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Influence of Drift on Efficiency of Optimal Analog Adaptive Communication Systems with Feedback and Its Compensation

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Abstract—**Results of analysis of drift influence on operation and efficiency of optimal analog adaptive communication systems with feedback and a new approach to drift compensation in these systems are presented in the paper. The proposed approach is based on application of the extended multidimensional adaptive algorithm that estimates simultaneously the value of a transmitted sample and the value of an unknown drift rate. The knowledge of the drift rate enables drift compensation and improves transmission efficiency of systems suffering problems related to drifts.**

Keywords—**communication systems, feedback, drift compensation, optimal adaptive estimation.**

I. Introduction

THE paper presents results of investigations on influence of drift on operation and transmission efficiency of optimal analog adaptive feedback communication systems (AFCS) and a new approach to drift compensation in these systems. The optimal analog AFCS have been recently considered in works by A.A. Platonov [1-3] and refer back to earlier investigations on optimization of transmission in analog AFCS from the 1960s, e.g. [4-7]. The key idea proposed in [1-3], not used earlier, is the so called statistical fitting condition, which determines the permissible values of AFCS parameters guaranteeing that the probability of overmodulation does not exceed a given level. This condition enables accurate formulation and solution of optimization task to find the best parameters of AFCS in case of the limited range of a transmitter. The main particularity distinguishing optimal analog AFCS is lack of digitizing and coding units in the peripheral transmission unit (TU) (see Fig. 1) which are replaced by the adaptive pulse-amplitude (PAM) modulator Σ+M1. This enables formulation and solution of optimization task for AFCS, as well as determines the optimal values of parameters of both parts of AFCS: the transmission unit (TU) and the base station (BS), and their operation ensuring the maximal quality and rate of data transmission.

This paper extends the investigations on the optimal AFCS whose results are presented in [1-3] and shows a new method of optimal drift estimation and compensation. The works [1-3] describe the fundamentals of analytical design of new classes of high-efficient low-energy and low cost AFCS for short-distance communication and data-transmission in such applications as wireless sensors, RFID, etc.

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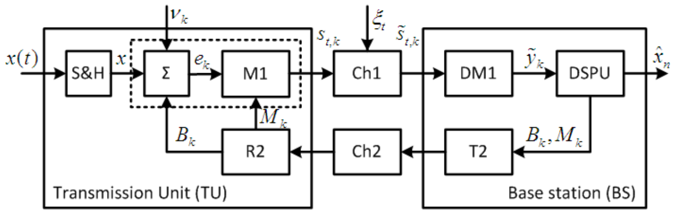

Fig. 1. Block diagram of optimal AFCS.

The method, proposed in this paper, robustifies AFCS on possible drifts occurring in the transmission system, which can cause abnormal errors and degrade dramatically transmission in AFCS. Drift-like errors appear in many electronic circuits and their components. As it is shown below, even small level of drift-errors in some components of AFCS may be of crucial importance for transmission efficiency. An especially sensitive component of AFCS is a sample-and-hold unit (S&H). Its output voltage may drift (droop) during a time needed to transmission of a single sample. The output voltage of S&H is not stable because of a leakage current flowing into or out of the hold capacitor which is connected with imperfections in the hold capacitor, switch or S&H output amplifier [8,9]. The level of drift errors can be reduced by increasing the value of the hold capacitor, but this increases also acquisition time and reduces the bandwidth of S&H [8,9] and, in consequence, AFCS. There are many differential circuit techniques used to reduce the influence of drifts in S&H circuits, but they usually cause an increase in sizes and production costs of transmission units, very often realized as integrated circuits. In this paper, another method of drift compensation based on application of the digital signal processing algorithm that estimates simultaneously the value of a transmitted sample and the value of an unknown drift rate is proposed. The method is based on the original approach to optimal estimation of signal parameters proposed in [10]. A similar method of drift-like errors compensation was used earlier in analog-to-digital converters [11]. The considerations, whose results are presented in the paper, were conducted for the simplest model of the communication channel, i.e. a memoryless stationary channel with Additive White Gaussian Noise (AWGN channel).

II. Optimal AFCS Principles

A. AFCS Operation

A simplified block diagram of optimal AFCS [1-3], which illustrates the principles of its functioning, is presented in

Fig. 1. The system consists of two parts: the peripheral transmission unit (TU) and the base station (BS). The input signal $x(t)$ is sampled in the sample-and-hold unit (S&H) of TU. Each sample $x^{(m)} = x(mT)$ formed by the S&H unit ($m = 1, 2, ...$ is a sample number and $T = 1/2F$ is a sampling interval) is transmitted to BS independently on the previous samples in $n = T/\Delta t_0 = F_0/F$ cycles. $\Delta t_0 = 1/2F_0$ is a duration of a single cycle of transmission, $2F_0$ is a bandwidth of forward (Ch1) and feedback (Ch2) channels. Further the superscript (m) is omitted, because samples $x^{(m)}$ are transmitted independently and analysis of AFCS can be reduced to the consideration of a single sample transmission.

The adaptive modulator Σ+M1 forms an analog signal emitted to BS in particular cycles of transmission. In our consideration, the following model of the double-sideband suppressed-carrier (DSB-SC) adaptive modulator, which takes into account possible overmodulation (saturation of the transmitter) during the transmission, is assumed:

$$s_{t,k} = A_0 \begin{Bmatrix} M_k(x_k - B_k) & \text{if } M_k|x_k - B_k| \le 1 \\ \text{sign}(x_k - B_k) & \text{if } M_k|x_k - B_k| > 1 \end{Bmatrix} \cos(2\pi f_0 t + \phi_k), \quad (1)$$

where $s_{t,k}$ is the signal emitted by TU in the k-th ($k = 1, 2, ..., n$) transmission cycle ($(k-1)\Delta t_0 < t \le k\Delta t_0$). A_0, f_0, ϕ_k are parameters of the carrier. B_k and M_k are the control signals of the adaptive modulator PAM (B_k – compensation, M_k – gain) in the k-th cycle of the sample transmission, B_k and M_k are computed in the digital signal processing unit (DSPU) in BS and delivered to TU through the feedback channel (Ch2). Signal $x_k = x + \nu_k$ is a sum of the sample value x and the analog noise ν_k assumed to be Additive White Gaussian Noise (AWGN) with the variance σ_ν^2. The noise ν_k is related to analog noises occurring in TU. For a non-ideal feedback channel, errors caused by its noise can be included into this noise as an additional component.

BS demodulates the received signal $\tilde{s}_{t,k} = (\gamma/r)s_{t,k} + \xi_t$ in the demodulation unit (DM1) and forms the signal:

$$\tilde{y}_k = A \begin{Bmatrix} M_k(x_k - B_k) & \text{if} & M_k|x - B_k| \le 1 \\ \text{sign}(x_k - B_k) & \text{if} & M_k|x - \hat{B}_k| > 1 \end{Bmatrix} + \xi_k, \quad (2)$$

where $A = A_0\gamma/r$, γ is the gain in the channel Ch1, r is the distance between TU and BS, ξ_k is assumed to be AWGN with the variance $\sigma_\xi^2 = N_\xi F_0$. The demodulated signal $\tilde{y}_k$ is routed to the input of DSPU, which computes iteratively the estimate $\hat{x}_k$ of the input sample x according to the following relationship:

$$\hat{x}_k = \hat{x}_{k-1} + L_k\tilde{y}_k, \quad (3)$$

where L_k is the coefficient which determines the convergence rate of estimation algorithm.

DSPU also computes new values of the control signals B_k and M_k that are transmitted to the TU through the feedback channel T2-Ch2-R2 and the next $(k+1)$-th cycle of transmission of the sample begins. Both forward and feedback channels Ch1, Ch2 are assumed to be memoryless.

B. Optimal AFCS

Optimization of AFCS [1-3] consists in finding the optimal values of the parameters B_k and M_k (for every cycle k) of the adaptive modulator and corresponding values of the coefficient L_k in (3), which minimize the mean square error (MSE) of the current estimates of the sample:

$$P_k = E[(x - \hat{x}_k)^2]. \quad (4)$$

Simultaneously, B_k and M_k should satisfy so called statistical fitting condition [10]:

$$\Pr_k^{over} = \Pr(M_k|x_k - B_k| > 1 \mid \tilde{y}_1^k, B_1^{k-1}, M_1^{k-1}) < \mu. \quad (5)$$

It means that, for each cycle of transmission, the parameters B_k and M_k of the modulator should have the values guaranteeing that the probability of overmodulation does not exceed a given small value μ. Parameter μ determines a probability of abnormal errors and total loss of information about the sample and should be considered as additional characteristic of AFCS performance. Typically, the value of μ (specified by designers) is from the interval $10^{-12} \le \mu \le 10^{-4}$ and depends on the requirements on a transmission system.

Minimization of the MSE (4) under the condition (5) and assumption that transmitted samples have a Gaussian distribution, with the known a priori mean value x_0 and variance σ_0^2, gives the optimal values of parameters for adjusting the transmission unit [1-3]:

$$B_k = \hat{x}_{k-1}, \qquad M_k = \frac{1}{\alpha\sqrt{\sigma_\nu^2 + P_{k-1}}}, \quad (6)$$

and optimal values of coefficients L_k used in the base station:

$$L_k = \frac{AM_kP_{k-1}}{\sigma_\xi^2 + A^2M_k^2(\sigma_\nu^2 + P_{k-1})}. \quad (7)$$

MSE P_k in (6) and (7) are calculated according to the following relationship:

$$P_k = P_{k-1} - \frac{A^2M_k^2P_{k-1}^2}{\sigma_\xi^2 + A^2M_k^2(\sigma_\nu^2 + P_{k-1})}. \quad (8)$$

Saturation factor α in (6) is determined by the permissible probability μ of overloading. In the Gaussian case, it satisfies the equation $\Phi(a) = (1 - \mu)/2$, where $\Phi(a)$ is the tabulated Gaussian error function. Initial conditions for the whole algorithm of the sample transmission in the optimal AFCS are as follows: $\hat{x}_0 = x_0$, $P_0 = \sigma_0^2$.

C. Efficiency of Optimal AFCS

The algorithm (1)-(3), (6)-(8) determines the work of the optimal statistically fitted AFCS which can transmit signals with minimal MSE (4). In [1-3] it was shown that the bit rate of data transmission R in the forward channel of the optimal AFCS attains the channel capacity C during the definite "threshold" number $n^\star$ of the initial cycles of sample transmission. The mean bit rate at AFCS output is determined by the following relationship [1-3]:

$$R_n = \frac{I(x;\hat{x}_n)}{n\Delta t_0} = \frac{F_0}{n}\log_2\left(\frac{\sigma_0^2}{P_n}\right) \quad \text{[bit/s]}, \quad (9)$$

where $I(x;\hat{x}_n) = H(x) - H(x|\hat{x}_n)$ is the amount of information in the estimate $\hat{x}_n$ about the input sample x, and $H(x)$, $H(x|\hat{x}_n)$ are the prior and posterior entropies.

For $1 \leq n \leq n^{\star}$, the bit rate is constant and determined by equation:

$$R_n = F_0 \log_2\left(1 + \frac{W^{sign}}{W^{noise}}\right) = F_0 \log_2\left(1 + \frac{W^{sign}}{N_\xi F_0}\right) = C, \tag{10}$$

where $W^{sign} = (A/\alpha)^2$ is the mean power of the information component of the received signal and $W^{noise} = N_\xi F_0 = \sigma_\xi^2$ is the power of the noise at the input of BS. For $n > n^{\star}$ the bit-rate diminishes monotonically. The most important result of research on the optimal AFCS [1-3], being a consequence of (10), is the fact that the power-bandwidth efficiency of transmission in the forward channel of the optimal AFCS, for $1 \leq n \leq n^{\star}$, attains the Shannon's capacity boundary [12,13] (compare Fig. 4) which expresses the ideal power-bandwidth trade-off in communication systems:

$$\frac{E^{bit}}{N_\xi} = \frac{F_0}{C}\left(2^{C/F_0} - 1\right), \tag{11}$$

where E^{bit} is the energy per bit of the signal received by BS.

The threshold point $n^{\star}$ determines the optimal number of cycles of sample transmission and can be assessed as a solution of the equation $P_{n^{\star}} = \sigma_\nu^2$:

$$n^{\star} = \frac{\log_2\left(\sigma_0^2/\sigma_\nu^2\right)}{\log_2\left(1 + W^{sign}/\sigma_\xi^2\right)}. \tag{12}$$

It is worth noticing that for $1 \leq n \leq n^{\star}$, MSE (4) decreases very fast (exponentially) with n according to the relationship:

$$P_n = \sigma_0^2\left(1 + W^{sign}/\sigma_\xi^2\right)^{-n}. \tag{13}$$

After the threshold point $n^{\star}$, MSE diminishes more slowly (hyperbolically).

III. Drift Compensation in Optimal AFCS

Now, let us consider the following model of the signal $\bar{x}_k$ including a drift component (to distinguish the drift case we use the upper dash over x_k, in Sect. II $x_k = x + \nu_k$):

$$\bar{x}_k = x + \beta(k-1) + \nu_k, \tag{14}$$

where x is the value of the transmitted sample and β is the unknown rate (amplitude) of a drift component, which has the known form, e.g. a linear drift: $k-1$, k is the cycle number ($k = 1, 2, , n$). The distorted value of the input sample at the k-th cycle of transmission is a sum of the actual value of the sample x and the drift component $\beta(k-1)$. As in Sect. II, ν_k is the analog additive noise, assumed to be zero-mean AWGN with the variance σ_ν^2. The sample (14) can be presented in the form of the regression type model:

$$\bar{x}_k = \boldsymbol{\theta}^T \boldsymbol{\varphi}_k + \nu_k, \tag{15}$$

where $\boldsymbol{\theta} = [x, \beta]^T$ is a vector of two unknown parameters to be estimated and a vector $\boldsymbol{\varphi}_k = [1, k-1]^T$ consists of two known deterministic components.

The proposed method of drift compensation consists in joint (simultaneous) optimal estimation of two unknown parameters to form the optimal controls B_k and M_k of the adaptive modulator (Σ+M1) guaranteeing the minimal MSE of the current estimates of the sample x and the drift rate β under the condition (5) related to overmodulation. This optimization problem, which is equivalent to minimization of the squared error: $S_k = E[(\bar{x}_k - \hat{\bar{x}}_k)^2] = \boldsymbol{\varphi}_k^T \boldsymbol{P}_k \boldsymbol{\varphi}_k$, where $\boldsymbol{P}_k = E[(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_k)(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_k)^T]$, can be solved using the analytical approach to optimization of adaptive estimation algorithms presented in [10]. According to the multidimensional version of the algorithm of optimal parameters estimation [10], the optimal value of the compensation signal B_k in the k-th cycle of sample transmission is calculated on the basis of optimal estimates $\hat{\boldsymbol{\theta}}_{k-1} = [\hat{x}_{k-1}, \hat{\beta}_{k-1}]^T$ obtained in the previous cycle:

$$B_k = \hat{\bar{x}}_{k,k-1} = \hat{\boldsymbol{\theta}}_{k-1}^T \boldsymbol{\varphi}_k = \hat{x}_{k-1} + \hat{\beta}_{k-1}(k-1). \tag{16}$$

The optimal gain of the adaptive modulator in the k-th cycle is calculated using the formula:

$$M_k = \frac{1}{\alpha\sqrt{\sigma_\nu^2 + \boldsymbol{\varphi}_k^T \boldsymbol{P}_{k-1} \boldsymbol{\varphi}_k}}, \tag{17}$$

where $\boldsymbol{P}_k$ is the correlation matrix calculated recursively as follows:

$$\boldsymbol{P}_k = \boldsymbol{P}_{k-1} - \frac{A^2 M_k^2 \boldsymbol{P}_{k-1} \boldsymbol{\varphi}_k \boldsymbol{\varphi}_k^T \boldsymbol{P}_{k-1}}{\sigma_\xi^2 + A^2 M_k^2 (\sigma_\nu^2 + \boldsymbol{\varphi}_k^T \boldsymbol{P}_{k-1} \boldsymbol{\varphi}_k)}. \tag{18}$$

The recursive equation for updating the vector of optimal estimates has the following form:

$$\hat{\boldsymbol{\theta}}_k = \hat{\boldsymbol{\theta}}_{k-1} + \boldsymbol{L}_k \tilde{y}_k, \tag{19}$$

where $\boldsymbol{L}_k = [L_k^{(1)}, L_k^{(2)}]^T$ is the vector of coefficients calculated according to the formula:

$$\boldsymbol{L}_k = \frac{A M_k \boldsymbol{P}_{k-1} \boldsymbol{\varphi}_k}{\sigma_\xi^2 + A^2 M_k^2 (\sigma_\nu^2 + \boldsymbol{\varphi}_k^T \boldsymbol{P}_{k-1} \boldsymbol{\varphi}_k)}. \tag{20}$$

Initial conditions for the extended two-dimensional algorithm (16)-(20) are: $\hat{\boldsymbol{\theta}}_0 = [x_0, \beta_0]^T$, $\boldsymbol{P}_0 = diag(\sigma_0^2, \sigma_\beta^2)$, where β_0 and σ_β^2 are the assumed a priori mean value and variance of the drift rate β.

IV. Results of Simulation Experiments

Theoretical analysis of drifts influence on efficiency of AFCS is very complicated and limited. Therefore, the simulation experiments based on mathematical models of the standard (Sect. II) and modified (Sect. III) AFCS were used in our research to analyse the drifts influence on AFCS operation as well as to verify usefulness of the proposed method. In the first group of experiments (Sect. IV.A), the basic properties of the optimal AFCS without drifts were investigated. These results serve as a reference point for the experiments related to behaviour of AFCS in case of drifts occurrence (Sect. IV.B).

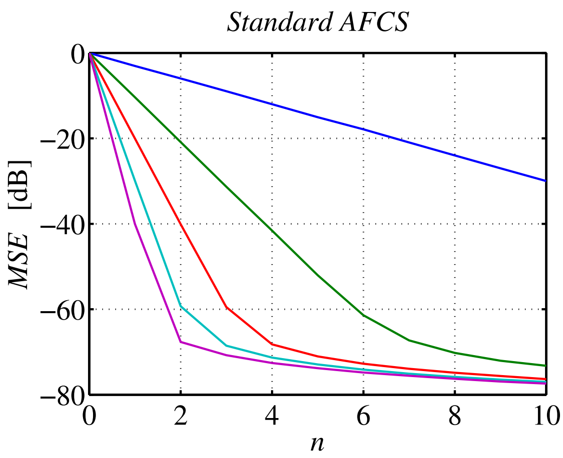

Fig. 2. MSE for optimal AFCS obtained for different values of SNR = $0, 10, 20, 30, 40$ dB (lines from top to bottom, respectively).

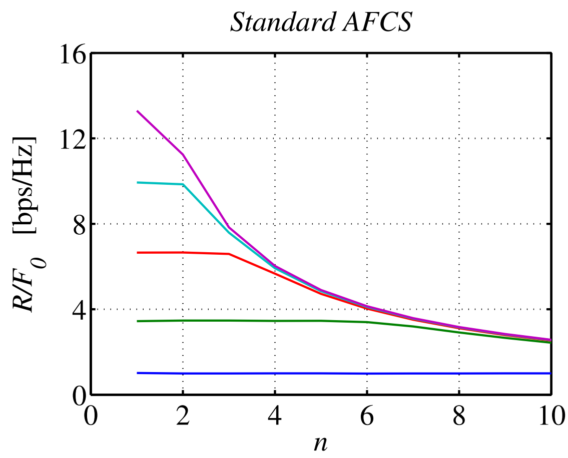

Fig. 3. Transmission bit-rates for optimal AFCS obtained for different values of SNR = $0, 10, 20, 30, 40$ dB (lines from bottom to top, respectively).

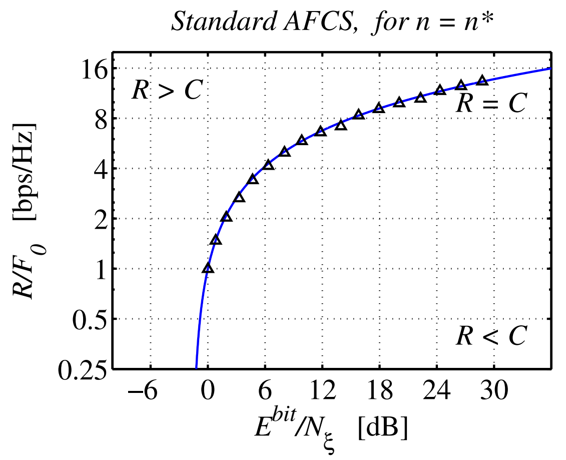

Fig. 4. Experimental power-bandwidth efficiency for optimal AFCS, triangles relate to SNR from 0 to 40 dB (from bottom to top, respectively).

A. *Optimal AFCS without Drifts*

In simulation experiments, input samples transmitted by AFCS $x^{(m)}(m = 1, 2, ..., M, M = 10000)$ were generated as sequences of random normally distributed values with the mean value $x_0 = 0$ V and the variance $\sigma_0^2 = 0.25$ V^2. The other values of parameters assumed in simulation experiments were as follows: $A = 10$ mV, $\alpha = 4$, $\sigma_\nu^2 = 10^{-8}$ V^2. Figure 2 shows the empirical values of the mean square errors (MSE) obtained for the optimal standard AFCS for different values of a number of cycles $n = 1, 2, ..., 10$ and different values of signal to noise ratio (SNR) at the input of BS defined as $W^{sign}/W^{noise} = (A/\alpha)^2/\sigma_\xi^2$ changing from 0 to 40 dB, SNR = $0, 10, 20, 30, 40$ dB, lines in Fig. 2 from top to bot-tom, respectively. The empirical values of MSE were calculated using the following formula:

$$MSE_n = \frac{1}{M}\sum_{m=1}^{M}\left[x^{(m)} - \hat{x}_n^{(m)}\right]^2. \tag{21}$$

The plots in Fig. 2 show how the level of noises influences on MSE of transmission. Using these results, we can easily determine experimentally the threshold points $n^\star$ and compare with their theoretical assessment (12) for particular values of SNR.

The next Fig. 3 presents the results of simulation experiments regarding the bit rate at which data are transmitted by AFCS under the assumption of AWGN channel, i.e. the trajectories of the bit rate (9) normalized by F_0 obtained for particular values of $n = 1, 2, ..., 10$ and for SNR = $0, 10, 20, 30, 40$ dB (lines in Fig. 3 from bottom to top, respectively). The values of $\hat{R}_n/F_0$ were calculated according to the following formula:

$$\frac{\hat{R}_n}{F_0} = \frac{1}{n}\log_2\left(\frac{\sigma_0^2}{MSE_n}\right) \quad \text{[bps/Hz]}. \tag{22}$$

The last figure in this series of experiments (Fig. 4) shows the results of simulation assessment of power-bandwidth efficiency of optimal AFCS in the form of the so called *bandwidth-efficiency diagram* [12], i.e. as points (corresponding to system parameters) located in the plane E^{bit}/N_ξ , $\hat{R}_n/F_0$, where E^{bit}/N_ξ is the signal energy per bit to noise power spectral density ratio, calculated as follows:

$$\frac{E^{bit}}{N_\xi} = 10\log_{10}\frac{W^{sign}/W^{noise}}{\frac{1}{n}\log_2\left(\frac{\sigma_0^2}{MSE_n}\right)} \quad \text{[dB]}. \tag{23}$$

The continuous blue line in Fig. 4 relates to the theoretical Shannon's capacity boundary [12,13] for which $R = C$, where C is the channel capacity. This line corresponds to the ideal systems with maximal efficiency theoretically achievable. The capacity boundary divides the plane into the part $R \le C$ achievable for real systems with error-free transmission, and the part $R > C$ unachievable for real systems with error-free transmission [12]. Locations of triangles in Fig. 4 relate to the empirical values of E^{bit}/N_ξ and $\hat{R}_n/F_0$ obtained for the optimal AFCS, calculated for $n = n^\star$ and for SNR changing from 0 to 40 dB with the step 2.5 dB. The simulation results confirm that the power-bandwidth efficiency of the optimal AFCS attains the Shannon's capacity boundary.

B. *AFCS with Drifts*

In the next experiment, it was investigated how the level of drifts affects AFCS performance and efficiency. Fig-

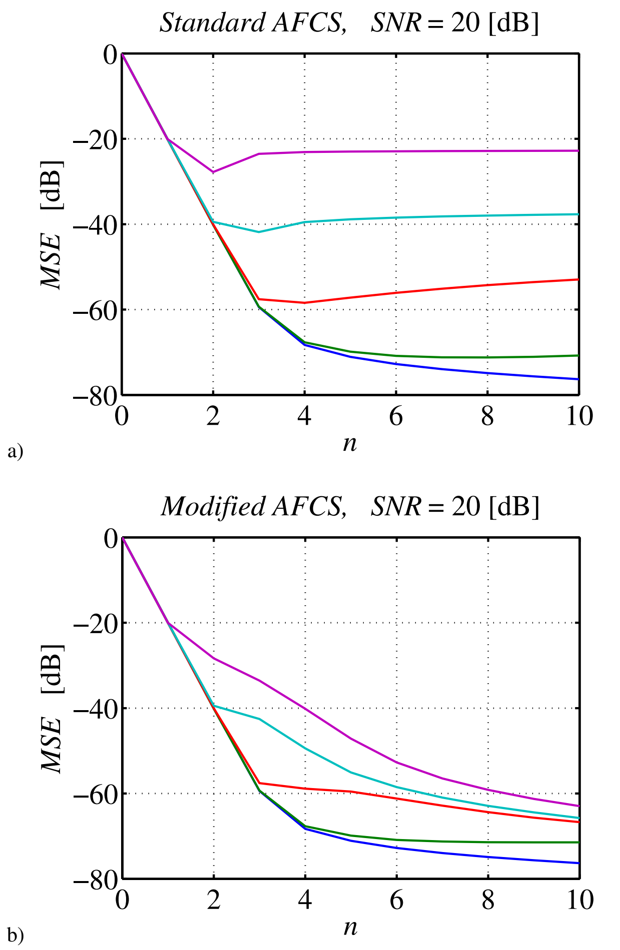

Fig. 5. MSE for standard (a) and modified (b) AFCS, for different values of drift rate $\sigma_\beta = 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}$ V (lines from bottom to top, respectively) under SNR = 20 dB.

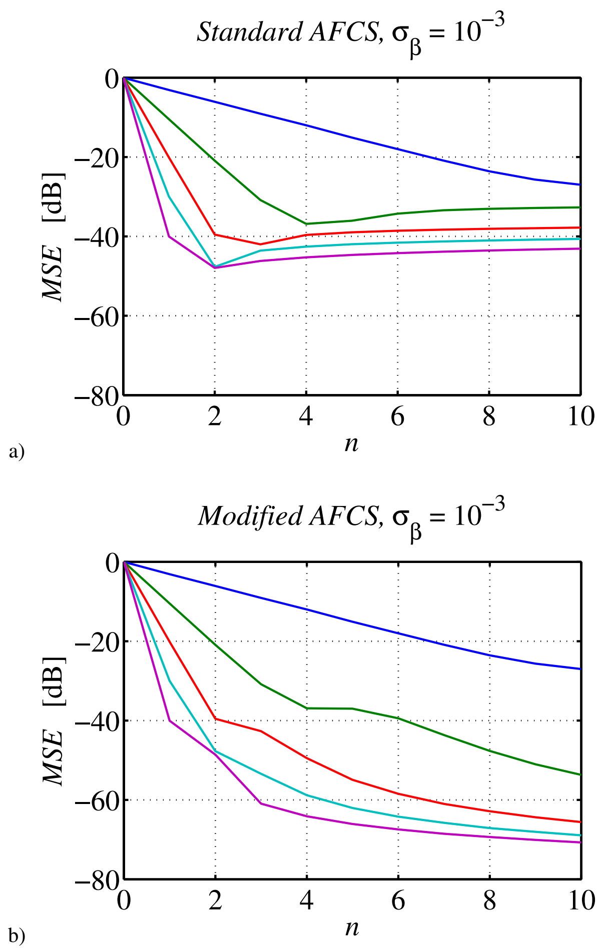

Fig. 6. MSE for standard (a) and modified (b) AFCS, for different values of SNR = $0, 10, 20, 30, 40$ dB (lines from top to bottom, respectively) and standard deviation of drift rate $\sigma_\beta = 10^{-3}$ V.

ure 5 presents the behaviour of MSE obtained for the standard (Fig. 5a) and modified (Fig. 5b) AFCS for different values of the standard deviation of the drift rate $\sigma_\beta = 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}$ V (lines from bottom to top, respectively). For every input sample $x^{(m)}$ concerned in simulation experiments, the value of the drift rate was generated randomly with the Gaussian distribution $N(0, \sigma_\beta^2)$. In this experiment, SNR $= W^{sign}/W^{noise} = 20$ dB was assumed.

The results obtained for the standard AFCS (Fig. 5a) show that the values of MSE increase proportionally to the drift rate, while the results for the modified AFCS (Fig. 5b) indicate that the two-dimensional algorithm improves the transmission quality in the case of drift occurrence in comparison with the use of one-dimensional algorithm as in the standard AFCS. Of course, we observe some worsening of the transmission quality, measured by MSE, for the modified AFCS in presence of drift compared to the situation when there is no drift (compare a plot for SNR = 20 dB in Fig. 2).

The next series of simulation experiments is devoted to analysis of the behaviour of the standard and modified AFCS for different values of SNR and the constant standard deviation of the drift rate $\sigma_\beta = 10^{-3}$ or $\sigma_\beta = 10^{-2}$ V. The experimental values of MSE for $\sigma_\beta = 10^{-3}$ V for both versions of AFCS are presented in Fig. 6. Figure 6a refers to the standard AFCS and Fig. 6b to the modified AFCS. The results presented in Fig. 6a show that, for this level of drift errors, the level of transmission errors expressed in MSE is very high and the samples transmission is totally degraded for all values of SNR analyzed in the experiment. The results obtained in the same conditions for the modified AFCS (Fig. 6b) indicate that the modified AFCS can work satisfactorily in the case of drift occurrence. As in the previous experiment, there is some worsening of transmission quality in comparison to the case, where there is no drift in AFCS (compare Fig. 2).

The similar results, but obtained for the standard deviation of the drift rate $\sigma_\beta = 10^{-2}$ V are presented in Fig. 7. In this case, the effects observed earlier for $\sigma_\beta = 10^{-3}$ V are even more visible. The difference between MSE obtained for the standard (Fig. 7a) and modified (Fig. 7b) AFCS for particular SNR is greater. For the standard AFCS, the values of MSE do not decrease after $n = 2$, while for the modified AFCS, the values of MSE continue the further decrease after the short break in the second cycle.

The next figures (Figs. 8 and 9) show the transmission bit rates trajectories calculated according to (22) on the basis of empirical values of MSE from Figs. 6 and 7, respectively. The

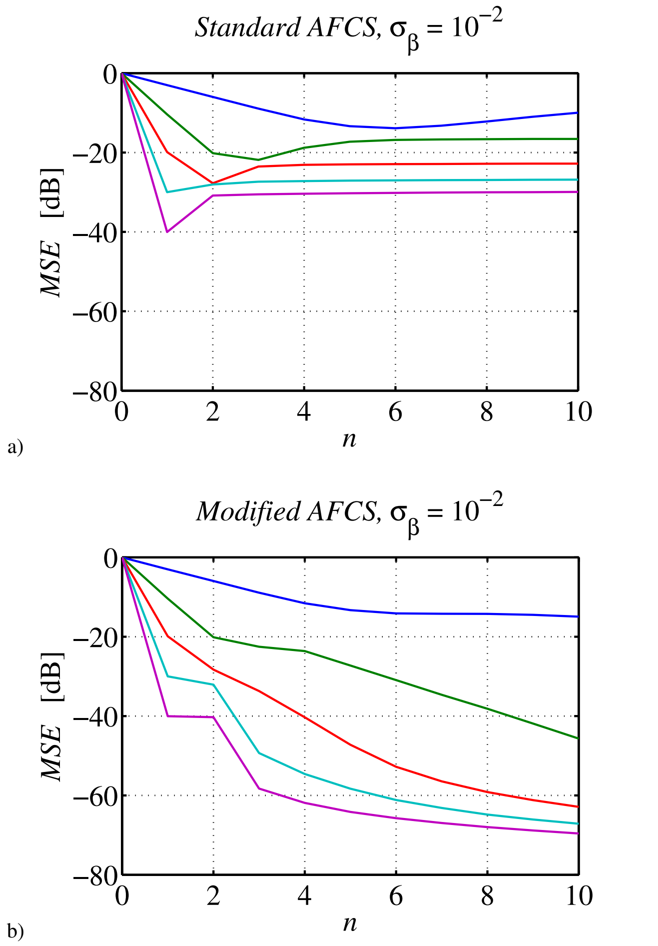

Fig. 7. MSE for standard (a) and modified (b) AFCS, for different values of SNR = $0, 10, 20, 30, 40$ dB (lines from top to bottom, respectively) and standard deviation of drift rate $\sigma_\beta = 10^{-2}$ V.

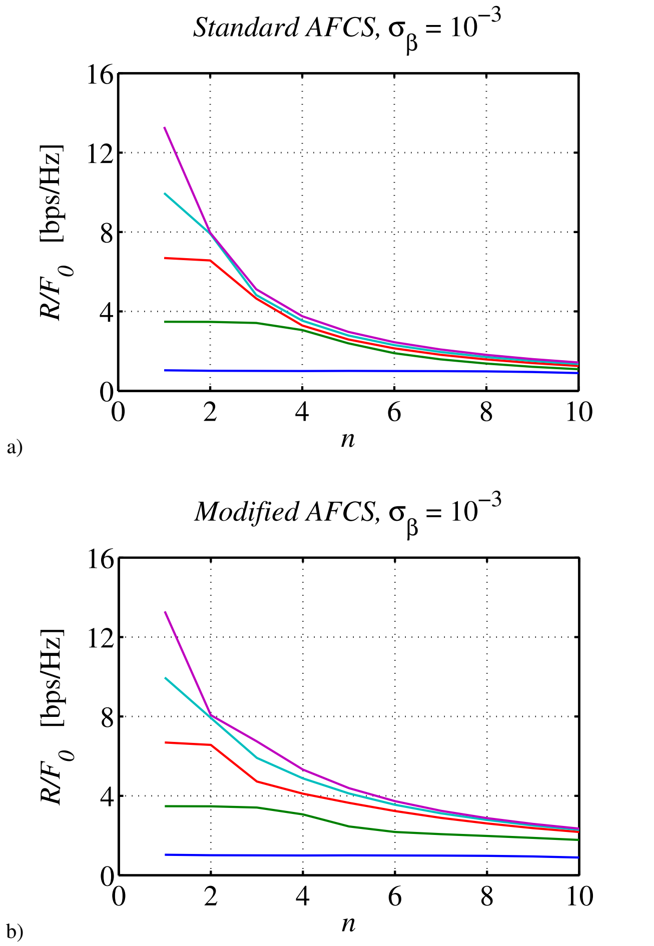

Fig. 8. Transmission bit-rates for standard (a) and modified (b) AFCS, for different values of SNR = $0, 10, 20, 30, 40$ dB (lines from bottom to top, respectively) and $\sigma_\beta = 10^{-3}$ V.

appropriate results obtained for optimal AFCS with no drift are presented in Fig. 3. In comparison to values of the transmission bit rates presented in Fig. 3, the bit rates obtained in the case of drift occurrence are significantly smaller for particular n, especially for $\sigma_\beta = 10^{-2}$ V. It means that the transmission time of the given information has to be longer in the case of the standard AFCS. Application of the modified versions of AFCS causes that the values of transmission bit rates $\hat{R}$ are greater and closer to the values obtained in the situation, when there is no drift (Fig. 3).

As a summary of simulation experiments, the results of experiments referring to worsening of the power-bandwidth efficiency of AFSC caused by drift are presented in Fig. 10. Figure 10 shows the empirical values of parameters E^{bit}/N_ξ (23) and $\hat{R}_n/F_0$ (22) (as coordinates of points) obtained for the standard and modified AFCS. Triangles in Fig. 10 relate to the empirical values of E^{bit}/N_ξ and $\hat{R}_n/F_0$ obtained for the standard optimal AFCS working without drift and calculated for $n = n^\star$ for different values of SNR changing from 0 to 40 dB with the step 2.5 dB (from bottom to top, respectively). Red stars and blue diamonds relate to the empirical values of E^{bit}/N_ξ and $\hat{R}_n/F_0$ obtained for the standard and modified AFCS, respectively, operating with drifts (Figure 10a for the standard deviations of the drift rate: $\sigma_\beta = 10^{-3}$ V and Fig. 10b for $\sigma_\beta = 10^{-2}$ V). The values of E^{bit}/N_ξ and $\hat{R}_n/F_0$ were calculated for the same $n = n^\star$ as in the case of the optimal standard AFCS operating without drift. The experiments were performed for relatively high levels of drift in order to show clearly differences in the E^{bit}/N_ξ, $\hat{R}_n/F_0$ plane.

Generally, the communication system is more efficient if the point E^{bit}/N_ξ, $\hat{R}_n/F_0$ corresponding to the system is closer to the Shannon's capacity boundary [12] determined by the equation $R = C$. The location of points obtained for the standard and modified AFCS shows that the use of the two-dimensional algorithm in AFCS operating within the conditions of drift occurrence moves the points E^{bit}/N_ξ, $\hat{R}_n/F_0$ closer to the capacity boundary in comparison to the points obtained for the standard AFCS. It proofs the improvement of power-bandwidth efficiency of the modified AFCS in comparison with the standard AFCS working in drift conditions.

V. Conclusion

The degradation of transmission efficiency in optimal analog adaptive communication systems with feedback (AFCS) caused by drift were investigated in the paper. The new

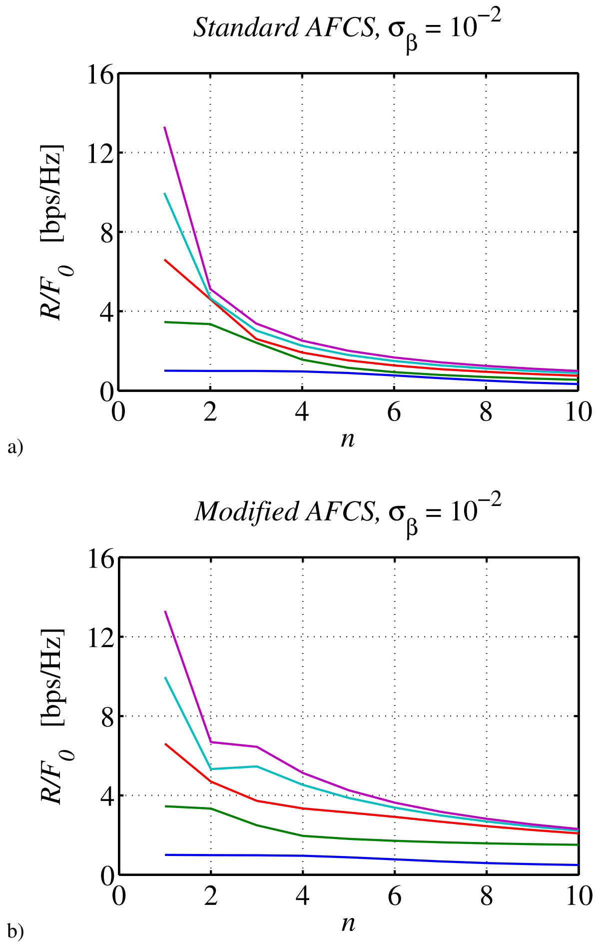

Fig. 9. Transmission bit-rates for standard (a) and modified (b) AFCS, for different values of SNR $= 0, 10, 20, 30, 40$ dB (lines from bottom to top, respectively) and $\sigma_\beta = 10^{-2}$ V.

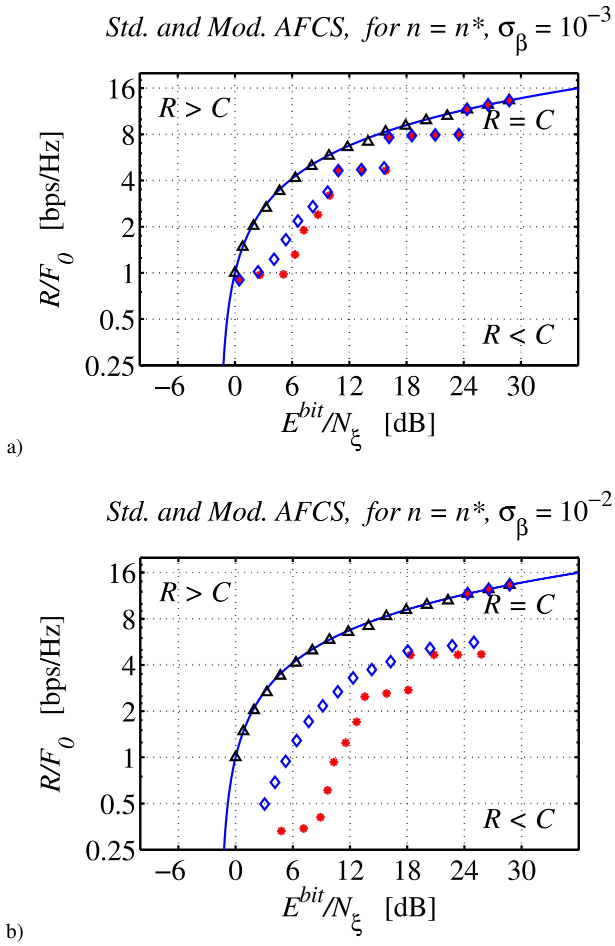

Fig. 10. Experimental power-bandwidth efficiency for standard (red stars) and modified (blue diamonds) AFCS, for $\sigma_\beta = 10^{-3}$ V (a), $\sigma_\beta = 10^{-2}$ V (b), and for SNR changing from 0 to 40 dB (from bottom to top).

approach to drift compensation in these systems, proposed in the paper, can significantly reduce the influence of drifts on transmission efficiency in AFCS. The developed method allows that the modified AFCS may operate satisfactorily even for the high levels of drifts while the standard AFCS does not work properly in the same conditions.

Implementation of the proposed method into AFCS consists only in changes in the algorithm performed by the base station. The base station uses the two-dimensional estimation algorithm which estimates simultaneously the value of a transmitted sample and the value of an unknown drift rate. The use of this algorithm does not cause any change in the architecture of AFCS.

The developed models of the standard and modified AFCS as well as the simulation software enable assessment of influence of the level of drifts, other parameters of AFCS and the transmission channel on the total AFCS efficiency. The results of investigations presented in the paper were obtained under assumption of the Gaussian (AWGN) transmission channel. Application of other channel models can be a subject of the further research, extending the obtained results.

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