \mathbf{x}_{(j)nom}'')(\mathbf{x}_{(j)}^{(i)} - \mathbf{x}_{(j)}^{(i-1)}), \quad (43)$$

$$\text{where: } \mathbf{J}(\mathbf{x}_{(j)}^{(i-1)}, \mathbf{x}_{(j)nom}'') = \frac{\partial \mathbf{f}(\mathbf{x}_{(j)}', \mathbf{x}_{(j)nom}'')}{\partial \mathbf{x}'} \bigg|_{\mathbf{x}_{(j)}^{(i-1)}},$$

By introducing additional markings

$$\Delta \mathbf{u}_{(j)}^{(i-1)} = \mathbf{u} - \mathbf{f}(\mathbf{x}_{(j)}^{(i-1)}, \mathbf{x}_{(j)nom}''), \quad (44)$$

$$\mathbf{J}_{(j)}^{(i-1)} = \mathbf{J}(\mathbf{x}_{(j)}^{(i-1)}, \mathbf{x}_{(j)nom}''), \quad (45)$$

$$\Delta \mathbf{x}_{(j)}^{(i)} = \mathbf{x}_{(j)}^{(i)} - \mathbf{x}_{(j)}^{(i-1)}, \quad (46)$$

the equation (42) takes the form

$$\mathbf{J}_{(j)}^{(i-1)} \Delta \mathbf{x}_{(j)}^{(i)} = \Delta \mathbf{u}_{(j)}^{(i-1)}. \quad (47)$$

and is called the iterated diagnostic equation. The iterated diagnostic equation is solved in each iteration by the normal equation method

$$[\mathbf{J}_{(j)}^{(i-1)}]^T \mathbf{J}_{(j)}^{(i-1)} \Delta \mathbf{x}_{(j)}^{(i)} = [\mathbf{J}_{(j)}^{(i-1)}]^T \Delta \mathbf{u}_{(j)}^{(i-1)} \quad (48)$$

each time updating the values of the elements of the vector $\mathbf{x}_{(j)}'$ taking into account the obtained values of the elements of the vector $\Delta \mathbf{x}_{(j)}^{(i)}$. The total deviations from the nominal values of the elements of the vector $\mathbf{x}_{(j)}'$ (determined after the iterative process is completed for $i = i$) are calculated using the formula

$$\Delta \mathbf{x}_{(j)}' = \sum_{i=1}^i \Delta \mathbf{x}_{(j)}^{(i)}. \quad (49)$$

Based on our knowledge of $\Delta \mathbf{x}_{(j)}'$ we determine $\mathbf{x}_{(j)}'$

$$\mathbf{x}_{(j)}' = \mathbf{x}_{(j)nom}' + \Delta \mathbf{x}_{(j)}'. \quad (50)$$

In the theoretical considerations in [10], absolute measurement accuracies of $1e-6$ were assumed. However, as it is shown by the practical example of diagnosing a real circuit in the frequency domain [10] (see Section V), it is possible to diagnose real systems with lower measurement accuracies. However, it should be emphasized, that the required measurement accuracy is significantly influenced by the sensitivity of the measured signal to the increments of the parameters considered in the diagnostic equation being solved. This is due to the fact that the columns of the Jacobian matrix represent these sensitivities for the parameter values associated with a given iteration. If the measurement accuracy is too low, it may prevent the diagnostic process from being carried out or distorts its results. This situation does not occur when the mathematical apparatus of the diagnostic procedure is used to select parameters of the circuit components in accordance with the adopted design assumptions. That is because in this case the reference signal

(a measured signal of the diagnosis method) against which the selection is performed is the waveform generated with using the simulation software with the maximum possible accuracy.

IV. TAKING INTO ACCOUNT PARAMETER TOLERANCES OF CIRCUIT COMPONENTS

The iterated diagnostic equation (47) corresponding to the diagnostic equation (41) for the given j has for the last iteration i the form

$$\mathbf{J}_{(j)}^{(i-1)} \Delta \mathbf{x}_{(j)}^{(i)} = \Delta \mathbf{u}_{(j)}^{(i-1)}. \quad (51)$$

where $\Delta \mathbf{x}_{(j)}^{(i)} \approx \mathbf{0}$ or $\Delta \mathbf{u}_{(j)}^{(i-1)} \approx \mathbf{0}$. Let us write equation (41) in a more general form

$$\mathbf{u} = \mathbf{f}(\mathbf{x}_{(j)}', \mathbf{x}_{(j)}''), \quad (52)$$

and expand it in the Taylor series in the neighbourhood of a point $(\mathbf{x}_{(j)}^{(i)}, \mathbf{x}_{(j)nom}'')$, omitting terms of order higher than the first, obtaining formula

$$\begin{aligned} \mathbf{u} - \mathbf{f}(\mathbf{x}_{(j)}^{(i)}, \mathbf{x}_{(j)nom}'') &= \\ &= \frac{\partial \mathbf{f}}{\partial \mathbf{x}_{(j)}'} \bigg|_{(\mathbf{x}_{(j)}^{(i)}, \mathbf{x}_{(j)nom}'')} \delta \mathbf{x}_{(j)}' + \frac{\partial \mathbf{f}}{\partial \mathbf{x}_{(j)}''} \bigg|_{(\mathbf{x}_{(j)}^{(i)}, \mathbf{x}_{(j)nom}'')} \delta \mathbf{x}_{(j)nom}'', \end{aligned} \quad (53)$$

which enables us to write the estimation

$$|\delta \mathbf{x}_{(j)}'| \leq \mathbf{L} |\delta \mathbf{x}_{(j)nom}''|, \quad (54)$$

where the matrix $\mathbf{L}$ is the matrix with non-negative elements and where: $\delta \mathbf{x}_{(j)nom}''$ is a vector directly related to the tolerances of the parameters contained in the vector $\mathbf{x}_{(j)nom}''$, $\delta \mathbf{x}_{(j)}'$ is the sought vector that allows estimating the influence of the tolerances of the parameters of the nonfaulty circuit components on the values of the elements of the vector $\mathbf{x}_{(j)}'$. Performing the above steps for all the solutions to the reduced diagnostic equations (52) and using appropriate verification procedure described in [10] allows us to estimate the ranges within which the deviations from the nominal values of the parameters of the faulty circuit components should fall [9], [10]. The verification criterion which enables us to find out the set of faulty components is also built with using equation (54) which leads to the estimation of the range in which the parameter of the unfaulty component should fall

$$0 \leq |\Delta \bar{x}_{\theta}| \leq \delta x_{\theta nom}' + (\delta x_{\theta}')_{\max}, \quad (55)$$

where $\delta x_{\theta nom}'$ is the nominal value of the parameter of the unfaulty component in each of the vectors $\mathbf{x}_{(j)}'$ containing the parameters of the sought faulty components, $(\delta x_{\theta}')_{\max}$ is the vector calculated based on formula (54) assuming the equality sign between the left hand side and the right hand side of it.

V. DIAGNOSIS OF THE REAL CIRCUIT IN THE FREQUENCY DOMAIN

The verification of the described method was carried out in real conditions. The circuit with the schematic presented in Figure 1 was diagnosed [9], [10]. The model of the operational amplifier is presented in Figure 2. The photograph of the measurement station is presented in Figure 3.

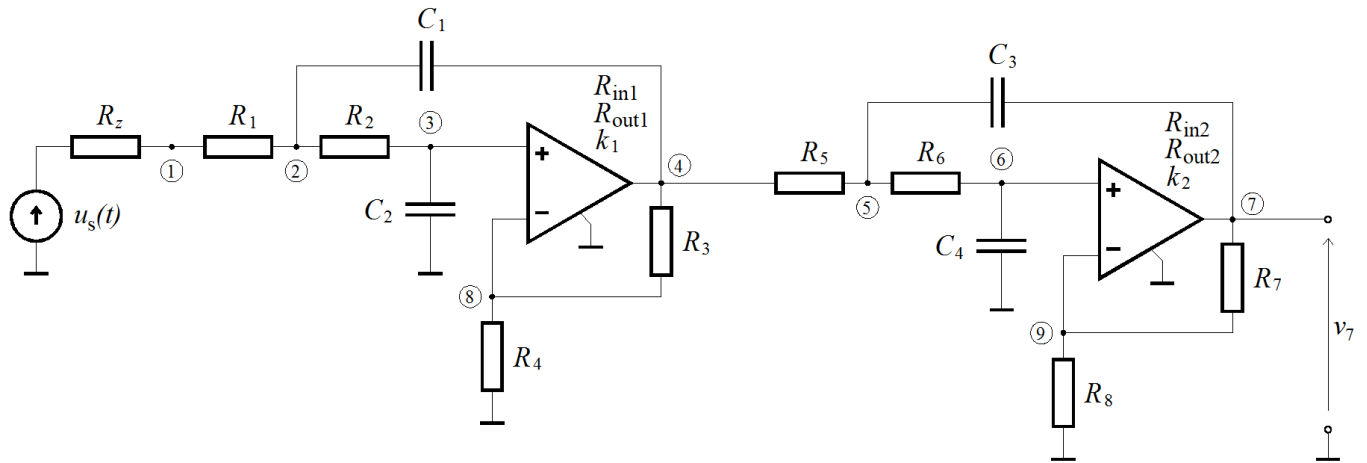


Fig. 1. Diagnosed dynamic circuit.

TABLE I
THE MEASUREMENT RESULTS OF THE DIAGNOSED CIRCUIT

	f	[Hz]	20	40	60	80	100	120	140	160	180
	ω	[rad/s]	125,66	251,33	376,99	502,65	628,31	753,98	879,65	1005,3	1131,0
Not faulty	$ U $	[V]	1,147	1,206	1,293	1,402	1,482	1,401	1,097	0,750	0,496
	ϕ	[°]	-17,94	-37,44	-57,60	-78,91	-108,00	-142,70	-179,93	-212,00	-231,30
$R_1 = 5,671 \text{ k}\Omega$	$ U $	[V]	1,143	1,197	1,280	1,392	1,491	1,445	1,171	0,837	0,580
	ϕ	[°]	-14,11	-31,68	-49,39	-69,70	-96,12	-128,90	-163,30	-190,08	-211,50
$R_2 = 12,170 \text{ k}\Omega$	$ U $	[V]	1,144	1,196	1,265	1,346	1,389	1,278	0,975	0,654	0,426
	ϕ	[°]	-17,14	-37,73	-58,00	-82,94	-113,40	-149,60	-185,98	-216,00	-236,39
$R_5 = 4,644 \text{ k}\Omega$	$ U $	[V]	1,143	1,189	1,253	1,330	1,392	1,370	1,183	0,883	0,610
	ϕ	[°]	-17,28	-24,85	-54,14	-74,59	-99,72	-133,20	-164,30	-195,26	-220,84
$R_6 = 7,071 \text{ k}\Omega$	$ U $	[V]	1,152	1,226	1,336	1,461	1,479	1,228	0,851	0,555	0,364
	ϕ	[°]	-17,86	-38,45	-58,03	-86,98	-123,84	-196,99	-194,54	-222,91	-239,00
$C_1 = 140,30 \text{ nF}$	$ U $	[V]	1,160	1,260	1,416	1,619	1,777	1,685	1,273	0,822	0,509
	ϕ	[°]	-15,55	-33,41	-55,87	-82,94	-111,60	-152,60	-195,05	-226,94	-252,07
$C_2 = 71,82 \text{ nF}$	$ U $	[V]	1,156	1,241	1,379	1,577	1,786	1,827	1,545	1,130	0,783
	ϕ	[°]	-11,52	-26,50	-43,34	-66,24	-90,00	-126,29	-162,29	-193,31	-218,51
$C_3 = 330,70 \text{ nF}$	$ U $	[V]	1,139	1,167	1,196	1,217	1,208	1,141	0,995	0,795	0,594
	ϕ	[°]	-15,84	-35,14	-55,00	-76,03	-101,52	-128,00	-155,20	-181,79	-206,06
$C_4 = 57,00 \text{ nF}$	$ U $	[V]	1,141	1,183	1,239	1,318	1,426	1,575	1,761	1,842	1,499
	ϕ	[°]	-12,38	-27,94	-42,48	-58,17	-78,12	-96,91	-121,97	-158,52	-200,00

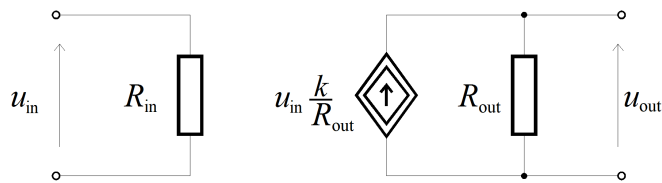


Fig. 2. Operational amplifier model

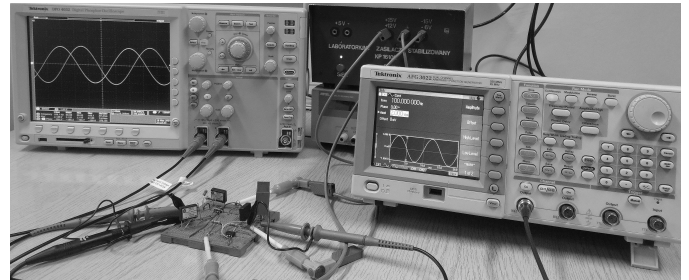


Fig. 3. Measurement station

Nominal parameters of the circuit elements: $R_z^n = 50\Omega$, $R_1^n = 10 \text{ k}\Omega$, $R_2^n = 10 \text{ k}\Omega$, $R_3^n = 4,7 \text{ k}\Omega$, $R_4^n = 10 \text{ k}\Omega$,

TABLE II
DIAGNOSIS RESULTS OF THE CIRCUIT UNDER CONSIDERATION

Simulated fault			Diagnosed fault	
No.	Faulty element	Parameter value	Diagnosed elements	Estimated range of the element value
1	R_1	5,671 k Ω	R_1	4,442 k Ω — 5,705 k Ω
2	R_2	12,170 k Ω	R_2	9,960 k Ω — 13,117 k Ω
			or C_2	100,592 nF — 114,719 nF
3	R_5	4,644 k Ω	R_5	4,171 k Ω — 4,990 k Ω
4	R_6	7,071 k Ω	R_6	6,472 k Ω — 7,695 k Ω
5	C_1	140,30 nF	C_1	127,753 nF — 152,002 nF
6	C_2	71,82 nF	C_2	64,994 nF — 77,913 nF
7	C_3	330,70 nF	C_3	310,584 nF — 358,103 nF
8	C_4	57,00 nF	C_4	53,318 nF — 60,492 nF

$R_5^n = 5,6 \text{ k}\Omega$, $R_6^n = 5,6 \text{ k}\Omega$, $R_7^n = 1 \text{ k}\Omega$, $R_8^n = 10 \text{ k}\Omega$, $C_1^n = 100 \text{ nF}$, $C_2^n = 100 \text{ nF}$, $C_3^n = 470 \text{ nF}$, $C_4^n = 100 \text{ nF}$.

Parameters of the operational amplifier equivalent circuit:

$R_{in1}^n = R_{in2}^n = 100 \text{ k}\Omega$, $R_{out1}^n = R_{out2}^n = 100 \text{ k}\Omega$, $k_1^n = k_2^n = 1 \cdot 10^5 \frac{V}{V}$.

Real parameters of the circuit elements (measured with an accurate RLC bridge): $R_z^r = 50 \Omega$, $R_1^r = 10,02 \text{ k}\Omega$, $R_2^r = 10,02 \text{ k}\Omega$, $R_3^r = 4,683 \text{ k}\Omega$, $R_4^r = 10,02 \text{ k}\Omega$, $R_5^r = 5,609 \text{ k}\Omega$, $R_6^r = 5,614 \text{ k}\Omega$, $R_7^r = 1,002 \text{ k}\Omega$, $R_8^r = 10,02 \text{ k}\Omega$, $C_1^r = 98,85 \text{ nF}$, $C_2^r = 99,00 \text{ nF}$, $C_3^r = 471,9 \text{ nF}$, $C_4^r = 99,1 \text{ nF}$.

The vector of the parameters of potentially faulty system components $\mathbf{x}' = [R_1 \ R_2 \ R_5 \ R_6 \ C_1 \ C_2 \ C_3 \ C_4]^T$.

The parameter tolerances of the unfaulty elements are: for resistors - 1%, for capacitors - 2%, for parameters of the components in the equivalent circuit of the operational amplifier - 10%.

The circuit was driven by the sinusoidal voltage source taken from the waveform generator Tektronix AFG3022. The output signal was the voltage of the node seven. Its rms value was measured with using digital voltmeter with accuracy $\pm 0,10\% \text{ rdg} \pm 0,05\% \text{ FS (fullscale)}$ and its phase angle was measured with using a Tektronix DPO4032 oscilloscope.

VI. CONCLUSIONS

This article presents in a descriptive way the mathematical basis of the diagnosis method of the linear analog circuits based on SAT approach. This method enables us to take into account in the diagnosis process the tolerances of the parameters of the circuit components. Potential applications of the mathematical apparatus of the presented diagnosis method and the conclusions drawn from the diagnostic analysis in the process of selection of the parameters of the components of the newly designed circuits are discussed. It seems that applying

the mathematical framework of the verification-type diagnosis methods — such as this one or those based on the similar concepts — to solve inverse problems constitutes a natural extension of the functionality of these methods.

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